

**Maria Laach 2016**  
**Lecture IV:**  
**QCD parton model: Jets and Top**

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## Ingredients for a parton calculation

Parton luminosity

Jet physics

Jet algorithms

Branching kinematics

Higgs and QCD branchings compared: angular separation

Higgs and QCD branchings compared: mass drop

Separating signal and background in  $VH(\rightarrow b\bar{b})$

Higgs tagging-filtering

Why top?

LO Top production

NLO Heavy quark production

Scale dependence

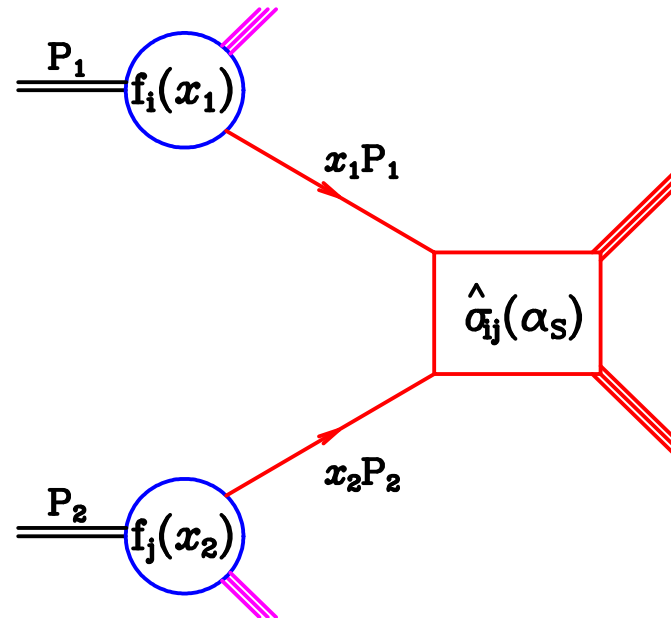
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## ■ Factorization formula

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_s(\mu^2), Q^2 / \mu^2)$$

- Non-perturbative parton distributions  $f_i(x, \mu^2)$  with calculable scale dependence.
- Short distance cross section that depends on  $\alpha_s$  and factorization scale  $\mu$ .
- Value of the coupling  $\alpha_s$  with known scale dependence.

