Gravitation und Kosmologie

IV: Quantengravitation und Quantenkosmologie

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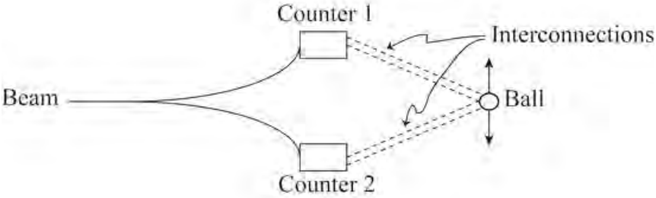
Contents

Richard Feynman 1957:

. . . if you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment. . . . It may turn out, since we've never done an experiment at this level, that it's not possible . . . that there is something the matter with our quantum mechanics when we have too much *action* in the system, or too much mass—or something. But that is the only way I can see which would keep you from the necessity of quantizing the gravitational field. It's a way that I don't want to propose. . . .



Why quantum gravity?

- Superposition principle
- Unification of all interactions
- Singularity theorems
    - Black holes
    - ‘Big Bang’
- Problem of time
- Absence of viable alternatives

Max Planck, Über irreversible Strahlungsvorgänge, *Sitzungsberichte der königlich-preußischen Akademie der Wissenschaften zu Berlin, phys.-math. Klasse*, Seiten 440–80 (1899)

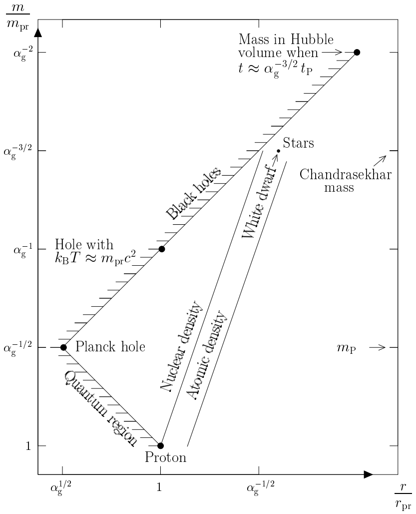
Planck units

$$
\begin{aligned}
l_{\mathrm{P}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ \mathrm{cm} \\
t_{\mathrm{P}} &= \frac{l_{\mathrm{P}}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ \mathrm{s} \\
m_{\mathrm{P}} &= \frac{\hbar}{l_{\mathrm{P}} c} = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ \mathrm{g} \approx 1.22 \times 10^{19}\ \mathrm{GeV}/c^2
\end{aligned}
$$

Max Planck (1899):

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

Structures in the Universe



$$\alpha_{\rm g} = \frac{Gm_{\rm pr}^2}{\hbar c} = \left(\frac{m_{\rm pr}}{m_{\rm P}}\right)^2 \approx 5.91 \times 10^{-39}$$

Main approaches to quantum gravity

No question about quantum gravity is more difficult than the question, "What is the question?"
(John Wheeler 1984)

- Quantum general relativity
  - Covariant approaches (perturbation theory, path integrals including spin foams, asymptotic safety, . . .)
  - Canonical approaches (geometrodynamics, connection dynamics, loop dynamics, . . .)
- String theory
- Fundamental discrete approaches
  (quantum topology, causal sets, group field theory, . . .);
  have partially grown out of the other approaches

Linearized quantum gravity

Pioneered by Matvei Bronstein (1936)

Perturbation theory:

$$\boxed{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}}$$

- $\bar{g}_{\mu\nu}$: classical background
- Perturbation theory with respect to $h_{\mu\nu}$ (Feynman rules)
- 'Particle' of quantum gravity: graviton (massless[1] spin-2 particle)
- perturbative non-renormalizability

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[1] $m_{\mathrm{g}} \lesssim 10^{-29}$ eV, cf. $m_\gamma \leq 10^{-18}$ eV

