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Physics Beyond the Standard Model

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III+IV: The Minimal Supersymmetric Standard Model

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1. Basics of Supersymmetry

Here, we give only basic facts needed later in the phenomenological discussion.

For details on theoretical issues, see basic textbooks like Drees, Godbole, Roy.

SUperSYmmetry: is a symmetry that relates scalars/vector bosons and fermions.

The SUSY generators $\mathcal{Q}$ transform fermions into bosons and vice–versa, namely:

$$\mathcal{Q}|\text{Fermion}> \, |\text{Boson}> \quad , \quad \mathcal{Q}|\text{Boson}> \, |\text{Fermion}>$$

$\mathcal{Q}$ must be an anti–commuting (and thus rather complicated) object.

$\mathcal{Q}^\dagger$ is also a distinct symmetry generator:

$$\mathcal{Q}^\dagger|\text{Fermion}> \, |\text{Boson}> \quad , \quad \mathcal{Q}^\dagger|\text{Boson}> \, |\text{Fermion}>$$

Highly restricted [e.g., no go theorem] theories and in 4-dimension with chiral fermions:

$\mathcal{Q}, \mathcal{Q}^\dagger$ carry spin–$\frac{1}{2}$ with left- and right- helicities and they should obey

.... The SUSY algebra: which schematically is given by

$$\{\mathcal{Q}, \mathcal{Q}^\dagger\} = P^\mu \ , \ \{\mathcal{Q}, \mathcal{Q}\} = 0 \ , \ \{\mathcal{Q}^\dagger, \mathcal{Q}^\dagger\} = 0,$$
$$[P^\mu, \mathcal{Q}] = 0, [P^\mu, \mathcal{Q}^\dagger] = 0, [T^a, \mathcal{Q}] = 0, [T^a, \mathcal{Q}^\dagger] = 0$$

P^μ: is the generator of space–time transformations.

T^a are the generators of internal (gauge) symmetries.

⇒ SUSY: unique extension of the Poincaré group of space–time symmetry to include a four–dimensional Quantum Field Theory...

The single–particle states of the theory are in irreducible representations of the SUSY algebra defined above, and which are called supermultiplets.

The fermions and bosons of the same supermultiplet are called superpartners.

They must have the same mass and the same gauge quantum numbers.

Three types of supermultiplets are needed to describe physics phenomena.

- Chiral (or "scalar") supermultiplet (denoted by ζ with $\zeta^c = \zeta$ and S):
  - 1 two–component Weyl fermion with spin $\pm\frac{1}{2}$ ($n_F = 2$)
  - 2 real spin–zero scalar = 1 complex scalar field ($n_B = 2$)
- Gauge (or "vector") supermultiplet (denoted by A_μ^a and λ_A):
  - 1 two–component Weyl gaugino–fermion with spin $\pm\frac{1}{2}$ ($n_F = 2$)
  - 1 real spin–1 massless gauge vector boson ($n_B = 2$)
- Gravitational supermultiplet:
  - 1 two–component Weyl gravitino–fermion with spin $\pm\frac{3}{2}$ ($n_F = 2$)
  - 1 real spin–2 massless graviton ($n_B = 2$)

Example: $\Psi = \binom{e_L}{e_R}$ with $e_{L/R}$ being 2–component Weyl left- and right fermions

Each spin-$\frac{1}{2}$ state has a complex spin–0 superpartner noted $\tilde{e}_L$ and $\tilde{e}_R$ and called sfermion.

One can write the fermion fields as $e \equiv e_L$ and $\bar{e} = e_R^\dagger$ so that one has:
two left-handed chiral supermultiplets for the electron: $(e, \tilde{e}_L)$, $(\bar{e}, \tilde{e}_R^*)$.
The same for all other leptons and quarks, except for the massless ν_L.

• All the fields involved have the canonical kinetic energies with Lagrangians

$$\mathcal{L}_{\rm kin} = \Sigma_i \{ (D_\mu S_i^*)(D^\mu S_i) + i\overline{\psi}_i D_\mu \gamma^\mu \psi_i \} + \Sigma_a \{ -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{i}{2} \overline{\lambda}_a D \lambda_a \}$$

with D the usual covariant derivative. Note that the fields $\psi(\lambda)$ have 4 (2) components.]

• The interactions are specified by SUperSYmmetry and gauge invariance only:

$$\mathcal{L}_{\rm interactions\ scalar-fermions-gauginos} = -\sqrt{2} \sum_{i,a} g_a [S_i^* T^a \overline{\psi}_{iL} \lambda_a + {\rm h.c.}]$$

$$\mathcal{L}_{\rm interaction\ quartic\,scalar} = -\frac{1}{2} \sum_a (\sum_i g_a S_i^* T^a S_i)^2$$

• All known interactions are determined by the gauge coupling constants g_1, g_2 and g_3 (fundamental prediction of SUSY: the same coupling g in gauge and Yukawa interactions)

• At this stage, all this is a very simple and minimalistic theory/extension of the SM: Everything in the theory is completely specified and there is no adjustable parameter. The only freedom: choice of Superpotential W which should be SUSY and gauge invariant. It gives the scalar potential and the Yukawa interactions (among fermions-scalars).

– $W \equiv$ function of the superfields z_i only (and not of the conjugate fields z_i^*);

– it should be an analytic function: there are no derivative interactions;

– renormalizability: only terms of dimension 2 and 3 are present.

$$\Rightarrow \mathcal{L}_W = -\sum_i \left| \frac{\partial W}{\partial z_i} \right|^2 - \frac{1}{2} \sum_{ij} \left[\overline{\psi}_{iL} \frac{\partial^2 W}{\partial z_i \partial z_j} \psi_j + {\rm h.c.} \right]$$

To obtain all the interactions explicitly: make the evaluation of W at $\partial W / \partial z_i |_{z_i = S_i}$.

The SUSY tree–level scalar potential has two distinct components $V_{\rm tree} = V_F + V_D$.

• The F–terms come in W through derivatives with respect to all scalar fields S_i:

$$V_F = \sum_i F_i F_i^* = \sum_i |W^i|^2 \text{ with } W^i = \partial W / \partial S_i \quad .$$

• The D–terms correspond to contributions of the U(1), SU(2), SU(3) gauge groups:

$$V_D = \frac{1}{2} \sum_i D_i D_i^* = \frac{1}{2} \sum_{a=1}^{3} \left(\sum_i g_a S_i^* T^a S_i \right)^2 \quad .$$

SUSY cannot be an exact symmetry of Nature since no scalars exist with the same mass as the known fermions (belong to same super multiplets) ⇒ SUSY must be broken.

Spontaneous SUSY breaking?

This would mean that the Lagrangian is invariant under SUSY global transformations but the ground state $|0>$ is not invariant: $\mathcal{Q}|0> \neq 0$ and $\mathcal{Q}^\dagger|0> \neq 0$.

However, recall that the Hamiltonian is related to the SUSY charges: $\{\mathcal{Q}, \mathcal{Q}^\dagger\} \sim P^\mu$

So that one has for the vacuum: $<0|H|0> \equiv <0|P^0|0> \propto <0|\mathcal{Q}\mathcal{Q}^\dagger|0> = E_{\rm vac} \neq 0$

In fact, the vacuum energy should always be positive: $E_{\rm vac} > 0$.

• $\langle 0|D|0 \rangle \neq 0$ or D–term breaking: leads to CCB minima ⇒ does not work!

• $\langle 0|F|0 \rangle \neq 0$ or F–term breaking: needs linear $a_i \Phi_i$ term in W ⇒ requires SM singlet!

Solution: SUSY–breaking occurs in a hidden sector of particles which have no (or else very tiny) couplings to the visible sector of the theory.

If the mediating interaction is flavor–blind, the SUSY-breaking terms are universal.

Examples: gravity (mSUGRA), gauge (GMSB) and anomaly (AMSB) mediation...

There are many breaking schemes but none is fully satisfactory at the present moment:

⇒ Explicit breaking by hand (but there are also several possibilities in this case).

• We need SUSY breaking at low energies to solve the known SM problems:

– the quadratic divergences in the Higgs sector.

– the unification of the coupling constants of $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$.

– the Dark Matter problem (the existence of a massive stable particle), etc.

• In the SUSY breaking process, we still need to preserve: gauge invariance, the renormalizability, and no quadratic divergence should appear (soft SUSY–breaking).

⇒ "Low energy SUperSYmmetry" ≡ effective theory at low energy.

As discussed earlier, in SUSY theories, we have that:

– for each SM particle, there is a SUSY partner with has a spin $\frac{1}{2}$ difference;

– SUSY must be broken at a relatively low scale $M_{\rm SUSY} = \mathcal{O}(1 \text{ TeV})$

In order to solve the unification, hierarchy and the dark matter problems of the SM.

The MSSM is the most economic low energy SUSY extension of SM.

2. The minimal Supersymmetric Standard Model

The MSSM is based on the following four basic assumptions:

- Minimal gauge group: $\mathrm{G_{SM}} = \mathrm{SU(3)_C \times SU(2)_L \times U(1)}$.

The SM spin–1 gauge bosons $[B, W_{1-3}$ and $g_{1-8}]$ and their spin–$\frac{1}{2}$ gaugino partners $[\tilde{b}, \tilde{w}_{1-3}, \tilde{g}_{1-8}]$ called binos, winos and gluinos are in vector superfields.

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle content
$\hat{G}^a$	8	1	0	$G^\mu, \tilde{g}$
$\hat{W}^i$	1	3	0	$W_i^\mu, \tilde{\omega}_i$
$\hat{B}$	1	1	0	$B^\mu, \tilde{b}$

- Minimal particle content:

Superfield	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Particle content
$\hat{Q}$	3	2	$\frac{1}{3}$	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{4}{3}$	$\overline{u}_R$, $\tilde{u}_R^*$
$\hat{D}^c$	$\overline{3}$	1	$\frac{2}{3}$	$\overline{d}_R$, $\tilde{d}_R^*$
$\hat{L}$	1	2	-1	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	2	$\overline{e}_R$, $\tilde{e}_R^*$
$\hat{H}_1$	1	2	-1	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	1	$(H_2, \tilde{h}_2)$

– Three fermion generations [but as in the SM no ν_R...] and their spin–0 SUSY partners, the two sfermions $\tilde{f}_L$, $\tilde{f}_R$, combined in chiral supermultiplets.

