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Physics Beyond the Standard Model

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I+II: Beyond the Standard Model: why and where?

1 The Standard Model in two and a half slides

2 Problems of the SM and Grand Unified Theories

3 The hierarchy problem and physics beyond the SM

4 Some elements for calculations in high-energy physics

5 Connection between divergences and symmetries

6 The supersymmetric solution

1. The Standard Model in two and a half slides

The Standard Model is based on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry.

It is a generalization of QED of electromagnetism, extending the $U(1)_Q$ abelian group.

- The local $SU(3)_C$ symmetry group describes the strong interactions:
  - the strong interaction between the quarks q,q,q which are color triplets of SU(3),
  - is mediated by 8 massless gluons, which correspond to 8 generators of SU(3).
- $SU(2)_L \times U(1)_Y$ is for the unified electromagnetic and weak interactions.

L is for left isospin $I^3_{f_L} = \pm\frac{1}{2} (I^3_{f_R} = 0)$ and Y_f the fermion hypercharge $Y_f = 2Q_f - 2I^3_f$.

- The group acts on quarks/leptons of isospin Left (doublets) and Right (singlets),

$$\binom{\nu}{\mathbf{e}}_{\mathbf{L}}, \ \mathbf{e}^-_{\mathbf{R}}, \ \binom{\mathbf{u}}{\mathbf{d}}_{\mathbf{L}}, \ \mathbf{u}_{\mathbf{R}}, \mathbf{d}_{\mathbf{R}}, \cdots$$

and idem for the two other families; the neutrinos are massless $\Rightarrow$ there are no $\nu_{\mathbf{R}}$;

- it is mediated by the gauge bosons W_1, W_2, W_3 generators of SU(2) and B of U(1):

the (W_1, W_2) and (W_3, B) then combine to form the (W^+, W^-) and (Z, γ) bosons.

Big problem: the photon is massless as it should but the weak bosons are massive!

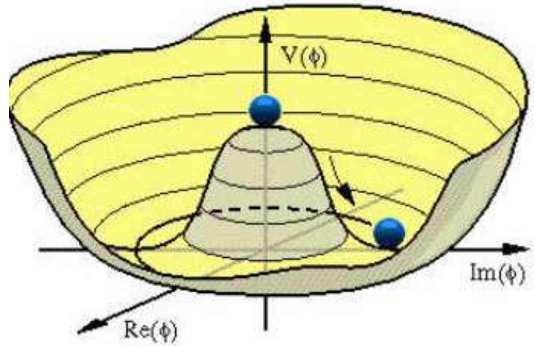
Naive inclusion of a mass for W/Z bosons and also fermions breaks invariance with respect to gauge symmetry and the renormalizability property of the theory is lost.

Former major problem of Particle Physics: how to nicely generate these masses?

$\Rightarrow$ the Higgs-Englert-Brout mechanism of electroweak symmetry breaking.

Introduce an SU(2) doublet of complex scalar fields

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \mathrm{Re}(\phi^+) + \mathrm{Im}(\phi^+) \\ \mathrm{Re}(\phi^0) + \mathrm{Im}(\phi^0) \end{pmatrix}, \quad Y_\phi = +1\,.$$



i.e. 4 degrees of freedom, with a scalar potential

$$V = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2 \text{ but with } \mu^2 < 0.$$

ϕ^0 gets vev: $\langle 0|\phi^0|0\rangle = v = (-\mu^2/\lambda)^{1/2} = 246$ GeV.

To the SM Lagrangian (where we ignore the strong interaction part) given by

$$\mathcal{L}_{\rm SM} = -\tfrac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} + \overline{L}\, i D_\mu \gamma^\mu\, L + \overline{e}_R\, i D_\mu \gamma^\mu\, e_R\ \cdots$$

with the field strengths $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ where g_2, g_1 are the SU(2) and U(1) couplings and the covariant derivative D_μ is

$$D_\mu = \partial_\mu - i g_2 T_a W^a_\mu - i g_1 \tfrac{Y_q}{2} B_\mu, \text{ with } [T^a, T^b] = i\epsilon^{abc} T_c \text{ and } [Y, Y] = 0\ .$$

We need to add to $\mathcal{L}_{\rm SM}$ the SU(2)×U(1) invariant terms of the scalar field part

$$\mathcal{L}_S = (D^\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2$$

After EWSB and a gauge transformation, the field Φ becomes $\Phi \to \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{pmatrix}$,

fully expand the term $|D_\mu \Phi|^2$ in $\mathcal{L}_S$, define the new fields $W^\pm_\mu$ and Z_μ, A_μ and then pick up the terms which are bilinear in the fields; one gets the gauge boson masses

$$M_W = \tfrac{1}{2} v g_2\ ,\ \ M_Z = \tfrac{1}{2} v \sqrt{g_2^2 + g_1^2}\ ,\ \ M_A = 0.$$

By spontaneously breaking $SU(2)_L \times U(1)_Y \to U(1)_Q$, 3 Goldstones are absorbed by $W^\pm, Z$ to form their longitudinal dofs and masses; $U(1)_Q$ is unbroken and A_μ remains massless.

With the same Φ and its cc, we can also generate fermion masses using the Yukawa $\mathcal{L}_f$

$$\mathcal{L}_f = -\lambda_e \bar{L}\Phi e_R - \lambda_d \bar{Q}\Phi d_R - \lambda_u \bar{Q}\tilde{\Phi} u_R + \text{ h. c.} \Rightarrow -\frac{1}{\sqrt{2}}\lambda_f (v+H)\bar{f}_L f_R\,; \; m_f = \frac{\lambda_f v}{\sqrt{2}}$$

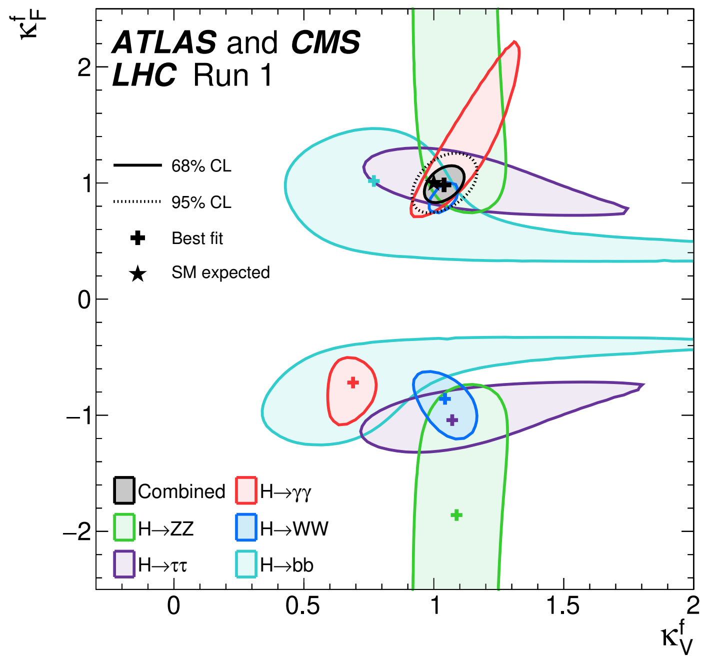
For the Higgs fields, its mass and self interactions come from the scalar Lagrangian

$$\mathcal{L}_S = \frac{1}{2}(\partial^\mu H)^2 - \lambda v^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 \Rightarrow \; M_H^2 = 2\lambda v^2 = -2\mu^2, \; g_{H^3} = 3i\,\frac{M_H^2}{v}.$$

The H couplings are related to the masses via $\mathcal{L}_{M_V} \sim M_V^2(1+H/v)^2, \mathcal{L}_{m_f} \sim -m_f(1+H/v)$

and one obtains the couplings: $g_{Hff} = i m_f/v, \; g_{HVV} = -2iM_V^2/v, \; g_{HHVV} = -2iM_V^2/v^2.$

This model is mathematically consistent and is extremely successful in describing nature!