Example: Transition rate from the $3d$ level to the $1s$ level in the hydrogen atom due to the emission of a graviton:

$$\Gamma_{\mathrm{g}} = \frac{G m_{\mathrm{e}}^3 c \alpha^6}{360 \hbar^2} \approx 5.7 \times 10^{-40}\ \mathrm{s}^{-1}$$

This corresponds to a life-time of

$$\tau_{\mathrm{g}} \approx 5.6 \times 10^{31}\ \mathrm{years}\ .$$

Gravitons from the early Universe

Gravitons are created out of the vacuum during an inflationary phase of the early Universe ($\sim 10^{-34}$ s after the big bang); the quantized gravitational mode functions $h_{\mathbf{k}}$ in de Sitter space obey

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{4}{k^3} \left(t_{\mathrm{P}} H\right)^2 \delta(\mathbf{k} + \mathbf{k}') \equiv P_{\mathrm{t}} \; \delta(\mathbf{k} + \mathbf{k}')$$

Power spectrum:

$$\Delta_{\mathrm{t}}^2(k) := \frac{k^3}{2\pi^2} P_{\mathrm{t}} = \frac{2}{\pi^2} \left(t_{\mathrm{P}} H\right)^2$$

(H is evaluated at Hubble-horizon exit, i.e. at $|k\eta| = 1$)

The BICEP2 experiment

“Background Imaging of Cosmic Extragalactic Polarization”

Figure credit: BICEP2 Collaboration

Most likely, the observed signal comes from a dust foreground (arXiv:1502.00612)

Quantum origin of perturbations

Power spectrum for the scalar modes (inflaton plus metric):

$$\Delta_{\mathrm{s}}^2(k) = \frac{1}{8\pi^2}\,(t_{\mathrm{P}}H)^2\,\epsilon^{-1} \approx 2\times 10^{-9}$$

ϵ: slow-roll parameter

Tensor-to-scalar ratio: $r := \frac{\Delta_{\mathrm{t}}^2}{\Delta_{\mathrm{s}}^2} = 16\epsilon$

The CMB spectrum from the PLANCK mission

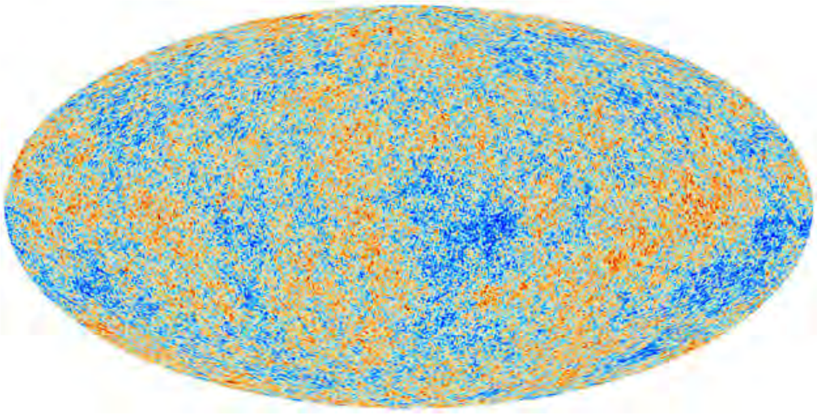


Figure credit: ESA/PLANCK Collaboration

First observational test of quantum gravity

- Within the inflationary scenario, the observed CMB fluctuations can only be understood from quantized metric plus scalar field modes.
- This is an indirect test of linearized quantum gravity.
- The observation of primordial B-modes would be an indirect confirmation of the existence of gravitons.
- The difference in the duration of inflation between the ‘cold spots’ and the ‘hot spots’ in the CMB spectrum is only of the order of the Planck time.

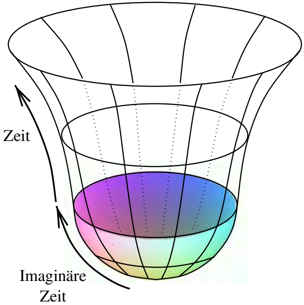
Path integrals

$$\boxed{Z[g] = \int \mathcal{D}g_{\mu\nu}(x)\ \mathrm{e}^{\mathrm{i}S[g_{\mu\nu}(x)]/\hbar}}$$

In addition: sum over all topologies?