– No chiral anomalies ($\sum_f Q_f \equiv 0$) and fermion mass generation in a SUSY invariant way (no conjugate H^* field for u–quarks) $\Rightarrow$ two chiral superfields with $Y=+1$ and $Y=-1$.

- R–parity conservation:

To eliminate terms violating B and L quantum numbers (thus leading to proton decay):
Introduce a discrete and multiplicative symmetry called R-parity or R_p defined as

$$R_p = (-1)^{2s+3B+L} \;\Rightarrow\; \begin{array}{l} R = +1 \text{ for all ordinary SMparticles} \\ R = -1 \text{ for all the SUSY particles} \end{array}$$

The phenomenological consequences of R_p conservation are extremely important:

– SUSY particles are always produced in pairs;

– SUSY particles decay into an odd number of SUSY particles;

– the lightest SUSY particle (LSP) is absolutely stable.

At this stage, we have a globally supersymmetric Lagrangian in which:

- everything is specified by SUSY and gauge invariance;
- there is no additional parameter compared to SM;
- the only freedom is the choice of the Superpotential.

The most general Superpotential compatible with supersymmetry, gauge invariance, renormalizability and R–parity conservation is given by:

$$W = \sum_{\rm i,j=gen} Y_{ij}^u \, \hat{u}_R^i \hat{H}_2 . \hat{Q}^j + Y_{ij}^d \, \hat{d}_R^i \hat{H}_1 . \hat{Q}^j + Y_{ij}^l \, \hat{l}_R^i \hat{H}_1 . \hat{L}^j + \mu \hat{H}_1 . \hat{H}_2$$

- $Y_{ij}^{u,d,l}$ denote the Yukawa couplings among the three generations of particles (and which are simply a generalization of the SM Yukawa interactions).
- μ: a supersymmetric Higgs–higgsino parameter with a dimension of mass (it is thus a really supersymmetric parameter, see discussion later....).
- Now, we have to introduce soft SUSY breaking:

To explicitly break SUSY without reintroducing the quadratic divergences (the so–called soft SUSY–breaking), we add by hand a collection of soft terms (of dimension 2 and 3):

$$\mathcal{L}_{\rm gaugino} = \frac{1}{2} \left[M_1 \tilde{b} \tilde{b} + M_2 \Sigma_{a=1}^3 \tilde{w}^a \tilde{w}_a + M_3 \Sigma_{a=1}^8 \tilde{g}^a \tilde{g}_a \ + \ {\rm h.c.} \right]$$

$$\mathcal{L}_{\rm sfermions} = \Sigma_i m_{\tilde{Q},i}^2 \tilde{Q}_i^\dagger \tilde{Q}_i + m_{\tilde{L},i}^2 \tilde{L}_i^\dagger \tilde{L}_i + m_{\tilde{u},i}^2 |\tilde{u}_{R_i}|^2 + m_{\tilde{d},i}^2 |\tilde{d}_{R_i}|^2 + m_{\tilde{l},i}^2 |\tilde{l}_{R_i}|^2$$

$$\mathcal{L}_{\rm Higgs} = m_2^2 H_2^\dagger H_2 + m_1^2 H_1^\dagger H_1 + B\mu (H_2 . H_1 + {\rm h.c.})$$

$$\mathcal{L}_{\rm trilinear} = \Sigma_{i,j} \left[A_{ij}^u Y_{ij}^u \tilde{u}_{R_i} H_2 . \tilde{Q}_j + A_{ij}^d Y_{ij}^d \tilde{d}_{R_i} H_1 . \tilde{Q}_j + A_{ij}^l Y_{ij}^l \tilde{l}_{R_i} H_1 . \tilde{L}_j + {\rm h.c.} \right]$$

This is a rather complicated and problematic potential indeed!

- It has too many parameters and thus it is not very predictive.
- It leads generically to a very problematic phenomenology.

In the most general case (mixing and phases allowed): 105 free parameters!

- complex gaugino masses M_1, M_2, M_3 : 6
- 3×3 hermitian mass matrices $m_{\tilde{F}}$: 45
- 3×3 complex trilinear coupling matrices A_f : 54
- 2×2 matrix for the bilinear B coupling : 4
- Higgs masses squared, $m^2_{H_1}, m^2_{H_2}$: 2

111–6 (due to constraints from symmetries and Higgs sector, see later)= 105.

For "generic" sets of these parameters, it leads to very severe problems:

- large and dangerous/unobserved flavor changing neutral currents [FCNC];
- an unacceptable amount of additional CP–violation;
- color and/or charge breaking minima that break SU(3) and U(1) symmetries;
- it leads to an incorrect value of the Z boson mass (predictable), etc......

We need more constraints on this MSSM!

3. The constrained MSSM's

A phenomenologically viable MSSM can be defined by making following assumptions:

- all soft SUSY–breaking parameters are real (no new source of CP violation);
- the mass and trilinear couplings for sfermions are diagonal (no FCNC);
- there is a 1st/2d sfermion generation universality (no problems with heavy quarks);

We define the phenomenological MSSM (pMSSM) with 22 free parameters:

$\tan\beta$: the ratio of the vevs of the two–Higgs doublet fields.
$m^2_{H_u}, m^2_{H_d}$: the Higgs mass parameters squared.
M_1, M_2, M_3: the bino, wino and gluino mass parameters.
$m_{\tilde{q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, m_{\tilde{l}}, m_{\tilde{e}_R}$: 1st/2d generation sfermion mass para.
$m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}, m_{\tilde{L}}, m_{\tilde{\tau}_R}$: third generation sfermion mass para.
A_t, A_b, A_τ: the third generation trilinear couplings.
A_u, A_d, A_e: the first/second generation trilinear couplings.

- You can trade the masses $m^2_{H_u}, m^2_{H_d}$ with the more "physical" parameters μ and M_A (in fact: μ^2 and $B\mu$ can be determined from requirement of ESWB, as seen later).
- The parameters A_u, A_d, A_e are in general not relevant for phenomenology (they enter only in "light" flavor physics: $(g-2)_\mu$, neutron edm, ...).
- If you focus on a given particle sector (Higgs, gauginos, sfermions): only few parameters to deal with and one can do model independent analyses....

$\Rightarrow$ a phenomenologically more viable model than the general MSSM.

• One can also use common soft–SUSY breaking terms in many concrete cases ($m_{\tilde{q}} = m_{\tilde{u}_R} = m_{\tilde{d}_R}$; $m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}$; A_t, A_b, A_τ; etc..)

and one ends with an even more restrictive set of parameters, possibly $\lesssim 10$.

$\Rightarrow$ much more predictive model than the general MSSM.

But almost all problems of the MSSM can be solved at once if soft SUSY–breaking parameters obey a set of universal boundary conditions at the scale $M_{\rm GUT}$.

Underlying assumption: SUSY–breaking occurs in a hidden sector communicating with the visible sector only through gravitational interactions.

$\Rightarrow$ some universal soft-SUSY breaking terms emerge if interactions are "flavor–blind":

Besides $g_{1,2,3}$ unification which fix the unification scale $M_{\rm GUT} \sim 2 \cdot 10^{16}$ GeV

Unification of gaugino, scalar masses and trilinear couplings at $Q = M_{\rm GUT}$

Universal gaugino masses: $M_1 = M_2 = M_3 \equiv m_{1/2}$

Universal scalar masses: $M_{\tilde{Q}_i} = M_{\tilde{L}_i} = M_{H_i} \equiv m_0$

Universal trilinear couplings: $A^u_{ij} = A^d_{ij} = A^l_{ij} \equiv A_0\, \delta_{ij}$

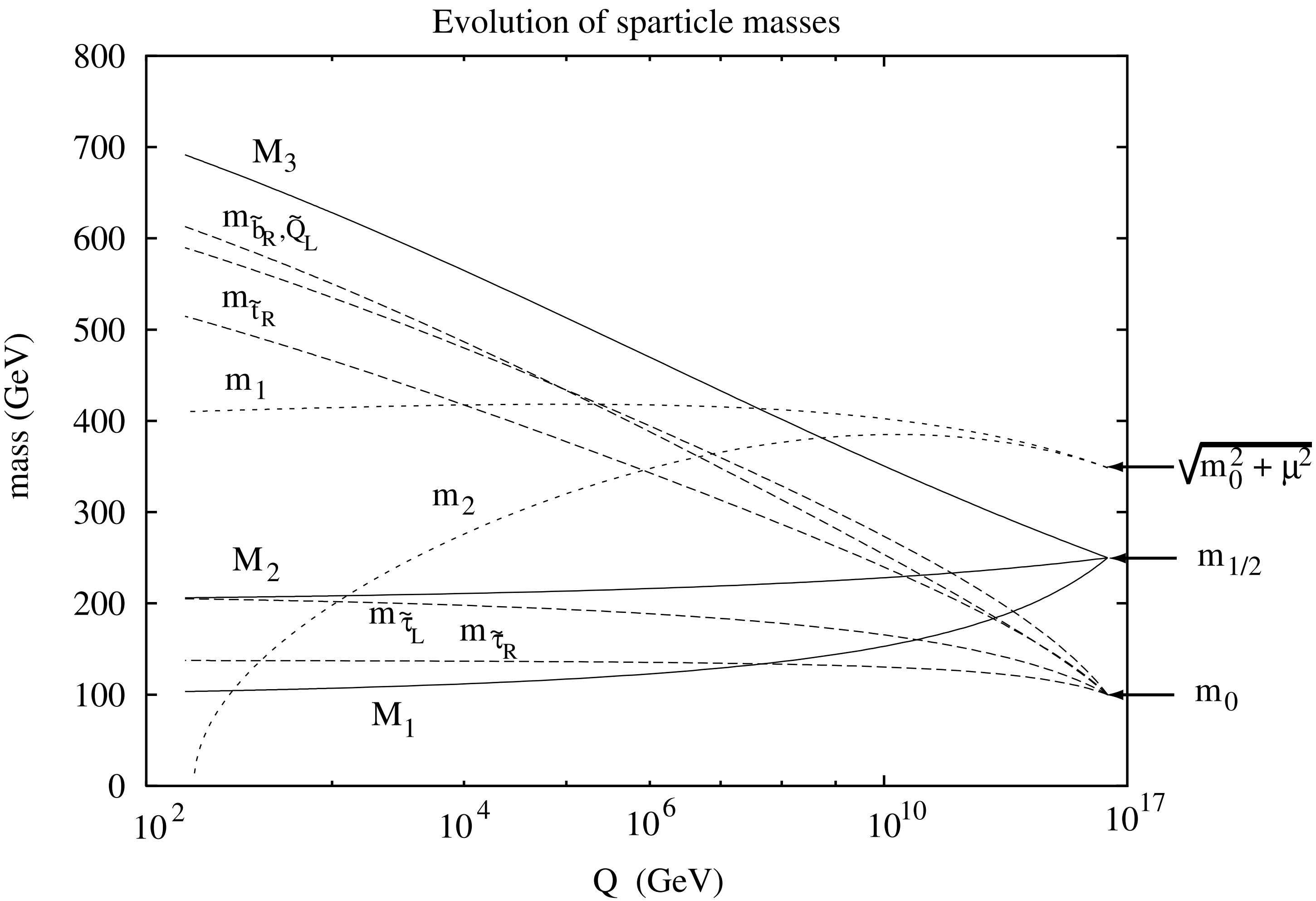
Also: B and μ^2 obtained from requiring EWSB and minimization of $V_{\rm Higgs}$