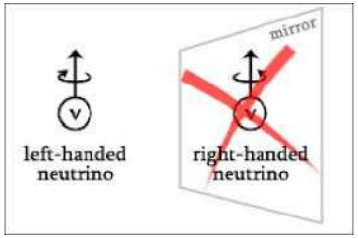


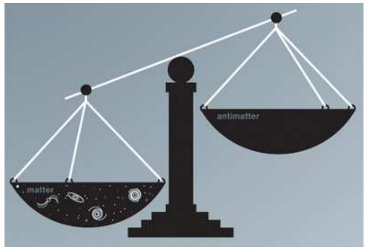
2. The main problems of the Standard Model and GUTs

• Although good to explain experimental data, the SM has severe shortcomings: it does not explain all the phenomena that are observed in Nature. In particular:

• The observed neutrinos are massless:

– SM: neutrinos are left-handed and massless,
– experiment: neutrinos oscillate ⇒ massive;
– their mass is not coming from the Higgs,
– we need right-handed neutrinos (≠ left).



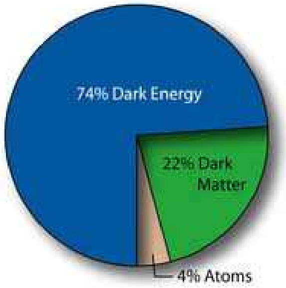


• No baryon asymmetry in the universe:

– there is a one billion p for a single $\bar{p}$,
– but at early times, CP conserved and $n_p = n_{\bar{p}}$,
– why there is such an asymmetry now?

• There is no Dark Matter particle:

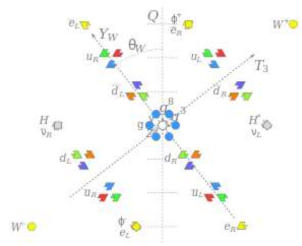
– known matter makes ≈4% of energy of Universe;
– ≈ 25% of it is a dark or invisible matter;
– astroparticle: must be massive and cold ($v \ll c$).
– In the SM, there is not a particle which is: neutral, weakly interacting, massive and stable.



There are also problems of aesthetic nature: the SM is far too complex and has too many ingredients. We want a theory with a few parameters and basic ingredients/principles.

- Too many ingredients that are put by hand:

– needs 19 parameters to describe everything;
– fermion masses very different from another;
– fermion mixing with P and CP violation puzzling;
– symmetry breaking is had-hoc/non-natural.

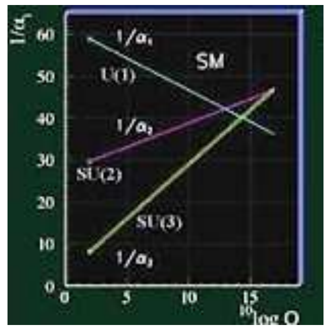


- Does not include gravitational interaction:

– this is desirable at very high energies (M_P);
– but there is no quantum gravity theory so far,
– the graviton of spin 2 is complicated object.

- Only partial unification of three interactions?

– 3 gauge groups with 3 different couplings,
– better: only one group and one coupling,
– coupling unification at a high scale?
– the three couplings do not converge.



There is at least one extension to go for: a Grand Unified Theory!

In the SM, we have 3 different gauge groups with 3 coupling constants:

Basic ideas: SU(3), SU(2) and U(1) are all subgroups of a bigger (unifying) gauge group.

Grand Unified Theory (GUT): SU(5), SO(10), E_6 etc....

These groups are (spontaneously) broken at a scale $M_{\rm GUT} = M_U$ down to $G_{\rm SM}$

but possibility of intermediate breaking, ex: $\rm SO(10) \to SU(5) \times U'(1) \to G_{SM} \times U'(1)$

- There is only one coupling constant at the unification (GUT) scale $M_{\rm GUT} = M_U$.
- The GUT group can have a fundamental representation that includes all SM fermions.

Ex: SO(10) has a representation of dimension 16 to incorporate the 15 SM fermions.

– There is space left for right-handed neutrinos: generation of m_ν via see–saw.

– If $\not{C}$P couplings, baryon asymmetry in the universe generated through leptogenesis.

– Can include a light stable axion (to explain small $\not{C}\mathrm{P}_{\rm strong}$): dark matter candidate.

$\Rightarrow$ Almost all SM model problems solved at once!

- Can also explain charge quantization; example: in SU(5), e, d in same multiplet.
- Can also relate the masses of the fermions at $M_{\rm U}$: Yukawa coupling unification.

This is the route that we will follow here: GUTs (and see later on, SUSY–GUTs).

But there are at least three big problems in normal GUTs:

gauge coupling unification, absence of cold dark matter, the hierarchy problem.

We start with gauge coupling unification, but first, what are the gauge couplings?

The SU(3) and SU(2) gauge couplings are $\alpha_3 = g_s^2/(4\pi)$ and $\alpha_2 = g_2^2/(4\pi) = g^2/(4\pi)$.
The case is more complicated for the U(1) coupling g': it needs a proper normalization.
Indeed, the scale is arbitrary: no physical change in $\frac{1}{2}Y g'$ if we use e.g. Ya and g'/a.
[remember that in SU(2), generators are normalized by the relation $\mathrm{Tr}(\frac{\tau^a}{2}\cdot\frac{\tau^b}{2}) = \frac{1}{2}\delta^{ab}$].

In a GUT like SU(5) for instance, $Y_G = a\frac{Y}{2}$ is the hypercharge generator and it has a normalization that is common to those of the SU(2) and SU(3) subgroups.

An example of normalization condition is such that $\mathrm{Tr}(Y_G)^2 = \mathrm{Tr}(a\frac{Y}{2})^2 = \mathrm{Tr}(T_3)^2$.

For one generation of u, d, e, ν_e fermions (recall that $\Sigma_f Q_f = \Sigma_f \frac{1}{2} Y_f + I_f^3 = 0$ in SM) :

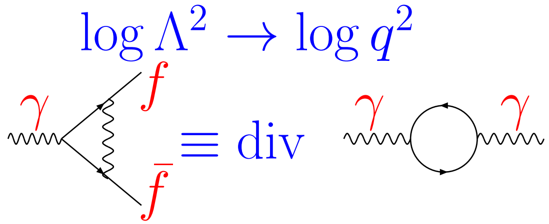
$$\mathrm{Tr}(T_3)^2 = 3(\tfrac{1}{4}+\tfrac{1}{4}) + \tfrac{1}{4} + \tfrac{1}{4} = 2 = \mathrm{Tr}(Y_G)^2 = \tfrac{1}{4}a^2[3(\tfrac{1}{9}+\tfrac{16}{9}+\tfrac{1}{9}+\tfrac{4}{9}) + 1 + 1 + 4] = a^2(\tfrac{10}{3}).$$

$$Y_G = \sqrt{\tfrac{3}{5}}(\tfrac{Y}{2}) \Rightarrow \alpha_1 = (\sqrt{\tfrac{5}{3}}g')^2/(4\pi) = g_1^2/(4\pi).$$

In fact, this is simply a prediction for the value of $\sin^2\theta_W$ at the unification scale:

$$g_2 \tan\theta_W = g' = \sqrt{\tfrac{5}{3}}g_1\ ;\ g_2(M_G) = g_1(M_U) \Rightarrow \sin^2\theta_W(M_U) = \tfrac{3}{8} = 0.375.$$

The running of the coupling constants comes from radiative corrections to interaction ($\gamma f \bar{f}$ in QED).
Ward identities $\Rightarrow$ renormalisation of 2-point function.
Logarithmic running: same coefficient as divergence.



Evolution determined by RGEs: $\frac{d\alpha_i}{d\log(q)} = -\frac{b_i}{2\pi}\alpha_i^2$

$b_i \equiv$ coefficients of the β functions related to divergences of two–point functions.

They depend on the relevant gauge group and the particle content of the theory.