- Euclidean path integrals
  (e.g. for Hartle–Hawking proposal or Regge calculus)
- Lorentzian path integrals
  (e.g. for dynamical triangulation)
- Asymptotic safety:
  search for non-trivial fixed points of renormalization group equations

Example: The proposal by Hartle and Hawking



Stephen Hawking, Vatican Conference 1982:

There ought to be something very special about the boundary conditions of the universe and what can be more special than the condition that there is no boundary.

Canonical quantum gravity

Central equations are constraints:

$$\boxed{\hat{H}\Psi = 0}$$

Different canonical approaches:

- Geometrodynamics –
  metric and extrinsic curvature
- Connection dynamics –
  connection (A_a^i) and coloured electric field (E_i^a)
- Loop dynamics –
  flux of E_i^a and holonomy

Erwin Schrödinger 1926:

We know today, in fact, that our classical mechanics fails for very small dimensions of the path and for very great curvatures. Perhaps this failure is in strict analogy with the failure of geometrical optics . . . that becomes evident as soon as the obstacles or apertures are no longer great compared with the real, finite, wavelength. . . . Then it becomes a question of searching for an undulatory mechanics, and the most obvious way is by an elaboration of the Hamiltonian analogy on the lines of undulatory optics.[2]

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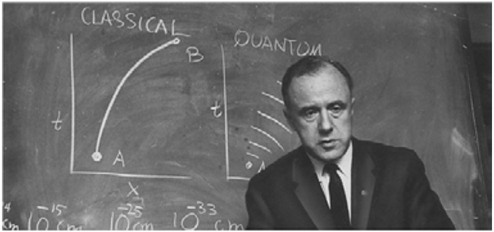
[2] *wir wissen doch heute, daß unsere klassische Mechanik bei sehr kleinen Bahndimensionen und sehr starken Bahnkrümmungen versagt*. Vielleicht ist dieses Versagen eine volle Analogie zum Versagen der geometrischen Optik . . . , das bekanntlich eintritt, sobald die ‘Hindernisse’ oder ‘Öffnungen’ nicht mehr groß sind gegen die wirkliche, endliche Wellenlänge. . . . Dann gilt es, eine ‘undulatorische Mechanik’ zu suchen – und der nächstliegende Weg dazu ist wohl die wellentheoretische Ausgestaltung des Hamiltonschen Bildes.

Quantum geometrodynamics



(a) John Archibald Wheeler

(b) Bryce DeWitt

Application of Schrödinger’s procedure to general relativity leads to

$$\hat{H}\Psi \equiv \left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - (16\pi G)^{-1}\sqrt{h}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0$$

Wheeler–DeWitt equation

$$\hat{D}^a\Psi \equiv -2\nabla_b \frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{ab}} = 0$$

quantum diffeomorphism (momentum) constraint

Problem of time

- External time t has vanished from the formalism
- This holds also for loop quantum gravity and probably for string theory
- Wheeler–DeWitt equation has the structure of a wave equation any may therefore allow the introduction of an ‘intrinsic time’
- Hilbert-space structure in quantum mechanics is connected with the probability interpretation, in particular with probability conservation *in time* t; what happens with this structure in a timeless situation?
- What is an observable in quantum gravity?

Recovery of quantum field theory in an external spacetime

An expansion of the Wheeler–DeWitt equation with respect to the Planck mass leads to the functional Schrödinger equation for non-gravitational fields in a spacetime that is a solution of Einstein's equations
(Born–Oppenheimer type of approximation)

(*Lapchinsky and Rubakov* 1979, *Banks* 1985, *Halliwell and Hawking* 1985, *Hartle* 1986, *C.K.* 1987, . . .)