$$\mu^2 = \tfrac{1}{2}[\tan 2\beta (m^2_{H_u} \tan\beta - m^2_{H_d} \cot\beta) - M_Z^2]$$

$$B\mu = \tfrac{1}{2} \sin 2\beta [m^2_{H_u} + m^2_{H_d} + 2\mu^2]$$

Only 4.5 param: $\tan\beta$, $m_{1/2}$, m_0 , A_0 , sign(μ)

All soft-SUSY breaking parameters at M_S are then obtained through RGEs.
With $M_{\rm GUT} \sim 2 \cdot 10^{16}$ GeV and the SUSY scale defined as $M_{\rm SUSY} \sim \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$:



Radiative EWSB occurs since one gets $M_{H_2}^2 < 0$ at the scale $Q = v$ (from $t/\tilde{t}$ loops)

$\Rightarrow$ EWSB is more natural in the MSSM ($\mu^2 < 0$ from RGEs) than in the SM!

Another possibility for a very constrained MSSM is gauge mediated SUSY breaking, GMSB: soft breaking transmitted to MSSM fields via SM gauge interactions.

• The hidden sector for SUSY–breaking contains messengers fields, $n_{\hat{q}}/n_{\hat{l}}$ quark/lepton-like pairs coupled to a gauge singlet chiral Superfield $\hat{S}$.

• The superpotential is $W = \lambda \hat{S}\hat{q}\hat{\bar{q}} + \lambda \hat{S}\hat{l}\hat{\bar{l}}$ with fields $\hat{S}$ having vevs. s and f_S

• soft SUSY-breaking are generated by (1 or 2) loop corrections at the scale $M_{\rm mes} = \lambda s$

$$M_G(M_{\rm mes}) = \frac{\alpha_G(M_{\rm mes})}{4\pi}\Lambda\, g(\frac{\Lambda}{M_{\rm mes}}) \sum_m N_R^G(m)$$

$$m_s^2(M_{\rm mes}) = 2\Lambda^2 f(\frac{\Lambda}{M_{\rm mes}}) \sum_{m,G} (\frac{\alpha_G(M_{\rm mes})}{4\pi})^2 N_R^G(m) C_R^G(s)$$

$A_f(M_{\rm mes}) \simeq 0$ (generated at the two–loop level).

with $\Lambda = f_s/s, G = {\rm U(1), SU(2), SU(3)}$, m and s label messengers and scalars; f/g are one/two loop functions; N/C are Dynkin indices / Casimirs etc....

Thus, in the GMSB model there are six basic input parameters to start with

$$tan\beta\ ,\ {\rm sign}(\mu)\ ,\ M_{\rm mes}\ ,\ \Lambda\ ,\ n_{\hat{q}}\ ,\ n_{\hat{l}}$$

and all soft-SUSY breaking terms are then obtained from the previous equations.

NB: there is also the mass of the gravitino which is an independent/input parameter (and it plays a rather important role: it is very light, LSP, and make DM).

A third possibility for a very constrained MSSM is anomaly mediated SUSY breaking.

In AMSB, SUSY breaking occurs also in a hidden sector (e.g. in an extra dimension) and is transmitted to the visible sector via, for instance, some super–Weyl anomalies.

The gaugino masses, the scalar masses and the trilinear couplings are simply related to the scale dependence of the gauge and matter kinetic functions.

In terms of the gravitino mass $m_{3/2}$, the β functions for the gauge g_a and Yukawa Y_i couplings and anomalous dimensions γ_i of chiral superfields, soft-SUSY breaking terms are:

$$M_a = \frac{\beta_{g_a}}{g_a} m_{3/2} \ , \ \ A_i = \frac{\beta_{Y_i}}{Y_i} m_{3/2}$$

$$m_i^2 = -\frac{1}{4} (\Sigma_a \frac{\partial \gamma_i}{\partial g_a} \beta_{g_a} + \Sigma_k \frac{\partial \gamma_i}{\partial Y_k} \beta_{Y_k}) m_{3/2}^2$$

RG invariant equations valid at any scale (which makes the model rather predictive).

(the μ^2 and $B\mu$ terms are obtained as usual by requiring radiative EWSB).

However, the picture is spoiled by tachyonic sleptons $m_{\tilde{L}}^2 < 0$ in general!

$\Rightarrow$ add a non anomalous contribution to soft masses $c_i m_0^2$ to m_i^2 to avoid that.

In minimal AMSB with a universal m_0, and coefficients $c_i = 1$, the inputs are:

$$m_0 \ , \ m_{3/2} \ , \ \tan\beta \ , \ \text{sign}(\mu) \text{ and } c_i$$

Makes AMSB also a rather predictive model.

4. The superparticle spectrum

The basic Lagrangian, gives the currents, interactions and the corresponding states: but we need to turn the current eigenstates into the mass (physical) eigenstates.

Terminology for the physical eigenstates in supersymmetric theories:

• The charginos: the mixtures of the charged higgsinos and the gauginos

$$\tilde{W}^{\pm}, \tilde{h}^{\pm}_{2/1} \longrightarrow \chi_1^{\pm}, \chi_2^{\pm}$$

• The neutralinos: the mixtures of the neutral higgsinos and the gauginos

$$\tilde{B}, \tilde{W}^0, \tilde{h}^0_2, \tilde{h}^0_2, \longrightarrow \tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_2, \tilde{\chi}^0_4$$

(for gluinos, there is no need of mixing, but include running and the hence the RC)..

• The sfermions: the mixed left- and right-handed sfermion states of same flavor

$$\tilde{f}_L, \tilde{f}_R \longrightarrow \tilde{f}_1, \tilde{f}_2$$

(mixing proportional to fermion mass: absent for sneutrinos and small for most sfermions).

• The Higgs bosons: two doublets of complex scalar fields $H_1, H_2 \equiv 4$ dofs:

⇒ 3 degrees of freedom go to generate M_{W^+}, M_{W^-}, M_Z as usual;

⇒ 5 degrees of freedom are left in the spectrum: h, H, A, H^+, H^-.

To determine the masses and the mixing angles of all the mass (physical) eigenstates: one needs to find the relevant mass matrices and diagonalize them; better include RC?

The general chargino mass matrix, in terms of M_2, μ and $\tan\beta$, is

$$\mathcal{M}_C = \begin{bmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{bmatrix}, \quad s_\beta \equiv \sin\beta \ etc$$

diagonalized by: $U\mathcal{M}_C V^{-1} \to U = \mathcal{O}_-, V = \begin{cases} \mathcal{O}_+ & \text{if } \det\mathcal{M}_C > 0 \\ \sigma_3\mathcal{O}_+ & \text{if } \det\mathcal{M}_C < 0 \end{cases}$

(Pauli σ_3 matrix to make the $\chi^\pm$ masses positive and $\mathcal{O}_\pm$ are rotation matrices)

Simple analytical formulae for the masses $m^\pm_{\chi 1,2}$ and mixing angles in terms of M_2, μ.

For limiting cases, interpretation much simpler. $\mu \gg M_2, M_W$:

$$m_{\chi_1^\pm} \simeq M_2 - M_W^2\mu^{-2}\left(M_2 + \mu s_{2\beta}\right)$$

$$m_{\chi_2^\pm} \simeq |\mu| + M_W^2\mu^{-2}\epsilon_\mu\left(M_2 s_{2\beta} + \mu\right)$$

IN the limit $|\mu| \to \infty$: $\chi_1^\pm$ wino with $m_{\chi_1^\pm} \simeq M_2$; $\chi_2^\pm$ higgsino with $m_{\chi_2^\pm} = |\mu|$

In the opposite limit, $M_2 \gg |\mu|, M_Z$, the roles of $\chi_1^\pm, \chi_2^\pm$ are simply reversed.

For neutralinos, the 4×4 mass matrix depends on parameters $\mu, M_2, \tan\beta, M_1$.

In the $(-i\tilde{B}, -i\tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0)$ basis, it is given by

$$\mathcal{M}_N = \begin{bmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{bmatrix}$$

and can be diagonalized by a single real matrix Z. Again for $|\mu| \gg M_{1,2} \gg M_Z$:

$$m_{\chi_1^0} \simeq M_1 - \frac{M_Z^2}{\mu^2}\left(M_1 + \mu s_{2\beta}\right) s_W^2$$

$$m_{\chi_2^0} \simeq M_2 - \frac{M_Z^2}{\mu^2}\left(M_2 + \mu s_{2\beta}\right) c_W^2$$

$$m_{\chi_{3/4}^0} \simeq |\mu| + \frac{1}{2}\frac{M_Z^2}{\mu^2}\epsilon_\mu(1 \mp s_{2\beta})\left(\mu \pm M_2 s_W^2 \mp M_1 c_W^2\right)$$

In the limit $|\mu| \to \infty$, χ_1^0 is bino (M_1), χ_2^0 wino (M_2) and χ_3^0, χ_4^0 higgsinos (μ).