Example of SU(N): $b_N = \frac{11}{3}N - \frac{2}{3}n_{\rm fermions} - \frac{1}{6}n_{\rm scalars}$

In the Standard Model: $b_3^{\rm SM} = 11 - \frac{2}{3} \times 6 = 7\,,\ b_2^{\rm SM} = \frac{22}{3} - 4 - \frac{1}{6} = \frac{19}{6}$;

for U(1): $b_1 = -\frac{2}{3}\Sigma_f Y_f^2 - \frac{1}{3}\Sigma_f Y_s^2 \Rightarrow b_1^{\rm SM} = -\frac{41}{10}$ (recall normalization).

Note that $b_3, b_2 > 0$ while $b_1 < 0$: asymptotic freedom of SU(N) gauge theories,

$\Rightarrow$ the couplings g_3, g_2 decrease with energy q while g_1 increases with energy q.

This explains why the strong coupling g_s becomes small at a few GeV scale.

For unification, it is more convenient to deal with α_i^{-1} and the RGEs become

$$\frac{d\alpha_i^{-1}}{d\log(q)} = +\frac{b_i}{2\pi}\alpha_i^2 \Rightarrow \alpha_i^{-1}(q) = \alpha_i^{-1}(q_0) + \frac{b_i}{2\pi}\log(q/q_0).$$

At the unification scale $q = M_U$, we need $\alpha_1(M_U) = \alpha_2(M_U) = \alpha_3(M_U) \equiv \alpha_U$;

with a starting point $q_0 = M_Z$, this implies the 3 relations between couplings:

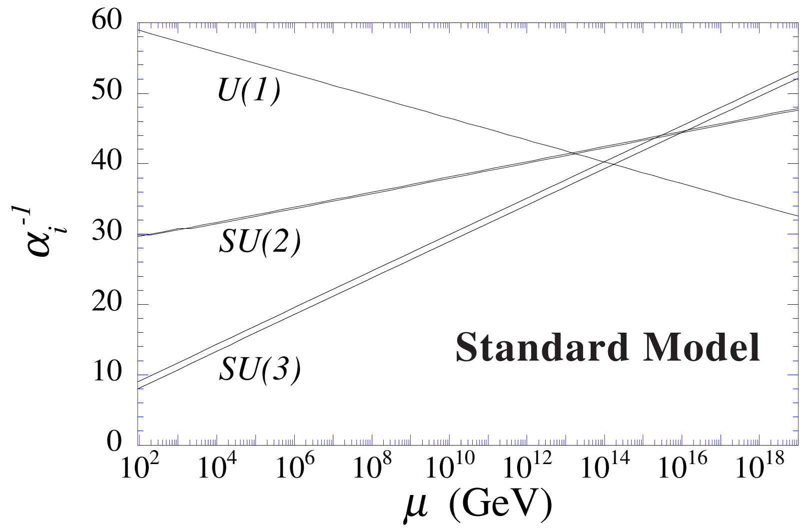
$$\alpha_U^{-1} = \alpha_{1,2,3}^{-1}(M_Z) + \frac{b_{1,2,3}}{2\pi}\log(M_U/M_Z),$$

$$B_{\rm exp} = \frac{\alpha_3^{-1}(M_Z) - \alpha_2^{-1}(M_Z)}{\alpha_2^{-1}(M_Z) - \alpha_1^{-1}(M_Z)} = \frac{b_2 - b_3}{b_1 - b_2} = B_{\rm th} = \frac{115}{218} = 0.528.$$

[Note: there also threshold corrections near the scale M_U but they should be small].

Let us now check with experiment if the gauge couplings really unify at a single point.

For $B_{\rm exp}$, experimental values from LEP:
$\sin^2\theta_W \simeq 0.231, \alpha_s \simeq 0.118, \alpha_{\rm EM} \simeq 1/129$
$\alpha_3^{-1} \simeq 8.40\,,$
$\alpha_2^{-1} = \alpha_{\rm EM}^{-1}\sin^2\theta_W \simeq 29.6\,,$
$\alpha_1^{-1} = \frac{3}{5}\alpha_2^{-1}\cot^2\theta_W \simeq 59.1,$
$\Rightarrow B_{\rm exp} \simeq 0.72 \neq B_{\rm th} = 0.528.$
No real unification of 3 gauge couplings!
(do not meet at a single point near $M_{\rm GUT}$).



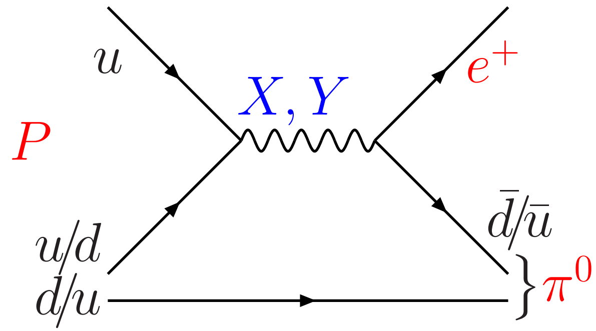
Another problem of GUTs: proton decays via exchange of heavy SU(5) gauge bosons X,Y.

To obtain the proton lifetime in this case:
– compute effective 4–fermion interaction,
– run vertices from $M_{X,Y} \sim M_{\rm GUT}$ to m_P,
– calculate hadronic ME of the 4f operator.
With input $M_{\rm GUT} \sim 10^{15}$ GeV from $g_i's$, get:

$$\tau_P^{\rm GUT} = 10^{30\pm1.7}\ \text{years}.$$

To be compared to $\tau_P^{\rm exp} \gtrsim 10^{33}$ years value.
The proton decays too fast in these GUTs.

$P \to e^+\pi^0, \cdots$



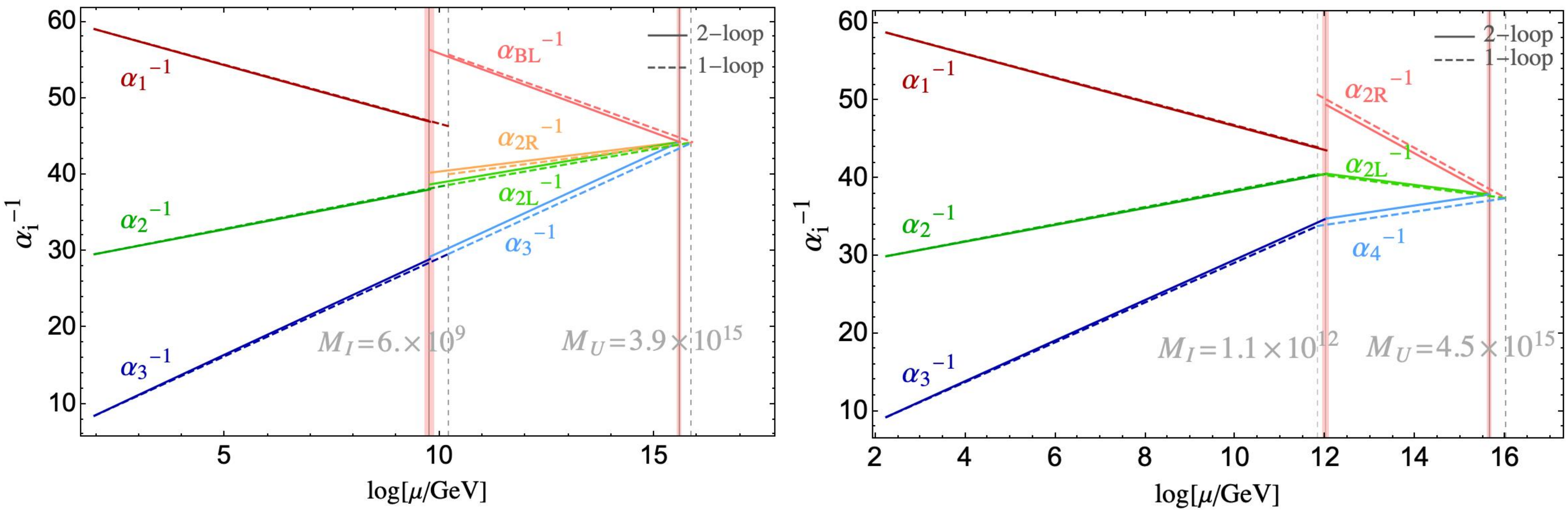
$\tau_P = \alpha_U^2 m_P^5 / M_{X,Y}^4.$

In conclusion: simple GUTs are problematic and something is needed to cure them.