Quantum gravitational corrections

Next order in the Born–Oppenheimer approximation gives

$$\hat{H}^{\mathrm{m}} \to \hat{H}^{\mathrm{m}} + \frac{1}{m_{\mathrm{P}}^{2}} \, (\text{various terms})$$

(*C.K. and Singh* (1991); *Barvinsky and C.K.* (1998))

- Quantum gravitational correction to energy values
- Possible contribution to the CMB anisotropy spectrum (*C.K. and Krämer* 2012, . . .)

Loop quantum gravity

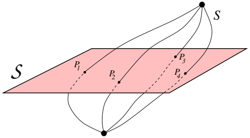
- new configuration variable: holonomy,

  $U[A, \alpha] := \mathcal{P} \exp\left(G \int_\alpha A\right)$;

- new momentum variable: densitized triad flux

  $E_i[\mathcal{S}] := \int_{\mathcal{S}} \mathrm{d}\sigma_a \, E_i^a$

Under some mild assumptions, the holonomy–flux representation is unique



Quantization of area:

$$\hat{A}(\mathcal{S})\Psi_S[A] = 8\pi\beta l_{\mathrm{P}}^2 \sum_{P \in S \cap \mathcal{S}} \sqrt{j_P(j_P+1)}\,\Psi_S[A]$$

Black-hole radiation

Black holes radiate with a temperature proportional to $\hbar$

('Hawking temperature'):

$$T_{\mathrm{BH}} = \frac{\hbar\kappa}{2\pi k_{\mathrm{B}} c}$$

Schwarzschild case:

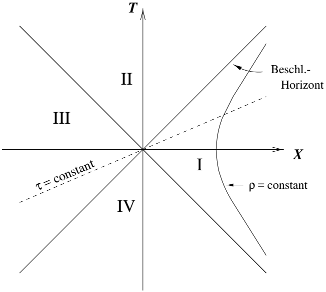
$$\begin{aligned} T_{\mathrm{BH}} &= \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M} \\ &\approx 6.17 \times 10^{-8} \left(\frac{M_\odot}{M} \right) \mathrm{K} \end{aligned}$$

Black holes also have an entropy

('Bekenstein–Hawking entropy'):

$$S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{A}{4 l_{\mathrm{P}}^2} \overset{\text{Schwarzschild}}{\approx} 1.07 \times 10^{77} k_{\mathrm{B}} \left(\frac{M}{M_\odot} \right)^2$$

Analogous effect in flat spacetime



Accelerated observer in the Minkowski vacuum experiences thermal radiation with temperature

$$T_{\mathrm{DU}} = \frac{\hbar a}{2\pi k_{\mathrm{B}} c} \approx 4.05 \times 10^{-23}\, a \left[\frac{\mathrm{cm}}{\mathrm{s}^2}\right]\ \mathrm{K}\ .$$

('Davies–Unruh temperature')

Is thermodynamics more fundamental than gravity?

Microscopic explanation of S_{BH}?

Cf. John Wheeler's "It from Bit"

$$S_{\mathrm{BH}} = -k_{\mathrm{B}} \mathrm{tr}\left(\rho \ln \rho\right)$$

Quantum gravity?

Information-loss problem

- Black holes have a finite lifetime:

$$\tau_{\mathrm{BH}} \approx 8895 \left(\frac{M_0}{m_{\mathrm{P}}}\right)^3 t_{\mathrm{P}} \approx 1.159 \times 10^{67} \left(\frac{M_0}{M_\odot}\right)^3 \mathrm{yr}$$

  from the emission of gravitons and photons (D. Page 1976)
- The semiclassical approximation breaks down if the black hole approaches the Planck mass m_{P}.
- If the black hole left only thermal radiation behind, a pure state for a closed system would evolve into a mixed system (information-loss problem)
- This would be in contradiction to ordinary quantum theory where the entropy

$$S = -k_{\mathrm{B}} \mathrm{Tr}(\rho \ln \rho)$$

  is conserved for a closed system (unitary evolution); the problem would more properly be called the “unitarity problem”.