In the opposite limit, $M_1, M_2 \to \infty$, the higgsino/wino roles are again reversed.

Finally, the gluino mass is identified with the gaugino mass parameter M_3 at tree–level

$$m_{\tilde{g}} = M_3$$

In constrained models with boundary conditions at the high energy scale M_U, the evolution of the gaugino masses given by RGEs

$$\frac{\mathrm{d}M_i}{\mathrm{d}\log(M_\mathrm{U}/Q^2)} = -\frac{g_i^2 M_i}{16\pi^2} b_i \ , \ b_1 = \frac{33}{5} \ , \ b_2 = 1 \ , b_3 = -3$$

where in b_i all sparticles contribute to the evolution from Q to M_U. RGEs related to those of the gauge couplings $\alpha_i = g_i^2/(4\pi)$. With inputs at scale M_Z and common value at $M_U \sim 2 \times 10^{16}$ GeV, one has for gaugino mass parameters at the SUSY scale M_S:

$$M_3 : M_2 : M_1 \sim \alpha_3 : \alpha_2 : \alpha_1 \sim 6 : 2 : 1$$

With normalization factor $\frac{5}{3}$ in α_1, we have the GUT relation $M_1 = \frac{5}{3}\tan^2\theta_W\, M_2 \simeq \frac{1}{2}M_2$.

$\mu \gg M_2 \Rightarrow m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim 2m_{\chi_1^0} \sim M_2,\ m_{\chi_3^0} \sim m_{\chi_4^0} \sim m_{\chi_2^\pm} \sim \mu$.

$\mu \ll M_2 \Rightarrow m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim m_{\chi_1^0} \sim \mu,\ m_{\chi_4^0} \sim 2m_{\chi_3^0} \sim m_{\chi_2^\pm} \sim M_2$.

The sfermion system is described by $\tan\beta$, μ and 3 parameters for each species: $m_{\tilde{f}_L}, m_{\tilde{f}_R}, A_f$.
For 3d generation, a mixing $\propto m_f$ has to be included. The sfermion mass matrices are:

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m_f^2 + m_{LL}^2 & m_f\, X_f \\ m_f\, X_f & m_f^2 + m_{RR}^2 \end{pmatrix}$$

with the various entries given by

$$m_{LL}^2 = m_{\tilde{f}_L}^2 + (I_f^{3L} - Q_f s_W^2)\, M_Z^2\, c_{2\beta}$$
$$m_{RR}^2 = m_{\tilde{f}_R}^2 + Q_f s_W^2\, M_Z^2\, c_{2\beta}$$
$$X_f = A_f - \mu(\tan\beta)^{-2I_f^{3L}}$$

They are diagonalized by 2×2 rotation matrices of angle θ_f, which turn the current eigenstates $\tilde{f}_L, \tilde{f}_R$ into the mass eigenstates $\tilde{f}_1, \tilde{f}_2$.

$$m^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4 m_f^2 X_f^2}\ \right]$$

- Note: mixing very strong in the stop sector, $X_t = A_t - \mu\cot\beta$ and generates mass splitting between $\tilde{t}_1, \tilde{t}_2$, leading to a light $\tilde{t}_1$ state
- The mixing in the sbottom sector can also be strong for large $X_b = A_b - \mu\tan\beta$.
- The same for the stau system and $\tilde{\tau}_1$ is in general the lightest sfermion at high $\tan\beta$!

In cMSSMs with universal m_0 and $m_{1/2}$ values at $M_{\rm GUT}$, the RGEs for the scalar masses are simple if the Yukawas are small ($c(\tilde{f})$ depend on I, Y, color):

$$m^2_{\tilde{f}_{L,R}} = m_0^2 + \sum_{i=1}^3 F_i(f) m^2_{1/2}\ , \ \ F_i = \frac{c_i(f)}{b_i}\left[1 - \left(1 - \frac{\alpha_U}{4\pi} b_i \log\frac{Q^2}{M_U^2}\right)^{-2}\right]$$

$$\tilde{L} : \begin{pmatrix} \frac{3}{10} \\ \frac{3}{2} \\ 0 \end{pmatrix}, \tilde{l}_R : \begin{pmatrix} \frac{6}{5} \\ 0 \\ 0 \end{pmatrix}, \tilde{Q} : \begin{pmatrix} \frac{1}{30} \\ \frac{3}{2} \\ \frac{8}{3} \end{pmatrix}, \ \tilde{u}_R : \begin{pmatrix} \frac{8}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}, \tilde{d}_R : \begin{pmatrix} \frac{2}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}$$

With the input values at $Q = M_Z$, $\alpha_U \simeq 0.041$ and $M_U \sim 2 \times 10^{16}$ GeV, one has

$$m^2_{\tilde{q}_i} \sim m_0^2 + 6m^2_{1/2}\, , \ \ m^2_{\tilde{\ell}_L} \sim m_0^2 + 0.52 m^2_{1/2}\, , \ \ m^2_{\tilde{e}_R} \sim m_0^2 + 0.15 m^2_{1/2}$$

For the third generation squarks, the large Yukawa couplings have to be included.

For instance, the approximate RGEs for top squarks [for small $\tan\beta$ values] are:

$$m^2_{\tilde{t}_L} = m^2_{\tilde{b}_L} \sim m_0^2 + 6m^2_{1/2} - \frac{1}{3} X_t$$
$$m^2_{\tilde{t}_R} = m^2_{\tilde{b}_L} \sim m_0^2 + 6m^2_{1/2} - \frac{2}{3} X_t$$

with the trilinear coupling given by the approximate value $X_t \sim 1.3 m_0^2 + 3m^2_{1/2}$.

⇒ In contrast to the first two generation sfermions, one has generically a sizable splitting between $m^2_{\tilde{t}_L}, m^2_{\tilde{t}_R}$ at the weak scale, due to the running of the large top Yukawa coupling.

⇒ Justifies the choice of different soft SUSY–breaking scalar masses and trilinear couplings for third generation and first/second generation sfermions [and for sleptons and squarks].

- Recall that already from mixing, $m^2_{\tilde{t}_1}$ lighter than all the other squarks

⇒ Special status for the top squark....

5. The Higgs sector of the MSSM

In the MSSM, we need two Higgs doublet fields $H_1 = \binom{H_1^0}{H_1^-}$ and $H_2 = \binom{H_2^+}{H_2^0}$.

The terms contributing to the scalar potential V come from three sources:

- D terms, quartic S interactions: $V_D = \frac{1}{2}\sum_a (\sum_i g_a S_i^* T^a S_i)^2$
- F terms of Superpotential: $V_F = \sum_i |\partial W(z_i)/\partial z_i|^2 \to \sum_i |\partial W(\phi_j)/\partial \phi_i|^2$
- Soft terms: $V_{\text{soft}} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + B\mu(H_2.H_1 + \text{h.c.})$

Adding all terms $\Rightarrow$ the scalar potential involving the Higgs bosons is

$$\begin{aligned} V_H &= \bar{m}_1^2|H_1|^2 + \bar{m}_2^2|H_2|^2 - \bar{m}_3^2\epsilon_{ij}(H_1^i H_2^j + \text{h.c.}) \\ &+ \frac{g_2^2+g_1^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^* H_2|^2 \end{aligned}$$

$$\text{with } \overline{m}_1^2 = |\mu|^2 + m_1^2,\ \overline{m}_2^2 = |\mu|^2 + m_2^2,\ \overline{m}_3^2 = B\mu$$

- Development in terms of components $H_1 = (H_1^0, H_1^-), H_2 = (H_2^+, H_2^0)$

$$\begin{aligned} V_H = &\overline{m}_1^2(|H_0^1|^2 + |H_1^+|^2) + \overline{m}_2^2(|H_2^0|^2 + |H_2^-|^2) - \overline{m}_3^2(H_1^+ H_2^- - H_1^0 H_2^0 + \text{h.c.}) \\ &+ \frac{g_2^2+g_1^2}{8}(|H_1^0|^2 + |H_1^+|^2 - |H_2^0|^2 - |H_2^-|^2)^2 + \frac{g_2^2}{2}|H_1^{+*}H_1^0 + H_2^{0*}H_2^-|^2 \end{aligned}$$

- Now require that the minimum of V_H breaks the symmetry $G_{\text{SM}} \to U(1)_{\text{QED}}$.
So at $V_H^{\min}$ we have $\langle H_1^+ \rangle = 0$ and at $\partial V/\partial H_1^+ = 0$ we have $\langle H_2^- \rangle = 0$; good for QED.

⟨

Ignoring the fields H_1^+, H_2^- to simplify, the relevant part of potential V_H is then simply:

$$V_H = \overline{m}_1^2|H_1^0|^2 + \overline{m}_2^2|H_2^0|^2 + \overline{m}_3^2(H_1^0 H_2^0 + \text{hc}) + (g_2^2 + g_1^2)(|H_1^0|^2 - |H_2^0|^2)^2/8$$

Some important remarks can be made on this scalar potential:

$$V_H = \overline{m}_1^2|H_1^0|^2 + \overline{m}_2^2|H_2^0|^2 + \overline{m}_3^2(H_1^0 H_2^0 + \text{hc}) + \frac{M_Z^2}{4v^2}(|H_1^0|^2 - |H_2^0|^2)^2$$

- Quartic couplings fixed in terms of the gauge couplings, only 3 free parameters: $\overline{m}_1^2, \overline{m}_2^2, \overline{m}_3^2$ (6 parameters and a phase in a general 2HDM).
- $m_{1,2}^2 + |\mu|^2$ real, only $B\mu$ can be complex. But any phase in $B\mu$ can be absorbed in phases of $H_1, H_2 \Rightarrow V_H$ (MSSM) conserves CP.
- If $B\mu$ is zero, all other terms are positive and thus $V_H = 0$ only if $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$. To have SSB (without CCB), we need $\overline{m}_{1,2,3} \neq 0$

$\Rightarrow$ Connection of gauge symmetry breaking and SUSY breaking!!