One option is to consider a larger symmetry group, eg SO(10) with intermediate breaking. For example, denoting the SM group as $\mathcal{G}_{321} = \mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$, we have the chains

$$\begin{aligned} \mathrm{PS}: &\quad \mathrm{SO}(10)|_{M_U} \Rightarrow \mathrm{SU}(4)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}|_{M_I} \Rightarrow \mathcal{G}_{321}|_{M_Z} \Rightarrow \mathcal{G}_{31}\,, \\ \mathrm{LR}: &\quad \mathrm{SO}(10)|_{M_U} \Rightarrow \mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_{\mathrm{B-L}}|_{M_I} \Rightarrow \mathcal{G}_{321}|_{M_Z} \Rightarrow \mathcal{G}_{31} \end{aligned} \quad (.1)$$

Some large Higgs representations are needed at the GUT and intermediate scales M_U, M_I. At M_I, the extra scalar+vector bosons that are present need to be included in the RGE; this is conveniently done using the so-called threshold corrections in the RGEs.
The slopes of RGEs of the gauge couplings are modified and one obtains a running:
Hence, there is indeed unification of the 3 gauge couplings in the two schemes at reasonably interesting (for neutrino physics) intermediate and GUT scales.



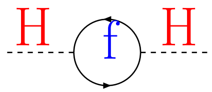
3. The hierarchy problem and physics beyond the SM

There is also a problem of theoretical consistency when one extrapolates to high scales: this is the hierarchy problem which is also connected with the criterium of naturalness.

- The Higgs should have mass of order of the W, Z boson masses i.e. $\mathcal{O}(100\,\text{GeV})$:

– it is required by mathematical consistency: perturbativity, triviality, unitarity, ...
– it is more natural to solve a problem at 100 GeV with an "object" of 100 GeV mass.

- And we should include all quantum corrections to the Higgs mass; in particular, as the contribution is divergent, should use a cut off Λ which naturally is M_G or M_P, $\Rightarrow$ the contributions to M_H are of order M_P while they should be of order $M_{W,Z}$..

$\Delta M_H^2 \equiv$ 

$\propto \Lambda^2 \approx (10^{18}\text{ GeV})^2.$

There is an enormous hierarchy between the two scales $M_P \gg M_{W,Z}$: not natural. But maybe this Λ^2 term is cancelled by higher orders ones? This is very unnatural.

- The problem might be that there is no symmetry to protect M_H from high scales?

– the gauge symmetry protects the photon for having a mass (vanishing corrections);
– the left–right or chiral symmetry protects the fermion masses (small/log corrections).

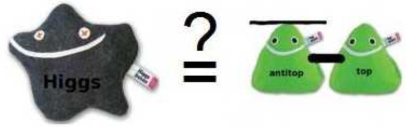
We will see that in some details later....

Three main avenues to solve the hierarchy problem of the SM.

I) The Higgs boson is not an elementary spin-0 particle, but it is composite.

The Higgs boson is the only known fundamental particle with spin equal to zero: if the Higgs is not a fundamental state ⇒ the hierarchy problem disappears.

• Only the Higgs can be composed of two fermions:
one can have bound states or condensates such as:
spin : $s=\frac{1}{2} \oplus s=\frac{1}{2}=0 \Rightarrow$ scalar (just like $\pi^0, \pi^\pm$).
But the constituent particle should be rather massive.
Only option in the SM: H is top-antitop condensate.



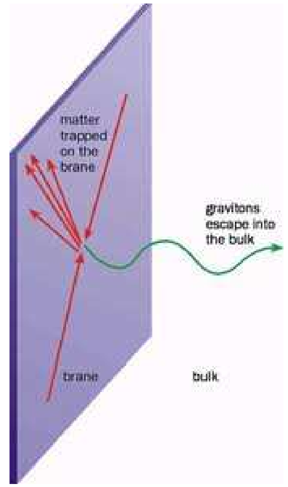
• An even more radical option is Technicolor:
all SM particles (fermions+bosons) are composite
(there is another layer in the onion/Nature);
≡ QCD but at a much higher scale $\Lambda = 1$ TeV.
⇒ The Higgs is a bound state of two techni-fermions.

• In both cases ⇒ the Higgs properties ≠ of those of the Standard Model H.
Both theories are of strong interactions ⇒ very constrained by various experiments.
In particular LHC states that the compositeness scale should be $\Lambda \gtrsim$ a few TeV.
Again, some unnaturalness is entering the description of Nature; very unlikely?

Three main avenues to solve the hierarchy problem of the SM.

II) Additional space-time dimensions at the scale of a few TeV?

We could have a fifth space-time dimension
in which at least the spin=2 gravitons propagate.
Gravity: the effective scale is $M_P^{\rm eff} \approx \Lambda \approx$ TeV,
and not $M_P = 10^{18}$GeV; gravity is now in the game!
Several possibilities to realize this extraD scenario:
large, warped, universal extra dimensions, ...



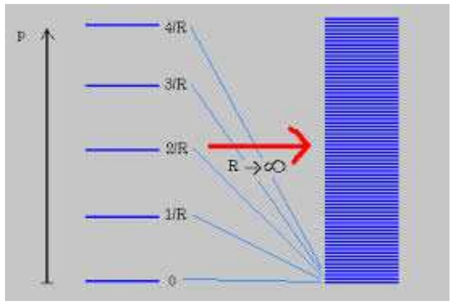
Enormous possible impact on particle physics
with also solutions to other SM problems
(unification, fermion masses, dark matter,..).

- But we still need EW symmetry breaking:

– use the same Higgs mechanism as in the SM,
– but also possibility of Higgs-less mechanism.

- The known particles are the zero modes of

– an infinite tower of Kaluza–Klein excitations,
– new heavy partners of the fermions/bosons.



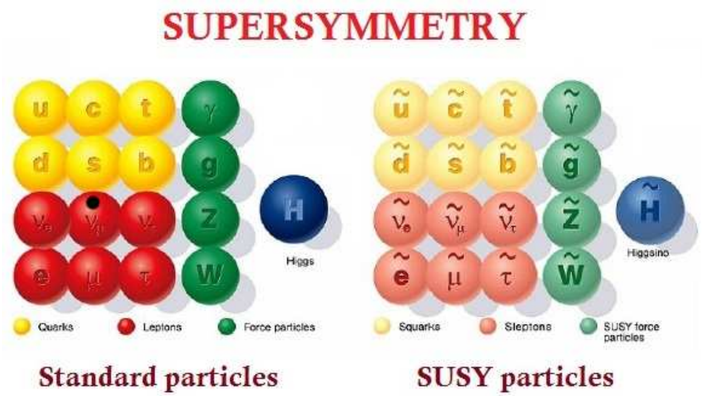
Plenty of new exotic particles to discover and study at the LHC and beyond.
However, again, LHC has searched for all these particles/interactions with no success:
the scale of this new physics is beyond few TeV or more ⇒ Naturalness is back again!

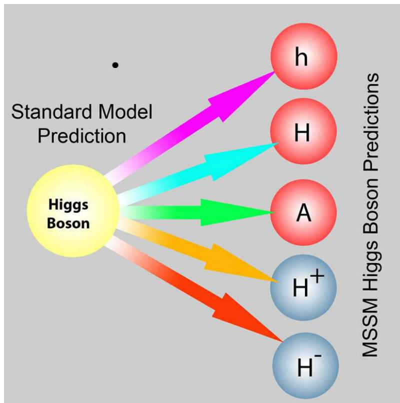
Three main avenues to solve the hierarchy problem of the SM.