Options

- Information is lost during the evaporation,

  $$\rho \to \$\rho \neq S\rho S^\dagger$$

  (Hawking's original opinion (1976))
- The full evolution is unitary, but this cannot be seen in the semiclassical approximation (now the most popular option)
- The black hole leaves a remnant carrying all the information

Final answer only within quantum gravity!

Most popular: second option

- At no point in the calculation by Hawking (and others) is an exact mixed (canonical) state used in the formalism. In fact, the corresponding quantum state is a *two-modes squeezed state* as known from quantum optics.
- The coherent superposition used by Hawking is indistinguishable from a local thermal mixture (L. Parker 1975).
- The reduced state of each mode in a two-mode squeezed state is a thermal state (canonical ensemble); in the special case of a black hole, the temperature is independent of k (universality).

Black holes and quantum gravity

Microscopic explanation of entropy?

$$\boxed{S_{\mathrm{BH}} = k_{\mathrm{B}} \frac{A}{4l_{\mathrm{P}}^2}}$$

- *Loop quantum gravity*: microscopic degrees of freedom are the spin networks; S_{BH} only follows under certain assumptions
- *String theory*: microscopic degrees of freedom are the "D-branes"; S_{BH} only follows for special (extremal or near-extremal) black holes
- *Quantum geometrodynamics*: e.g. $S \propto A$ in particular models

Georges Lemaître 1931:

Wenn die Welt mit einem einzigen Quantum beginnt, kommt den Begriffen Raum und Zeit insgesamt am Anfang keine Bedeutung zu … Falls diese Annahme zutreffend ist, geschah der Anfang der Welt ein wenig vor dem Anfang von Raum und Zeit …

Quantum cosmology

- Wheeler–DeWitt equation

$$\frac{1}{2}\left(\frac{G\hbar^2}{a^2}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right)-\frac{\hbar^2}{a^3}\frac{\partial^2}{\partial\phi^2}-G^{-1}a+G^{-1}\frac{\Lambda a^3}{3}+m^2a^3\phi^2\right)\psi(a,\phi)=0$$

- Loop quantum cosmology

  Difference equation instead of differential equation:
  scale factor a can assume only discrete values; $a=0$ can be avoided (Bojowald 2000 etc.)

  one can also derive modified Friedmann equations

SINGULARITY AVOIDANCE?

Interpretation of quantum cosmology

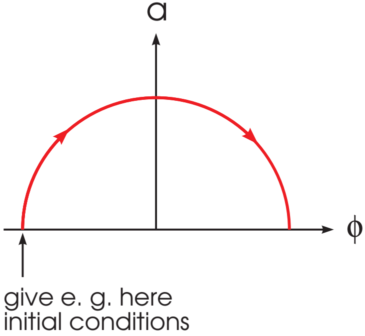
Both quantum general relativity and string theory preserve the linear structure for the quantum states
consequence: strict validity of the superposition principle

only interpretations so far: Everett interpretation (with decoherence as an essential part) and Bohm interpretation

B. S. DeWitt 1967:
Everett's view of the world is a very natural one to adopt in the quantum theory of gravity, where one is accustomed to speak without embarassment of the 'wave function of the universe.' It is possible that Everett's view is not only natural but essential.

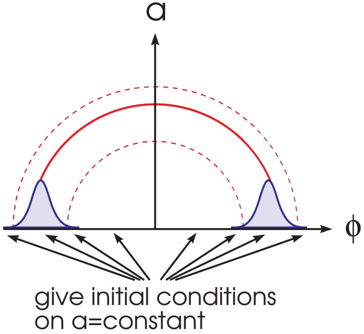
Determinism in classical and quantum theory

Classical theory



Recollapsing part is deterministic successor of expanding part

Quantum theory



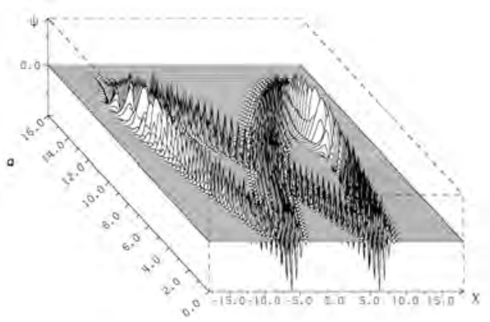
‘Recollapsing’ wave packet must be present ‘initially’

No intrinsic difference between ‘big bang’ and ‘big crunch’!