More precisely: in SM, symmetry breaking takes place with ad hoc choice $\mu^2 < 0$.

In the MSSM, $m_{H_i}^2 > 0$ at $Q = M_{\text{GUT}}$ but $t/\tilde{t}$ in RGEs make $m_{H_i}^2 < 0$ at $Q = M_Z$:

radiative breaking of the electroweak symmetry (i.e. through radiative corrections).

$\Rightarrow$ Symmetry breaking more natural and elegant than in SM.

To obtain the physical Higgs fields and their masses from potential V_H, develop

$H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ into real (CP-even+charged Higgses) and imaginary (CP-odd Higgs+Goldstone bosons) parts and diagonalize the 2×2 mass matrices:

$$\mathcal{M}_{ij}^2 = \frac{1}{2}\partial^2 V_H / \partial H_i \partial H_j |_{\langle \text{Re}(H_{1,2}^0) \rangle = v_{1,2}, \langle \text{Im}(H_{1,2}^0) \rangle = 0, \langle H_{1,2}^\pm \rangle = 0}$$

To obtain masses M_1, M_2 and the mixing angle θ, two useful relations are:

$$\mathrm{Tr}(\mathcal{M}^2) = M_1^2 + M_2^2 \quad , \quad \mathrm{Det}(\mathcal{M}^2) = M_1^2 M_2^2$$

$$\sin 2\theta = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2}} \ , \ \cos 2\theta = \frac{\mathcal{M}_{11}-\mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2}}$$

First note that if you perform the first derivative of the scalar potential V_H:

$$V_H = \overline{m}_1^2 |H_1^0|^2 + \overline{m}_2^2 |H_2^0|^2 + \overline{m}_3^2 (H_1^0 H_2^0 + \mathrm{hc}) + \frac{M_Z^2}{4v^2} (|H_1^0|^2 - |H_2^0|^2)^2$$

you have, at the minimum, $\partial V_H / \partial H_{1,2} = 0$, leading to the two relations:

$$\bar{m}_1^2 = -\bar{m}_3^2 \tan\beta - \tfrac{1}{2} M_Z^2 \cos(2\beta) \ , \ \bar{m}_2^2 = -\bar{m}_3^2 \mathrm{cot}\beta + \tfrac{1}{2} M_Z^2 \cos(2\beta)$$

The second derivative of V_H gives you the relevant mass matrices:

$$\mathcal{M}_R^2 = \begin{bmatrix} -\bar{m}_3^2 \tan\beta + M_Z^2 \cos^2\beta & \bar{m}_3^2 - M_Z^2 \sin\beta\cos\beta \\ \bar{m}_3^2 M_Z^2 \sin\beta\cos\beta & -\bar{m}_3^2 \mathrm{cot}\beta + M_Z^2 \sin^2\beta \end{bmatrix}$$

$$\mathcal{M}_I^2 = \begin{bmatrix} -\bar{m}_3^2 \tan\beta & \bar{m}_3^2 \\ \bar{m}_3^2 & -\bar{m}_3^2 \mathrm{cot}\beta \end{bmatrix}$$

For the CP–odd case, since $\mathrm{Det}\mathcal{M}_I^2 = 0$, one eigenvalue is zero (the Goldstone boson) and the other one corresponds to the CP-odd Higgs (A boson) with a mass:

$$M_A^2 = -\bar{m}_3^2 (\tan\beta + \mathrm{cot}\beta) = -2\bar{m}_3^2 / \sin 2\beta$$

The mixing angle θ is, in fact, just the angle β:

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}(H_1^0) \\ \mathrm{Im}(H_2^0) \end{pmatrix}$$

In the case of the CP–even Higgs bosons, the determinant and trace give:

$$\mathrm{Det}\mathcal{M}_R^2 = M_A^2 M_Z^2 c_{2\beta}^2 \equiv M_h^2 M_H^2$$

$$\mathrm{Tr}\mathcal{M}_R^2 = M_A^2 + M_Z^2 \equiv M_h^2 + M_H^2$$

To obtain the CP–even Higgs masses, solve the second order equation:

$$M_h^2(M_A^2+M_Z^2-M_h^2) = M_A^2 M_Z^2 c_{2\beta}^2 \Rightarrow M_h^4 - M_h^2(M_A^2+M_Z^2) + M_A^2 M_Z^2 c_{2\beta}^2 = 0$$

The two solutions are then (h is the lightest CP-even Higgs boson):

$$M_{h,H}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \mp \sqrt{(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\right]$$

The mixing angle α which rotates the fields is given by (and obeys $-\frac{\pi}{2} \leq \alpha \leq 0$)

$$\tan 2\alpha = \frac{2\mathcal{M}_{12}}{\mathcal{M}_{11}-\mathcal{M}_{22}} = \frac{-(M_A^2+M_Z^2)\sin 2\beta}{(M_Z^2-M_A^2)\cos 2\beta} = \tan 2\beta \, \frac{M_A^2+M_Z^2}{M_A^2-M_Z^2}$$

In the case of the charged Higgs bosons, one obtains similarly to the A boson case:

$$M_{H^\pm}^2 = M_A^2 + M_W^2$$

and the mixing angle θ is, also as for the A boson, simply the angle β.

In the MSM, we have an important constraint on the lightest MSSM h boson mass:

$$M_h \leq \min(M_A, M_Z) \cdot |\cos 2\beta| \leq M_Z$$

besides some other (also important) relations for the H, A and $H^\pm$ bosons:

$$M_H > \max(M_A, M_Z) \;\; \text{and} \;\; M_{H^\pm} > M_W$$

If we send M_A to infinity, we will have for the Higgs masses and the angle α:

$$M_h \sim M_Z|\cos 2\beta| \; , \; M_H \sim M_{H^\pm} \sim M_A \; , \qquad \alpha \sim \frac{\pi}{2} - \beta$$

This is the decoupling regime: all the Higgs bosons are heavy except for h.

The lightest h boson should be lighter than M_Z and should have been seen at LEP2 (as we had $\sqrt{s}_{\rm LEP2} \sim 200$ GeV $\Rightarrow$ masses $M_h + M_Z \sim 180$ GeV were accessible)

So what happened in this case? Maybe the MSSM was already ruled out at LEP2?

No! This relation holds only at first order (tree–level) and there are strong couplings involved in the theory, in particular the htt and $h\tilde{t}\tilde{t}$ couplings of the top/stop quarks.

$\Rightarrow$ The calculation of radiative corrections to M_h necessary.

More generally, radiative corrections very important in the MSSM Higgs sector.

A large activity for the calculation of radiative corrections in the last 30 years.

- The dominant corrections are due to top and stop quarks at one-loop level (they can be calculated easily using the material given before for top/stop loops).

$$\Delta M_h^2 = \frac{3g^2}{2\pi^2}\frac{m_t^4}{M_W^2}\log\frac{m_{\tilde{t}}^2}{m_t^2}$$

It depends on m_t^4 (quadratic) and $\log(m_{\tilde{t}}^2/m_t^2)$; thus large: $M_h^{\rm max} \to M_Z + 40$ GeV.

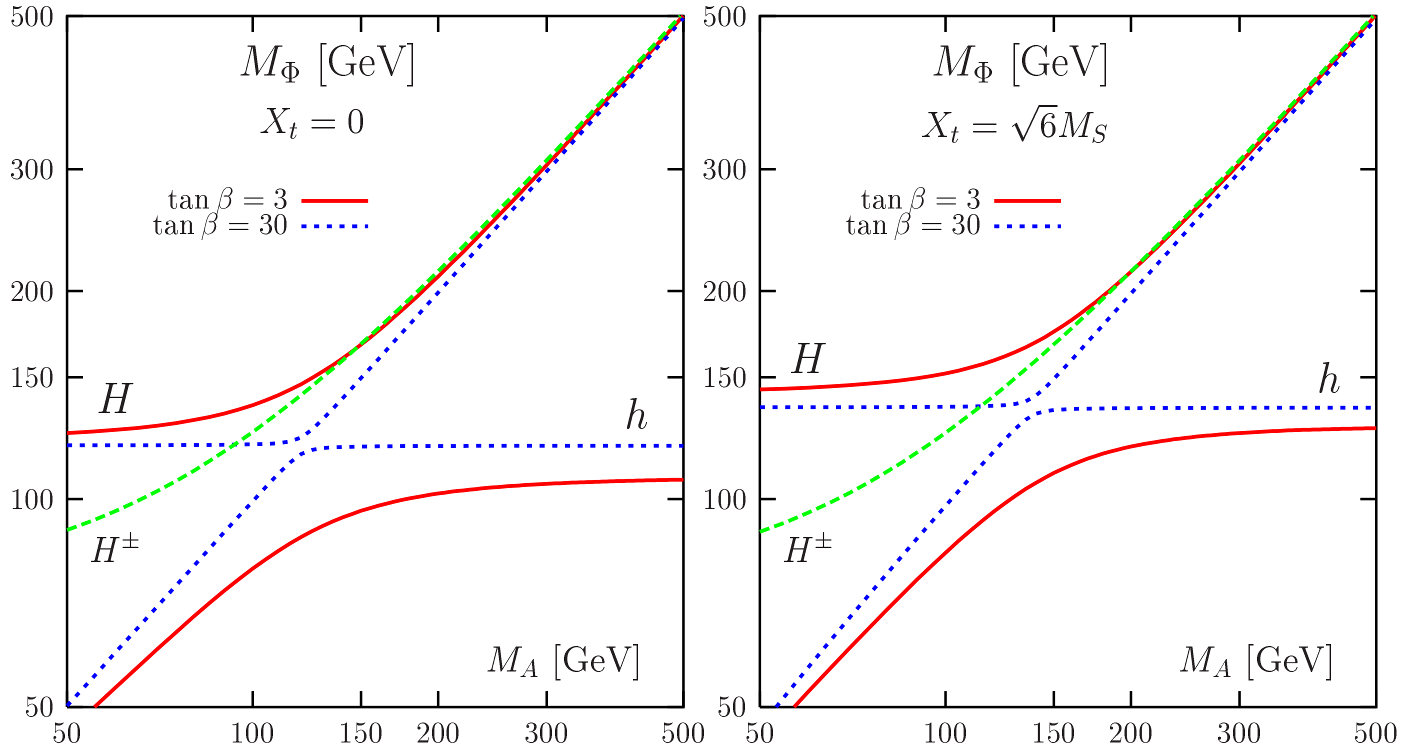
This explains why the h boson has not been observed at LEP2; heavier than 90 GeV.