III) Supersymmetric theories (SUSY) or how to double the world.

Supersymmetry is considered to be the most attractive extension of the SM, and there are many very good reasons for that:

- relates the $s=\frac{1}{2}$ fermions to s=0,1 bosons;
- relates internal and space-time symmetries;
- if SUSY is made local, we recover gravity;
- is naturally present in Superstrings theory.
- To each particle ⇒ a superparticle (sfermions of s=0 and gauginos of s=$\frac{1}{2}$).
- Enlarged Higgs sector: h,H,A,H^+, H^- (two doublets of scalar Higgs fields).
- Cancels the quadratic Λ^2 divergences and fixes the hierarchy problem;
- $\mu^2 < 0$ naturally via quantum effects; (radiative electroweak symmetry breaking);
- leads to unification of gauge couplings;
- has the ideal candidate for Dark Matter.



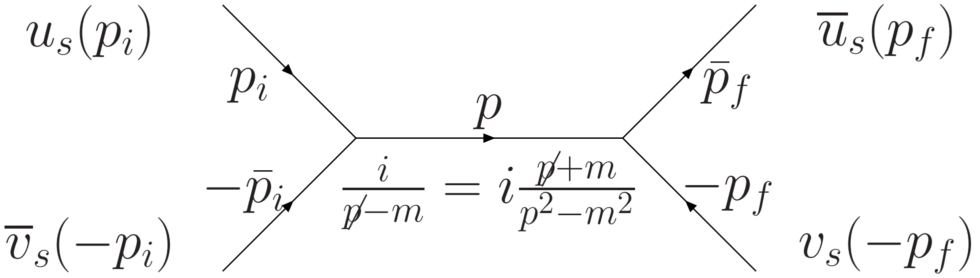


And it has only one drawback: no supersymmetric particle has been found at LHC!

4. Some elements for calculations in high-energy physics

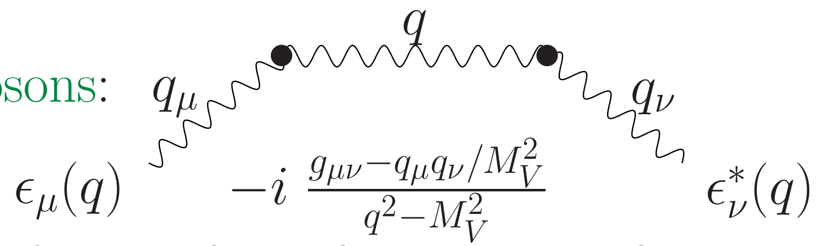
A. Feynman rules for vertices and propagators

Fermions:



$$\Sigma_s\, u_s(p)\, \bar{u}_s(p) = \not p + m$$
$$\Sigma_s\, v_s(p)\, \bar{v}_s(p) = \not p - m$$

Gauge bosons:



Photon: discard $q^\mu q^\nu$ everywhere

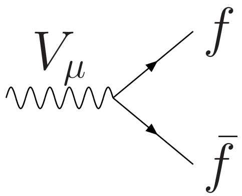
$$\Sigma_{\rm pol}\epsilon_\mu\epsilon^*_\nu = -(g_{\mu\nu} - q_\mu q_\mu / M_V^2)$$

Photon transversality: $\epsilon_\mu \cdot q^\mu = 0$

Higgs bosons:

$i/(p^2 - M_H^2)$

Interaction vertices:



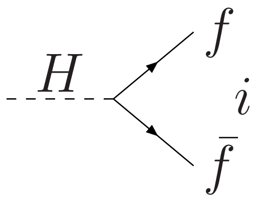
$$-ie\gamma_\mu(v_f - a_f\gamma_5)$$

$Z\ :\ v_f = (2I_f^3 - 4e_f s_W^2)/(4s_W c_W)$

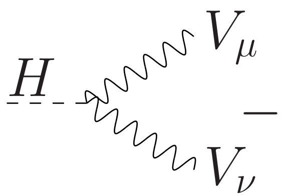
$\quad\ a_f = 2I_f^3/(4s_W c_W)$

$W\ :\ v_f = a_f = 1/(2\sqrt{2}s_W)$

$\gamma\ :\ v_f = e_f\ ,\ a_f = 0$



$i\frac{m_f}{v}$



$-\frac{2iM_V^2}{v}\, g_{\mu\nu}$

B. Diracology: contractions and traces of γ matrices

Basic relations : $\{\gamma_\mu\,,\,\gamma_\nu\} = \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$ and $\not p = p_\mu\gamma^\mu$

$$\gamma_5 = \frac{i}{4!}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = i\gamma^0\gamma^1\gamma^2\gamma^3 \text{ and } \{\gamma_\mu, \gamma_5\} = 0$$

$$\mathrm{Tr}(\mathbf{1}) = 4\,,\ \mathrm{Tr}(\gamma_\mu) = 0\,,\quad \mathrm{Tr}(\gamma_5) = 0\,,\ \mathrm{Tr}(A_1A_2\cdots A_N) = \mathrm{Tr}(A_2\cdots A_NA_1)$$

Contractions :

$$\begin{aligned} \gamma_\mu\gamma^\nu &= 2g^\nu_\mu - \gamma^\nu\gamma_\mu \Rightarrow \gamma^\mu\gamma_\mu = \delta^\mu_\mu = 4 \\ \gamma^\mu\gamma_\nu\gamma_\mu &= \gamma^\mu(2g_{\mu\nu} - \gamma_\mu\gamma_\nu) = 2\gamma_\nu - 4\gamma_\nu = -2\gamma_\nu \\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu &= (2g^{\mu\nu} - \gamma^\nu\gamma^\mu)(2g^\rho_\mu - \gamma_\mu\gamma^\rho) \\ &= 4g^{\nu\rho} - 2\gamma^\nu\gamma^\rho - 2\gamma^\nu\gamma^\rho + 4\gamma^\nu\gamma^\rho = 4g^{\nu\rho} \end{aligned}$$

Traces : $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = \mathrm{Tr}(2g^{\mu\nu} - \gamma^\mu\gamma^\nu) = 2g^{\mu\nu}\mathrm{Tr}(1) - \mathrm{Tr}(\gamma^\mu\gamma^\nu) \Rightarrow \mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu}$

$$\begin{aligned} &\mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5\gamma^5) = (-1)\,\mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^5\gamma^{\mu_{2n+1}}\gamma^5) \\ &= (-1)^{2n+1}\,\mathrm{Tr}(\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}\gamma^5) = -\,\mathrm{Tr}(\gamma^5\gamma^5\gamma^{\mu_1}\cdots\gamma^{\mu_{2n+1}}) = 0 \end{aligned}$$

$$\begin{aligned} &\mathrm{Tr}(\gamma^\mu\gamma_5) = \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\sigma\gamma_5) = \mathrm{Tr}(\gamma^{\mu_1}\cdots\gamma^{\mu_n}\gamma^5) = 0 \\ &\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = \frac{1}{4}\mathrm{Tr}(\gamma^\alpha\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5) = (1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma_5\gamma^\alpha) \\ &= -(1/4)\,\mathrm{Tr}(\gamma_\alpha\gamma^\mu\gamma^\nu\gamma^\alpha\gamma_5) = -\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma_5) = 0 \end{aligned}$$

Proof that :

$$\begin{aligned} &\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma}) \\ &\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = -4i\epsilon^{\mu\nu\sigma\rho} \end{aligned}$$

C. Cross sections and decay widths

The differential cross section for a $2 \to n$ process $ab \to f_1 \cdots f_n$ can be written as

$$\mathrm{d}\sigma = \frac{|M(ab \to f_1..f_n)|^2}{4[(p_a \cdot p_b)^2 - m_a^2 m_b^2]^{1/2}} \left(\Pi_n \frac{d^3 p_f}{(2\pi)^3 E_f} \right) (2\pi)^4 \delta^4(\Sigma p_i - \Sigma p_f)\, S$$

- In $|M|^2$: average (sum) on all degrees of freedom of initial (final) particles.
- There is a symmetry factor $S = 1/n!$ for n identical initial/final particles.
- Flux factor is $2(p_a + p_b)^2 = 2s$ for $2 \to n$ ($m_a = m_b = 0$) and $2M$ for $1 \to n$ (mass M).