Example

Indefinite Oscillator

$$\hat{H}\psi(a,\chi) \equiv (-H_a + H_\chi)\psi \equiv \left(\frac{\partial^2}{\partial a^2} - \frac{\partial^2}{\partial \chi^2} - a^2 + \chi^2\right)\psi = 0$$



C. K. (1990)

How special is the Universe?

Penrose (1981):

Entropy of the observed part of the Universe is maximal if all its mass is in one black hole; the probability for our Universe would then be (updated version from C.K. arXiv:0910.5836)

$$\frac{\exp\left(\frac{S}{k_{\mathrm{B}}}\right)}{\exp\left(\frac{S_{\max}}{k_{\mathrm{B}}}\right)} \sim \frac{\exp\left(3.1 \times 10^{104}\right)}{\exp\left(1.8 \times 10^{121}\right)} \approx \exp\left(-1.8 \times 10^{121}\right)$$

Arrow of time from quantum cosmology

Fundamental asymmetry with respect to "'intrinsic time"':

$$\hat{H}\Psi = \left(\frac{\partial^2}{\partial\alpha^2} + \sum_i \left[-\frac{\partial^2}{\partial x_i^2} + \underbrace{V_i(\alpha, x_i)}_{\to 0 \text{ for } \alpha\to-\infty} \right] \right) \Psi = 0$$

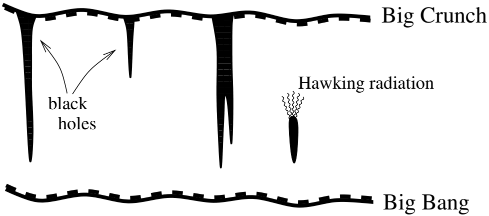
Is compatible with simple boundary condition:

$$\Psi \overset{\alpha\to-\infty}{\longrightarrow} \psi_0(\alpha) \prod_i \psi_i(x_i)$$

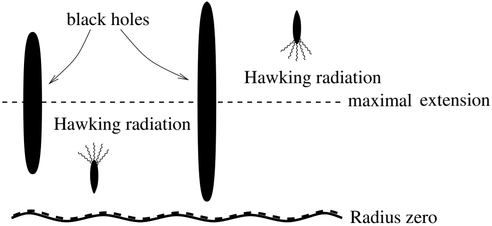
Entropy increases with increasing α, since entanglement with other degrees of freedom increases;
this defines the direction of time

Is the expansion of the Universe a tautology?

Arrow of time in a recollapsing quantum universe



(Penrose 1979)



(C.K. and Zeh 1995)

Quo vadis?

Albert Einstein 1953:

Es hat schweren Ringens bedurft, um zu dem für die theoretische Entwicklung unentbehrlichen Begriff des selbständigen und absoluten Raumes [und der Zeit] zu gelangen. Und es hat nicht geringerer Anstrengung bedurft, um diesen Begriff nachträglich wieder zu überwinden – ein Prozeß, der wahrscheinlich noch keineswegs beendet ist.

Literature

- C.K., *Quantum Gravity*, third edition (Oxford University Press, Oxford, 2012);
- C.K., Conceptual Problems in Quantum Gravity and Quantum Cosmology, *ISRN Math.Phys.* 2013 (2013) 509316, see also arXiv:1401.3578 [gr-qc];
- C.K., *Der Quantenkosmos* (S. Fischer, Frankfurt am Main, 2008).