- The full one–loop corrections have been calculated:

– other important parameters such as μ, A_t and A_b appear at the subleading level.

– the h boson mass is maximal (minimal) for a stop mixing parameter $A_t \sim 2M_{\tilde{Q}}(0)$.

- Approximate calculation for the dominant two–loop radiative corrections (in the effective potential approach where momenta transfert small compared to mass of internal particles):
  - dominant QCD corrections large but are absorbed by making $m_t|^{\rm pole} \to m_t|^{\overline{\rm MS}}$.
  - the Yukawa corrections are rather small in the limit $M_h = 0$.
- Using full 1–loop and the 2–loop corrections in the effective potential approach:
  - $\mathcal{O}(\alpha_t \alpha_S)$: including squark mixing and the gluino loops.
  - $\mathcal{O}(\alpha_t^2)$: including mixing and $\mathcal{O}(\alpha_b \alpha_S), \mathcal{O}(\alpha_\tau \alpha_S)$.



Calculation of radiative corrections to M_h

Let us do the calculation, but with some simplifications:

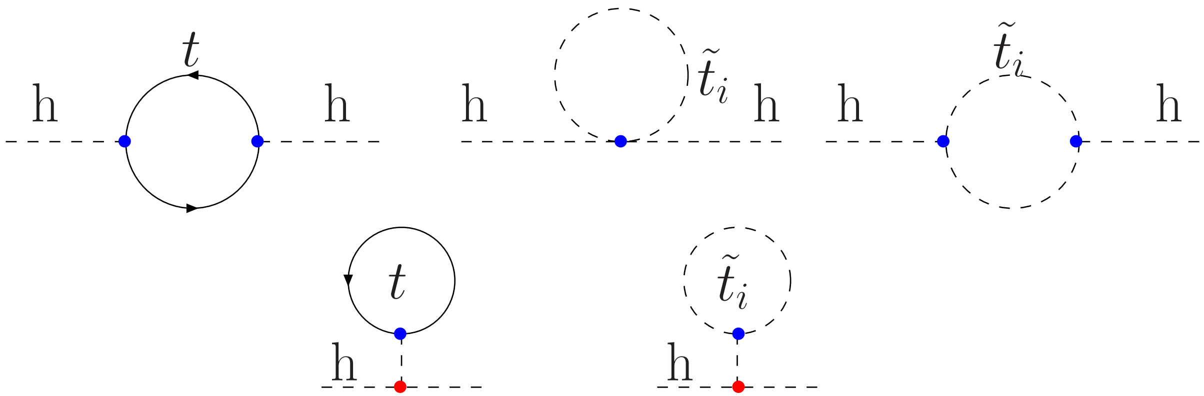
– take the (decoupling) limit $M_A \to 0$ and use $\tan\beta \gg 1$ ($M_h^{\max}$)

– assume no stop mixing and same masses, $m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{t}}$

– simple couplings: $h\bar{t}t \sim h\tilde{t}\tilde{t} \sim \lambda_t$, $hh\tilde{t}^*\tilde{t} \sim \lambda_t^2$ with $\lambda_t = \sqrt{2}m_t/v$

– work in the limit $M_h \ll m_t, m_{\tilde{t}}$.

In addition to two–point functions including fermion/scalar loops, we have also tadpole contributions (counterterm corrections):



• The calculation is almost already done: for two–point function:

$$\Delta M_h^2|_2 = \frac{3\lambda_t^2}{4\pi^2}\left[(m_t^2 - m_{\tilde{t}}^2)\log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) + 3m_t^2\log\left(\frac{m_{\tilde{t}}}{m_t}\right)\right]$$

- For the tadpole contributions, the calculation is very simple:

$$
\begin{aligned}
\Delta M_h^2|_1 &= iN_f \left(\frac{-iM_H^2}{v}\right) \frac{i}{-M_h^2} \left(-i\frac{\lambda_f}{\sqrt{2}}\right) (-4mi) \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{k^2 - m_f^2} \\
&+ iN_S \left(\frac{-iM_H^2}{v}\right) \frac{i}{-M_h^2} (iv\lambda_S) i \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{k^2 - m_S^2} \\
&= \frac{4N_f m_f \lambda_f}{\sqrt{2} v 16\pi^2} \int_0^{\Lambda^2} dy \frac{y}{y + m_f^2} + \frac{N_S \lambda_S}{16\pi^2} \int_0^{\Lambda^2} dy \frac{y}{y + m_S^2}
\end{aligned}
$$

Using λ_S=-λ_f^2=-$2m_f^2/v$ and $\int \ldots = \Lambda^2 - m^2 \log(\Lambda^2/m^2)$, one obtains

$$
\Delta M_h^2|_1 = -\frac{3\lambda_t^2}{4\pi^2 v^2} \left[m_{\tilde{t}}^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) - m_t^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) \right]
$$

- The total correction to the h boson mass is then given by:

$$
\Delta M_h^2 = \frac{3m_t^4}{2\pi^2 v^2} \log \frac{m_{\tilde{t}}}{m_t} = \frac{3g^2}{2\pi^2} \frac{m_t^4}{M_W^2} \log \frac{m_{\tilde{t}}^2}{m_t^2}
$$

Its depends on m_t^4 and $\log(m_{\tilde{t}}^2/m_t^2)$, and is large: $M_h^{\max} \to M_Z + 40$ GeV! This explains why the h boson has not been seen at LEP2.

The Higgs boson couplings

Trilinear and Quartic scalar couplings obtained from scalar potential V_H by making:

$$\lambda^2_{ijk} = \left. \frac{\partial^3 V_H}{\partial H_i \partial H_j \partial H_k} \right|_{\langle H_1^0 \rangle = v_1, \langle H_2^0 \rangle = v_2, \langle H_{1,2}^{\pm} \rangle = 0}$$

$$\lambda^2_{ijkl} = \left. \frac{\partial^4 V_H}{\partial H_i \partial H_j \partial H_k \partial H_l} \right|_{\langle H_1^0 \rangle = v_1, \langle H_2^0 \rangle = v_2, \langle H_{1,2}^{\pm} \rangle = 0}$$

with the H_i expressed in terms of the fields $h, H, A, H^{\pm}$ and $G^0, G^{\pm}$ with rotations of angles β et α. Examples (unit: $\lambda_0 = -iM_Z^2/v$):

$$\lambda_{hhh} = 3 \cos 2\alpha \sin(\beta + \alpha) + \text{rad. corr.}$$
$$\lambda_{hhhh} = 3 \cos^2 \alpha / M_Z^2 + \text{rad. corr.} \qquad (\text{in units of } \lambda_0^2)$$

In the decoupling limit, $M_A \gg M_Z$ we have $\alpha \to \beta - \pi/2$:

$$\lambda_{hhh} \to 3 \cos^2(2\beta) = 3M_h^2/M_Z^2 = \lambda^3|_{\rm MS}$$
$$\lambda_{hhhh} \to 3 \cos^2(2\beta)/M_Z^2 = 3M_h^2/M_Z^4 = \lambda^4|_{\rm MS}$$

$\Rightarrow$ In the decoupling limit, $M_A \gg M_Z$, the Higgs potential of the MSSM becomes like the one of the SM: only one light Higgs with a mass $M_h \lesssim 130$ GeV and with standard interactions. All other Higgsses are heavy and decouple (but self–couplings are non–zero).

Couplings to gauge bosons: from kinetic terms of H_1, H_2 in SU(2)xU(1) Lagrangian:

$$\mathcal{L}_{\rm kin.} = (D^\mu H_1)^\dagger (D_\mu H_1) + (D^\mu H_2)^\dagger (D_\mu H_2)$$

Develop D_μ and get masses eigenstates via rotatios of angles θ_W, β, α

$$\begin{aligned} g_{h_i VV} &\equiv \text{coefficients de } h_i V_\mu V_\mu \quad (g_{\mu\nu}) \\ g_{h_i h_j V} &\equiv \text{coefficients de } h_i h_j V_\mu \quad (\partial_\mu \to p_\mu) \\ g_{h_i h_j VV} &\equiv \text{coefficients de } h_i h_j V_\mu V_\mu \quad (g_{\mu\nu}) \end{aligned}$$

Some very important couplings for Higgs phenomenology:

$$\begin{aligned} Z^\mu Z^\nu h \; &: \; \frac{igM_Z}{\cos\theta_W} \sin(\beta-\alpha) g^{\mu\nu} \;\; , \;\; Z^\mu Z^\nu H \; : \; \frac{igM_Z}{\cos\theta_W} \cos(\beta-\alpha) g^{\mu\nu} \\ W^\mu W^\nu h \; &: \; igM_W \sin(\beta-\alpha) g^{\mu\nu} \;\; , \;\; W^\mu W^\nu H \; : \; igM_W \cos(\beta-\alpha) g^{\mu\nu} \\ Z^\mu hA : &\frac{g\cos(\beta-\alpha)}{2\cos\theta_W} (p+p')^\mu \;\; , \;\; Z^\mu HA : -\frac{g\sin(\beta-\alpha)}{2\cos\theta_W} (p+p')^\mu \end{aligned}$$

– γ massless: no coupling with the neutral Higgsses at tree–level.
– CP invariance: no ZZA and Zhh, ZHh, ZHH couplings e.g.
– Couplings of h and H complementary: $g^2_{hZZ} + g^2_{HZZ} = g^2_{\rm MS}$!
– Decoupling limit ($M_A \to \infty, \alpha \to \beta - \frac{\pi}{2}$): $\sin(\beta-\alpha) \to 1, \cos(\beta-\alpha) \to 0$:

$$\Rightarrow g_{hVV} = g_{H_{\rm MS}VV} \; , g_{HVV} = 0 \; (= g_{AVV}) \qquad .$$