Calculation of phase–space for a two–body process $a + b \to f_1 + f_2$:

$$\mathrm{dPS2} = \frac{1}{16\pi^2} \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \delta^4(p_a + p_b - p_1 - p_2)$$

$$\int \frac{d^3 p_2}{E_2} \delta^4(p_a + p_b - p_1 - p_2) = \frac{1}{E_2} \delta(E_a + E_b - E_1 - E_2) \ \text{ with } \ \begin{array}{l} |\vec{p}_2| = |\vec{p}_a + \vec{p}_b - \vec{p}_2|\,,\ E_2^2 = |\vec{p}_2|^2 + m_2^2 \\ \mathrm{d}^3 p_1 = \mathrm{d}\Omega |p_1|^2 \mathrm{d}|p_1|\,,\ E_1^2 = |\vec{p}_1|^2 + m_1^2 \end{array}$$

In the c.m. frame we have $w = E_a + E_b$, $w' = E_1 + E_2 = (m_2^2 + p^2)^{\frac{1}{2}} (m_1^2 + p^2)^{\frac{1}{2}}$:

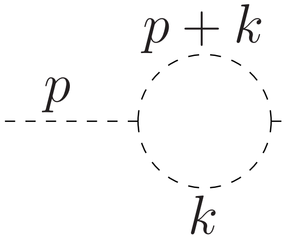
$$\frac{\mathrm{d}w'}{\mathrm{d}p} = p \left(\frac{1}{E_1} + \frac{1}{E_2} \right) \Rightarrow \mathrm{d}w' = p\mathrm{d}p \left(\frac{1}{E_1} + \frac{1}{E_2} \right) = E_1 \mathrm{d}E_1 \frac{E_1 + E_2}{E_1 E_2}$$

$$= \frac{\mathrm{d}\Omega}{16\pi^2} |p| \frac{E_1 \mathrm{d}E_1}{E_1 E_2} \delta(w - w') = \frac{\mathrm{d}\Omega}{16\pi^2} |p| \frac{\mathrm{d}w'}{w'} \delta(w - w') \stackrel{\text{after integral}}{\Rightarrow} \frac{\mathrm{d}\Omega}{16\pi^2} \frac{|p|}{\sqrt{s}}$$

Differential cross section : $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \dfrac{1}{2\mathrm{s}} \times \Sigma |\mathrm{M}(\mathrm{ab} \to \mathrm{f_1 f_2})|^2 \times \dfrac{1}{16\pi^2} \left(\dfrac{|\mathrm{p}|}{\sqrt{\mathrm{s}}} \right) \times \mathrm{S}$

Note the useful relation: $|p| = \frac{1}{2}\sqrt{s}\lambda = \frac{1}{2}\sqrt{s}[1 - m_1^2/s - m_2^2/s)^2 - 4m_1^2 m_2^2/s^2]^{\frac{1}{2}}$.

D. Calculation of loop integrals



$= -i\Pi(p^2)$

Measure of loop integral over the internal momentum: $\int \mathrm{d}^4k/(2\pi)^4$. (For fermion loops: take trace and factor (-1) for Fermi statistics).

$$-i\Pi = (ig)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{i}{(p+k)^2 - m^2} \frac{i}{k^2 - m^2} = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{(p+k)^2 - m^2} \frac{1}{k^2 - m^2}$$

- Symmetrize the integrand using the relation $1/ab = \int_0^1 \mathrm{d}x/[a + (b-a)x]^2$

$$\Pi(p^2) = ig^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int_0^1 \mathrm{d}x \frac{1}{(k^2 + 2pkx + p^2x - m^2)^2}$$

- Perform shift of variable $k \to k' = k + px$ (then, integrand becomes k^2 symmetric)

$$(k^2 + 2pkx + p^2x - m^2)^2 \longrightarrow (k^2 + p^2x(1-x) - m^2)^2$$

- Make a Wick rotation $k_0 \to ik_0$ to go to the Euclidean space $(k^2 \to -k^2)$

$$(k^2 + p^2x(1-x) - m^2)^2 \longrightarrow (k^2 - p^2x(1-x) + m^2)^2$$

- Move to polar coordinates for d^4k: $\int_{-\infty}^{+\infty} d^4k\, F(k^2) = \pi^2 \int_0^\infty dk^2\, k^2\, F(k^2)$

$$\Pi(p^2) = -\frac{g^2}{16\pi^2} \int_0^1 \mathrm{dx} \int_0^\infty y\mathrm{d}y \frac{1}{(y - p^2x(1-x) + m^2)^2}$$

- Perform the integrals over the variables y and x. If the integral is divergent, use cut-off at the maximal energy Λ ($\int_0^{\Lambda^2} \mathrm{d}k^2$). Eventually, use the on–shell mass relation $p^2 = m^2$.

5. Connection between divergences and symmetries

Let us first recall a few elements of scalar/fermionic Lagrangians and interactions.

- Take the QED Lagrangian for a fermion of charge e and of mass m:

$$\mathcal{L}_{\rm QED} = -(1/4)F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu\partial_\mu\psi - m\overline{\psi}\psi + e\overline{\psi}\gamma^\mu A_\mu\psi \tag{2}$$

with A_μ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the usual electromagnetic field and tensor. U(1) gauge invariance tells us that there is no $A_\mu A^\mu$ term so that the photon is massless.

- Let us add a scalar charged field ϕ into the theory with a Lagrangian

$$\mathcal{L}_\phi = |\partial_\mu\phi|^2 - m_S^2|\phi|^2 + \lambda(\phi^+\phi)^2 \tag{3}$$

leading to symmetry breaking, $\langle\phi\rangle = \frac{v}{\sqrt{2}}$; write $\phi = (H+v)/\sqrt{2}$ with H being physical.

- And couple this field to a fermion f (à la Yukawa) to give it a mass

$$\mathcal{L}_f = -\lambda_f.\,\overline{\psi}\psi\phi \overset{\rm SSB}{\Longrightarrow} m_f = \lambda_f v/\sqrt{2} \tag{4}$$

- Let us now introduce two scalar fields ϕ_1 and ϕ_2 to the theory:

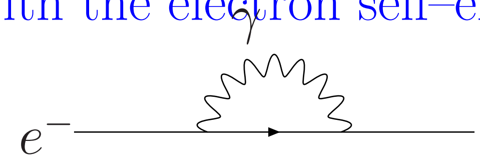
$$\mathcal{L}_{\rm kin} = |\partial_\mu\phi_1|^2 + |\partial_\mu\phi_2|^2 - m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2 \tag{5}$$

they will have a coupling to the scalar field ϕ after spontaneous symmetry breaking:

$$\mathcal{L}_S = \lambda_S|\phi|^2(|\phi_1|^2 + |\phi_2|^2) + 2v\lambda_S\phi(|\phi_1|^2 + |\phi_2|^2) \tag{6}$$

Plus, eventually, bilinear terms in $\phi_1\phi_2$ that we take zero for simplicity.