Yukawa couplings to fermions:

The Higgs couplings to fermions come from the Superpotential W:

$$W = \sum_{\rm i,j=gen} Y_{ij}^u \, \hat{u}_R^i \hat{H}_2 . \hat{Q}^j + Y_{ij}^d \, \hat{d}_R^i \hat{H}_1 . \hat{Q}^j + Y_{ij}^l \, \hat{l}_R^i \hat{H}_1 . \hat{L}^j + \mu \hat{H}_1 . \hat{H}_2$$

with $\mathcal{L}_{\rm Yuk} = -\frac{1}{2} \sum_{ij} [\overline{\psi}_{iL} \frac{\partial^2 W}{\partial z_i \partial z_j} \psi_j + {\rm hc}]$ evaluated in terms of H_1, H_2. Taking bilinears out, digagonal Y matrices and relations to masses, expressings $H_{1,2}$ in terms of physical fields:

$$\begin{aligned} \mathcal{L}_{\rm Yuk} &= -g m_u / (2 M_W \sin\beta) \left[\bar{u} u (H \sin\alpha + h \cos\alpha) - i \bar{u} \gamma_5 u \, A \cos\beta \right] \\ &\quad - g m_d / (2 M_W \cos\beta) \left[\bar{d} d (H \cos\alpha - h \sin\alpha) - i \bar{d} \gamma_5 d \, A \sin\beta \right] \\ &\quad + g / (2\sqrt{2} M_W) [H^+ \bar{u} [m_d \tan\beta (1+\gamma_5) + \frac{m_u}{\tan\beta} (1+\gamma_5)] d + {\rm hc}] \end{aligned}$$

The couplings, in termsof those of $H_{\rm SM}$ [normalisation factor $-(i) g m_f / 2 M_W = -i m_f / v$] and their values in the decoupling limit [$\alpha \to \beta - \frac{\pi}{2} \Rightarrow \cos\alpha \to \sin\beta, \sin\alpha \to -\cos\beta$]:

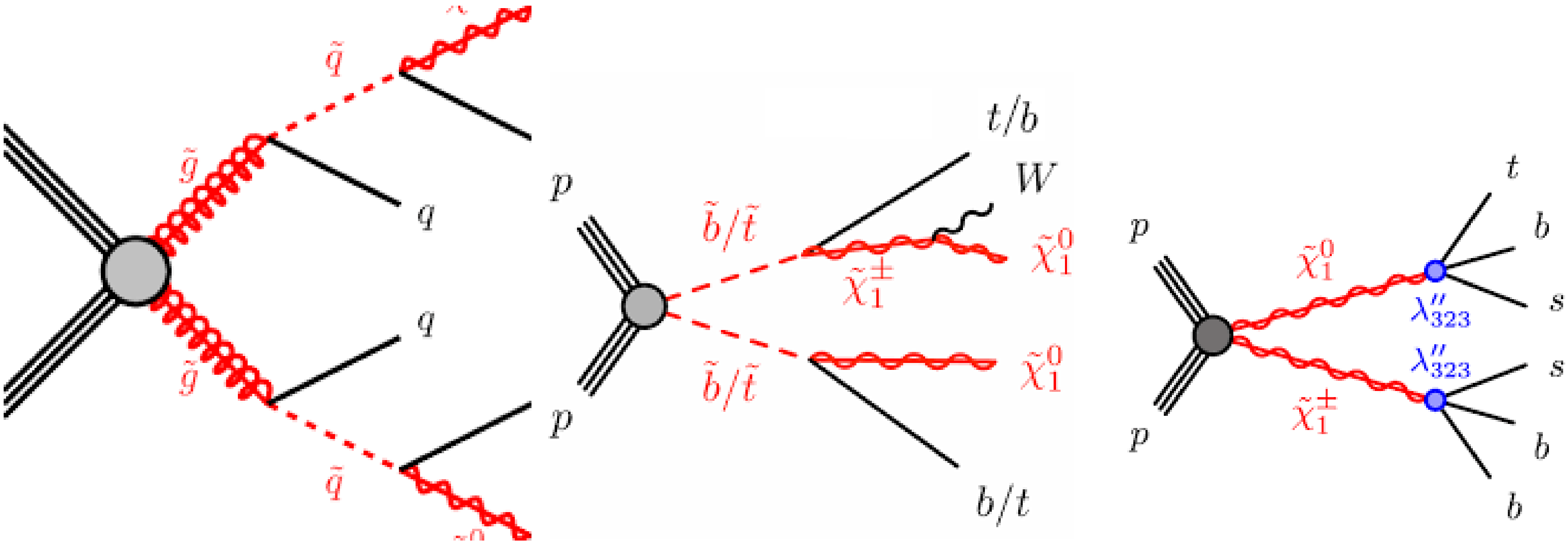
f	g_{ffh}	g_{ffH}	g_{ffA}
u	$\cos\alpha / \sin\beta \to 1$	$\sin\alpha / \sin\beta \to -\tan\beta$	$\cot\beta$
d	$-\sin\alpha / \cos\beta \to 1$	$\cos\alpha / \cos\beta \to \tan\beta$	$\tan\beta$

– The couplings of $H^\pm$ have the same intensity as those of A.
– For $\tan\beta > 1$: Cplgs to d enhanced, cplgs to u suppressed.
– For $\tan\beta \gg 1$: couplings to b quarks b $(m_b \tan\beta)$ very strong.
– For $M_A \gg M_Z$: h couples like the SM Higgs boson and H like A.

6. Verification of the SUSY and MSSM dogmas

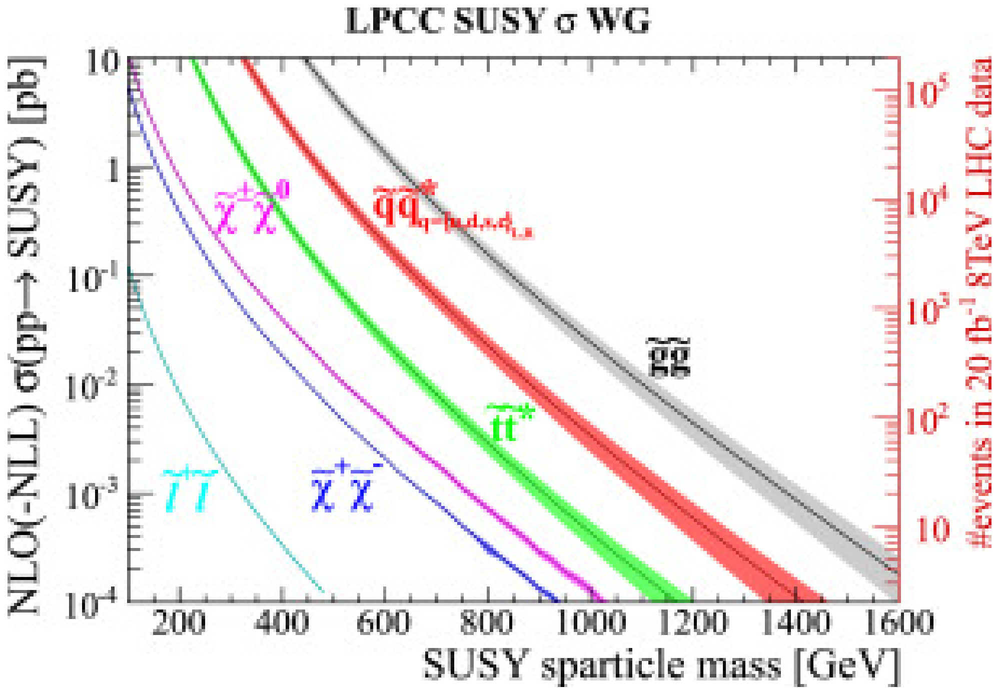
SUSY is nice but it is not enough: it needs to be checked experimentally!

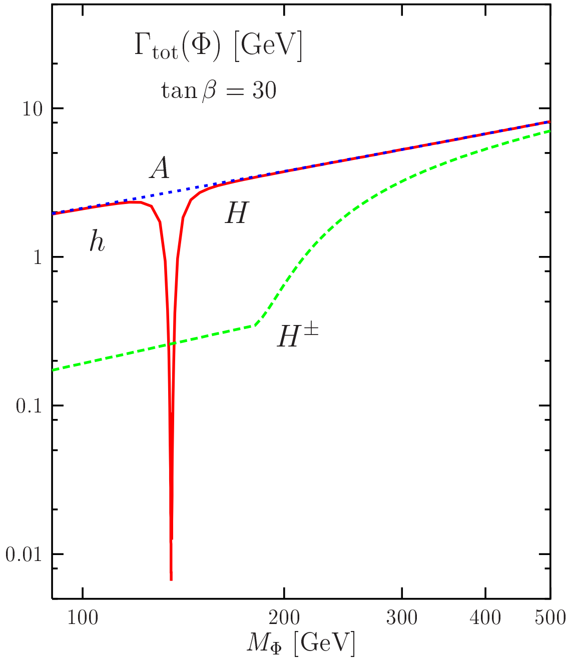
Once you have masses/couplings you can calculate production/decay rates.

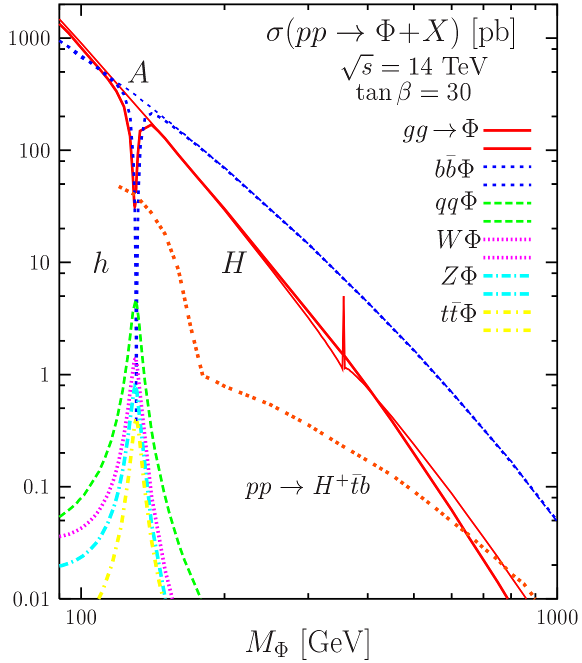


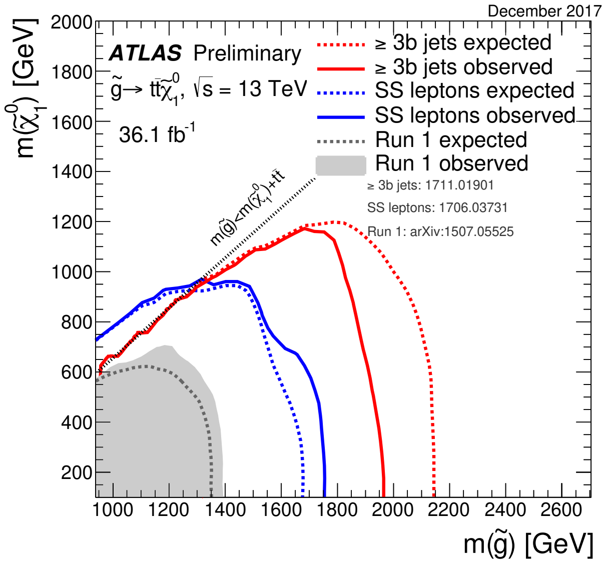
Predict what you should observe at the LHC in the various channels,

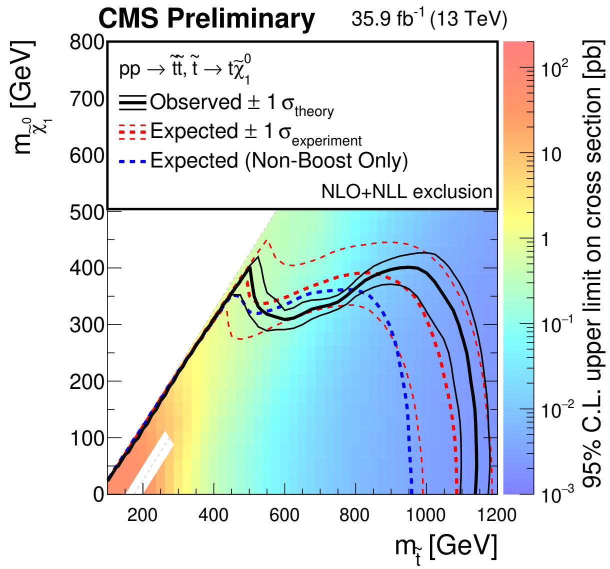
.... and compare with the exprimental data....

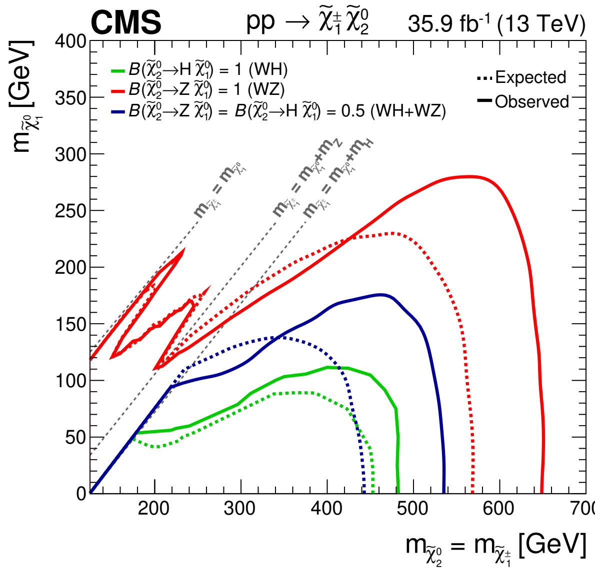

LPCC SUSY σ WG
NLO(-NLL) σ(pp→ SUSY) [pb]
#events in 20 fb⁻¹ 8TeV LHC data
SUSY sparticle mass [GeV]
g̃g̃
t̃t̃*
χ̃⁺χ̃⁰
χ̃⁺χ̃⁻
τ̃τ̃
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁵
10⁴
10³
10²
10
200
400
600
800
1000
1200
1400
1600

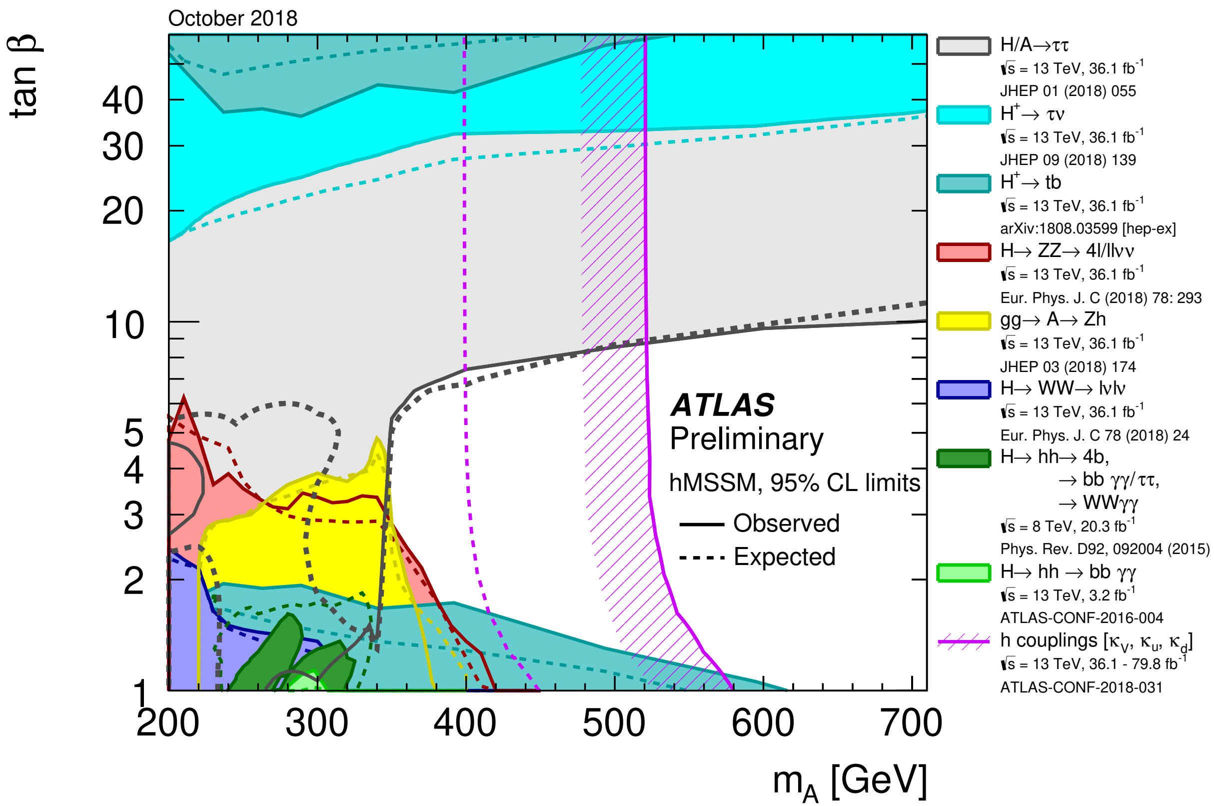

Γtot(Φ) [GeV]
tan β = 30
A
H
h
H±
10
1
0.1
0.01
100
200
300
400
500
MΦ [GeV]


σ(pp → Φ+X) [pb]
√s = 14 TeV
tan β = 30
gg → Φ
bb̄Φ
qqΦ
WΦ
ZΦ
tt̄Φ
A
h
H
pp → H⁺t̄b
1000
100
10
1
0.1
0.01
100
1000
MΦ [GeV]


December 2017
ATLAS Preliminary
36.1 fb-1
≥ 3b jets expected
≥ 3b jets observed
SS leptons expected
SS leptons observed
Run 1 expected
Run 1 observed
≥ 3b jets: 1711.01901
SS leptons: 1706.03731
Run 1: arXiv:1507.05525
m(g̃) [GeV]
m(χ̃^0_1) [GeV]


CMS Preliminary
35.9 fb-1 (13 TeV)
Observed ± 1 σ_theory
Expected ± 1 σ_experiment
Expected (Non-Boost Only)
NLO+NLL exclusion
95% C.L. upper limit on cross section [pb]
m_t̃ [GeV]


CMS
35.9 fb-1 (13 TeV)
Expected
Observed
(WH)
(WZ)
(WH+WZ)
[GeV]


October 2018
tan β
m_A [GeV]
ATLAS
Preliminary
hMSSM, 95% CL limits
Observed
Expected
H/A→ττ
√s = 13 TeV, 36.1 fb-1
JHEP 01 (2018) 055
H+→ τν
√s = 13 TeV, 36.1 fb-1
JHEP 09 (2018) 139
H+→ tb
√s = 13 TeV, 36.1 fb-1
arXiv:1808.03599 [hep-ex]
H→ ZZ→ 4l/llνν
√s = 13 TeV, 36.1 fb-1
Eur. Phys. J. C (2018) 78: 293
gg→ A→ Zh
√s = 13 TeV, 36.1 fb-1
JHEP 03 (2018) 174
H→ WW→ lνlν
√s = 13 TeV, 36.1 fb-1
Eur. Phys. J. C 78 (2018) 24
H→ hh→ 4b,
→ bb γγ/ττ,
→ WWγγ
√s = 8 TeV, 20.3 fb-1
Phys. Rev. D92, 092004 (2015)
H→ hh → bb γγ
√s = 13 TeV, 3.2 fb-1
ATLAS-CONF-2016-004
h couplings [κV, κu, κd]
√s = 13 TeV, 36.1 - 79.8 fb-1
ATLAS-CONF-2018-031