Let us see what happens with the electron self–energy



$$\equiv -i\Sigma_e(\not p)$$

$$-i\Sigma_e(p) = \int \frac{d^4k}{(2\pi)^4}(-ie\gamma_\mu)\frac{i}{\not p + \not k - m}(-ie\gamma_\nu)\ \frac{-ig^{\mu\nu}}{k^2}$$

Perform the Dirac algebra $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$, etc...; propagator symmetrization $1/ab \rightarrow$ .. and change of variable, Euclidean space with Wick rotation $k_0 \rightarrow ik_0$, $k^2 \rightarrow -k^2$; integrate over momentum (symmetric integrand and regularization) $\int_{-\infty}^{+\infty} d^4k \rightarrow \int_0^{\Lambda^2} dk^2$; we get

$$\Rightarrow \delta m_e = \Sigma_e(\not p)|_{p=m} = \frac{m_e e^2}{8\pi^2}\int_0^1 dx(1+x)\int_0^{\Lambda^2} dyy[y + m_e^2 x^2]^{-2} = \frac{3\alpha}{4\pi} m_e \log\frac{\Lambda^2}{m_e^2} + \cdots (7)$$

As QED is valid at least up to $M_{\rm GUT}$ and even M_P, we can set $\Lambda = M_P \sim 10^{19}$ GeV.
The ultraviolet divergence (UV, at large k^2) is logarithmic and $\propto m_e \Rightarrow$ small ($\delta m_e \sim 0.2$).
(but in principle $\Lambda = \infty \Rightarrow$ renormalizable theory: $m_e^{\rm phys} = m_e^{\rm bare} + \delta m_e \Rightarrow$ finite).

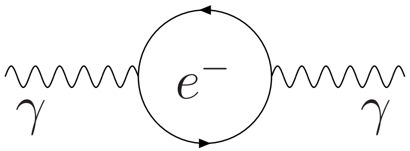
At the more fundamental level: the correction is small because of chiral symmetry:
if $m_e = 0$, $\mathcal{L}_{\rm QED}$ is invariant under the chiral transformation:

$$\psi_L \rightarrow e^{i\theta_L}\psi_L \text{ and } \psi_R \rightarrow e^{i\theta_R}\psi_R \text{ with } \psi_{L,R} = 1/2(1 \mp \gamma_5)\psi.$$

But m_e breaks the chiral symmetry $\Rightarrow$ the correction is proportional to the mass.

$\Rightarrow$ Symmetry $\equiv$ protection for the mass.

Let also see what happens to the photon self–energy:



$$\equiv -i\Pi^{\gamma\gamma}_{\mu\nu}(p^2)$$

$$-i\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = \int \frac{d^4k}{(2\pi)^4}(-1)\mathrm{Tr}(-ie\gamma_\mu)\frac{i}{\not{k}-m}(-ie\gamma_\nu)\frac{i}{\not{p}+\not{k}-m}$$

We already started the calculation before and reached the level where we had

$$\Pi^{\gamma\gamma}_{\mu\nu}(p^2) = \frac{e^2}{2\pi^2}\int_0^1 \mathrm{d}x \int_0^\infty ydy \frac{[\frac{1}{2}y^2 + m^2 - x(1-x)p^2]g_{\mu\nu} + 2x(1-x)[g_{\mu\nu}p^2 - p_\mu p_\nu]}{[y+m^2-p^2x(1-x)]^2}$$

Using the usual tricks and applying a cut–off Λ for the integral over k^2, one gets

$$\delta m_\gamma = \frac{1}{4}g^{\mu\nu}\Pi^{\gamma\gamma}_{\mu\nu}(0) = \frac{e^2}{16\pi^2}\int_0^1 \mathrm{d}x \int_0^{\Lambda^2} dy \frac{y^2+2m^2y}{(y+m^2)^2} \sim \frac{\alpha}{4\pi}\Lambda^2 \quad !?$$

Divergence, while we must have $m_\gamma \equiv 0$ at all orders because of gauge invariance.

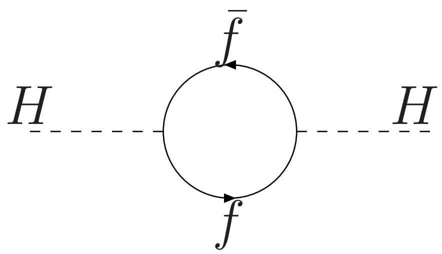
Problem: the cut–off Λ^2 violates the QED gauge invariance and should not be used.

Solution: dimensional regularization, work in n=4$-\epsilon$ dimensions, preserves invariance:

- Internal momenta in n dim: $\int d^nk/(2\pi)^n$.. $\qquad A = \int [k^2-m^2]^{-1} \sim am^2/\epsilon + \cdots$
- Diracology: $\mathrm{Tr}(\mathrm{I}) = n$, $\gamma_\mu\gamma^\mu = n\mathrm{I}$, $g^\mu_\mu = n$,.. $\quad \Rightarrow \quad m^2\partial A/\partial 2m^2 \sim am^2/\epsilon$
- UV divergence: poles in $\frac{1}{n-4} = \frac{1}{\epsilon}$ with $\epsilon \to 0$. $\qquad \delta m_\gamma = 0$ at all orders.

Another example of a protection for a mass from higher orders by a symmetry...

Let us now come to the Higgs boson self–energy: fermionic contributions:



$$-i\Sigma_H(p^2) = N_f \int \frac{d^4k}{(2\pi)^4} (-1)\mathrm{Tr}\,(-\frac{i\lambda_f}{\sqrt{2}})\frac{i}{\not{k} - m}(-\frac{i\lambda_f}{\sqrt{2}})\frac{i}{\not{p} + \not{k} - m} \tag{8}$$

Simplification: $p^2 = M_H^2 = 0$ $(m_f \gg M_H)$; using a cut–off Λ for k^2 integral one has:

$$\Sigma_H(p^2 = 0) = 4N_f \left(\frac{\lambda_f}{\sqrt{2}}\right)^2 \frac{1}{16\pi^2} \int_0^1 dx \int_0^{\Lambda^2} dy\, \frac{y(-y + m_f^2)}{(y + m_f^2)^2} \tag{9}$$

$$\delta M_H^2 = N_f \frac{\lambda_f^2}{8\pi^2} \left[-\Lambda^2 + 6m_f^2 \log\frac{\Lambda}{m_f} - 2m_f^2 \right] + \mathcal{O}(1/\Lambda^2) \tag{10}$$

We have thus a quadratic divergence, $\delta M_H^2 \sim \Lambda^2$.

Divergence independent of M_H, stays if $M_H \to 0$ (does not increase symmetry of $\mathcal{L}_{\mathrm{SM}}$).

Cut–off does not break symmetry and problem unsolved with dimensional regularization.

If we fix the cut–off Λ to M_{GUT} or to $M_P \Rightarrow$ Higgs mass $M_H \sim 10^{14}$ to 10^{17} GeV.

This is the hierarchy problem:

the Higgs mass prefers to be close to the very high scale rather to the input value $\mathcal{O}(v)$.

But we want a light Higgs ($\lesssim 1$ TeV) for unitarity, perturbativity, etc... reasons.

We need thus to arrange that $M_H^2|^{\rm physical} = M_H^2|^{\rm bare} + \Delta M_H^2 + $ countreterm,

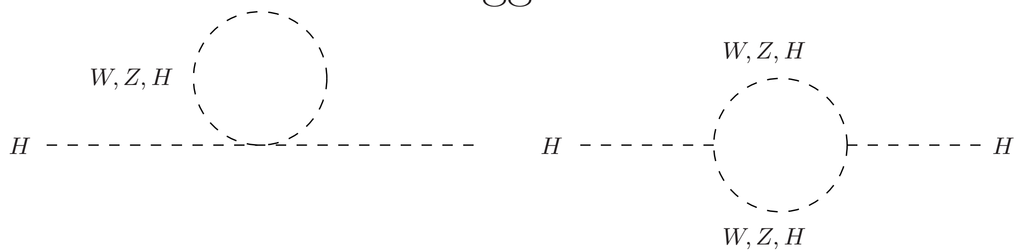
and adjust this counterterm (for renormalisation) with a precision of 10^{30} (30 digits)!

which looks highly unnatural $\Rightarrow$ this is the naturalness problem.

In a complete theory, no problem formally: we adjust bare M_H and counterterm which are infinite, to have a physical finite mass. This works for the log divergence of m_e in QED.

However, we want to give a physical meaning to the cut–off Λ (the scale at which the theory is not valid) and the logarithmic and quadratic divergences are of different nature.

In the Standard Model: besides the fermions, there are also contributions to M_H from the massive gauge bosons and from the Higgs boson itself:



The total contribution of fermions and bosons in the SM at the one–loop level is

$$\Delta M_H^2 \propto [3(M_W^2 + M_Z^2 + M_H^2)/4 - \sum m_f^2](\Lambda^2/M_W^2) \qquad (11)$$

Adjust unknown M_H for Λ^2 terms to disappear (a prediction for $M_H \sim 200$ GeV)?

Does not work at 2-loop or higher orders; problem of quadratic divergences to M_H stays.

There is no symmetry which protects M_H in the SM.

6. The supersymmetric solutions

Imagine now that one can add a new contribution of (2=L+R) additional scalars:

$$\Delta M_H^2 = \Sigma_H(p^2) = (i) N_S \int \frac{d^4k}{(2\pi)^4} (i\lambda_S) \left[\frac{i}{k^2 - m_1^2} + \frac{i}{k^2 - m_2^2} \right]$$
$$+ N_S (i\lambda_S v)^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{i}{k^2 - m_1^2} \frac{i}{(k+p)^2 - m_1^2} + m_1 \leftrightarrow m_2 \right] \quad (12)$$

$$\Delta M_H^2 = \frac{\lambda_S N_S}{16\pi^2} \left[-2\Lambda^2 + 2m_1^2 \log(\Lambda/m_1) + 2m_2^2 \log(\Lambda/m_2) \right]$$
$$- \frac{\lambda_S^2 v^2 N_S}{16\pi^2} \left[-2 + 2\log(\Lambda/m_1) + 2\log(\Lambda/m_2) \right] + \mathcal{O}\left(1/\Lambda^2\right) \quad (13)$$

Again, quadratic divergences appear. But let us now assume (as in supersymmetry) that:

- the couplings of the two scalars are related to the fermion couplings: $\lambda_f^2 = -\lambda_S$;
- the multiplicative factors are the same: $N_S = N_f$ (but there are two scalars);
- to make the situation simpler, the scalars have the same mass: $m_1 = m_2 = m_S$.

$$\text{f+}\tilde{\text{f}}\text{ cont.} \Rightarrow \Delta M_H^2\big|^{\text{tot}} = \frac{\lambda_f^2 N_f}{4\pi^2} \left[(m_f^2 - m_S^2) \log(\Lambda/m_S) + 3m_f^2 \log(m_S/m_f) \right] \quad (14)$$

The quadratic divergences have disappeared in the sum. The log divergence still there, but even with $\Lambda = M_P$, the contribution is small. It disappears also if we have $m_S = m_f$.

$\Rightarrow$ Symmetry fermions–scalars $\Rightarrow$ no divergence in Λ^2: M_H is protected!

Let us see what happens for the problem of gauge coupling unification:

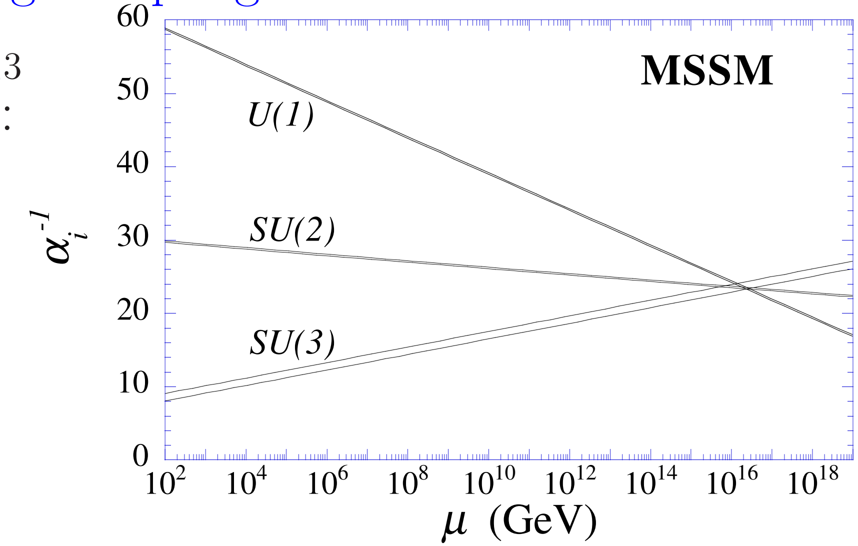
Superparticles contribute to the running of $g_{1,2,3}$ (ex: contribution of $b_N = -2N/3$ for gauginos):

$b_3^{\rm SUSY} = (11 - 2\frac{6}{3} - 2 - \frac{12}{6} = 7 - 5 = 3$

$b_2^{\rm SUSY} = (\frac{22}{3} - 4 - \frac{1}{6} + (-\frac{4}{3} - \frac{12}{6} - \frac{2}{3} - \frac{1}{6}) = -1$

$b_1^{\rm SUSY} = -3\frac{10}{5}(f + \tilde{f}) - \frac{3}{5}4\frac{1}{4}(H + \tilde{h}) = -\frac{33}{5}$

We get after all calculations: $B_{\rm th} = \frac{5}{7} = 0.74$, to be compared with experiment: $B_{\rm exp} \simeq 0.72$. small discrepancy could be due to errors on α_i.



Alternative view: the running couplings meet at a single point $M_U \sim 2 \cdot 10^{16}$ GeV, a value that is obtained from $\log(M_U/M_Z) = \frac{10\pi}{28}\left[\alpha_1^{-1}(M_Z) - \alpha_2^{-1}(M_Z)\right] \simeq 33.1$

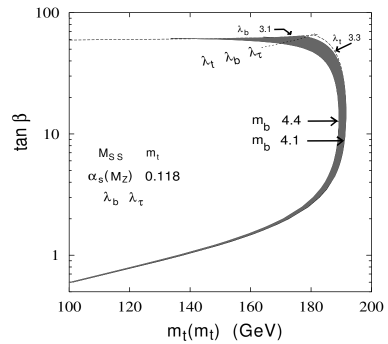
One also needs the small threshold corrections and two-loop corrections to be included.

Note also that the larger M_U value ($=20\ M_U^{\rm "SM"}$) is good to prevent proton decay.

In SUSY–GUTs, there also additional colored Higgs/higgsino exchange in the diagrams:

$\Rightarrow \tau_P^{\rm SUSY} \propto 1/M_{\rm GUT}^4 > 10^{33}$ years. To be observed soon?

One can also unify (3d gen) Yukawa couplings at $M_{\rm GUT}$:



$$\frac{dh_t}{d\log Q} = \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$

$$\frac{dh_b}{d\log Q} = \frac{h_b}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right],$$

$$\frac{dh_\tau}{d\log Q} = \frac{h_\tau}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right].$$

And also what supersymmetry does for the problem of dark matter:

To eliminate some terms that violate B and L numbers and could lead to proton decay, one introduces a discrete and multiplicative symmetry called R-parity or R_p:

$$R_p = (-1)^{2s+3B+L} \qquad \begin{array}{l} R = +1 \text{ for all ordinary SM particles} \\ R = -1 \text{ for all the SUSY particles} \end{array}$$

The consequences of R_p conservation are very important for SUSY phenomenology:

– the SUSY particles will always be produced in pairs;

– the SUSY particles decay into an odd number of SUSY particles;

– the lightest SUSY particle (LSP) cannot decay; it is absolutely stable.

• Experimental constraints show that this LSP must be quite heavy, $M_{\rm LSP} \gtrsim 1$–50 GeV, and thus is non–relativistic (it can thus be a cold dark matter candidate).

• In most areas of the SUSY parameter space, this LSP is electrically neutral and interacts very weakly with other particles.

• For some values of its mass and couplings, this LSP can have a relic abundance which is within the range given by experimental measurements.

The LSP is an ideal candidate for dark matter!

These are at least three good reasons (plus the link to gravity) to believe in supersymmetry.

We will discuss this theory briefly (and mostly its phenomenology) in the next two lectures.