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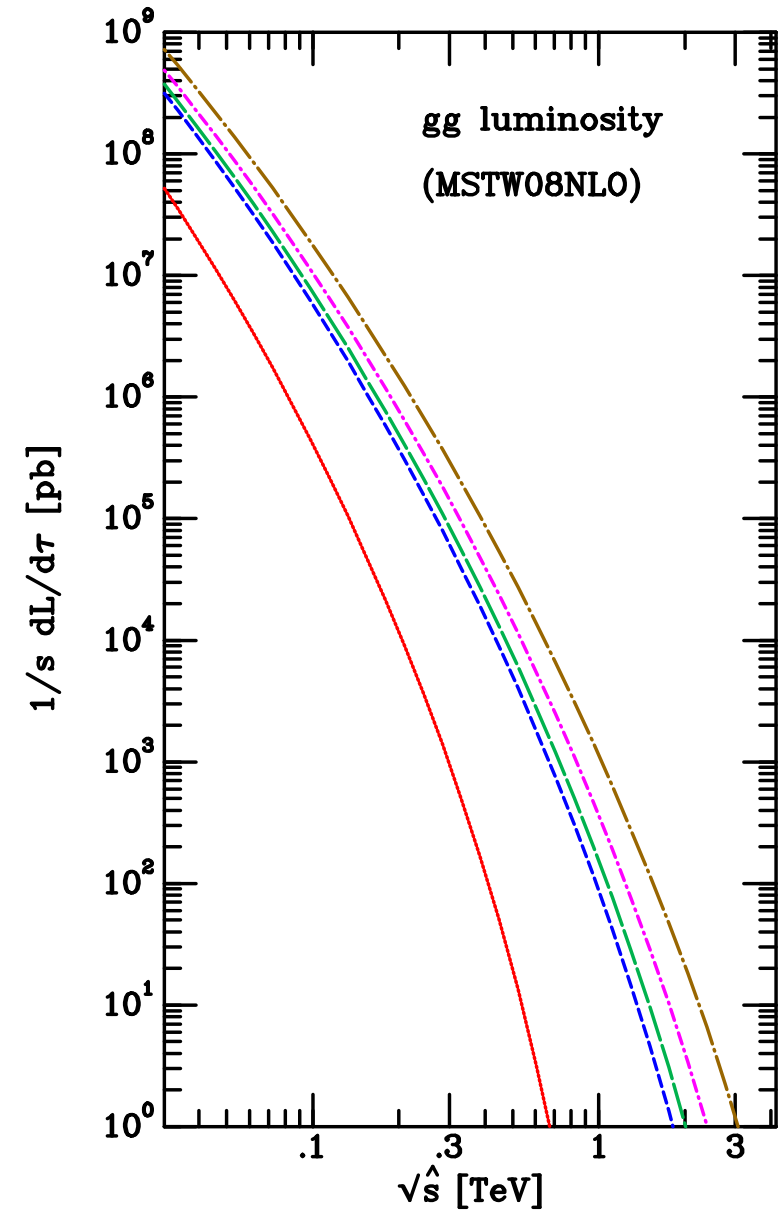
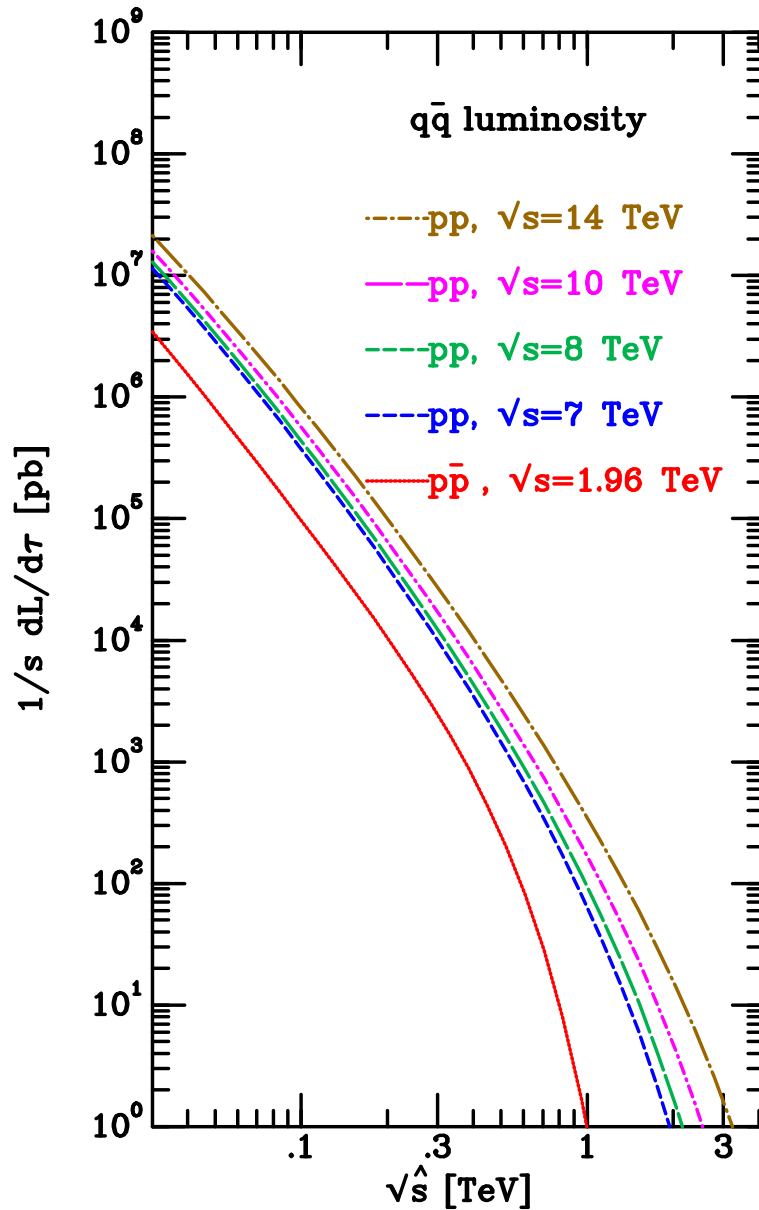
- Parton luminosity is determined by the parton distribution functions,  $f_i(x_1, \mu^2)$  and  $f_j(x_2, \mu^2)$ .
- $f_j(x_i, \mu^2)$  need to be determined by data.
- the available centre-of-mass energy-squared of the parton-parton collision,  $\hat{s}$ , is less than the overall hadron-hadron collision energy,  $s$ , by a factor of  $x_1 x_2 \equiv \tau$ .
- Define differential parton luminosities

$$\tau \frac{dL_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_0^1 dx_1 dx_2 \times \left[ (x_1 f_i(x_1, \mu^2) x_2 f_j(x_2, \mu^2)) + (1 \leftrightarrow 2) \right] \delta(\tau - x_1 x_2).$$

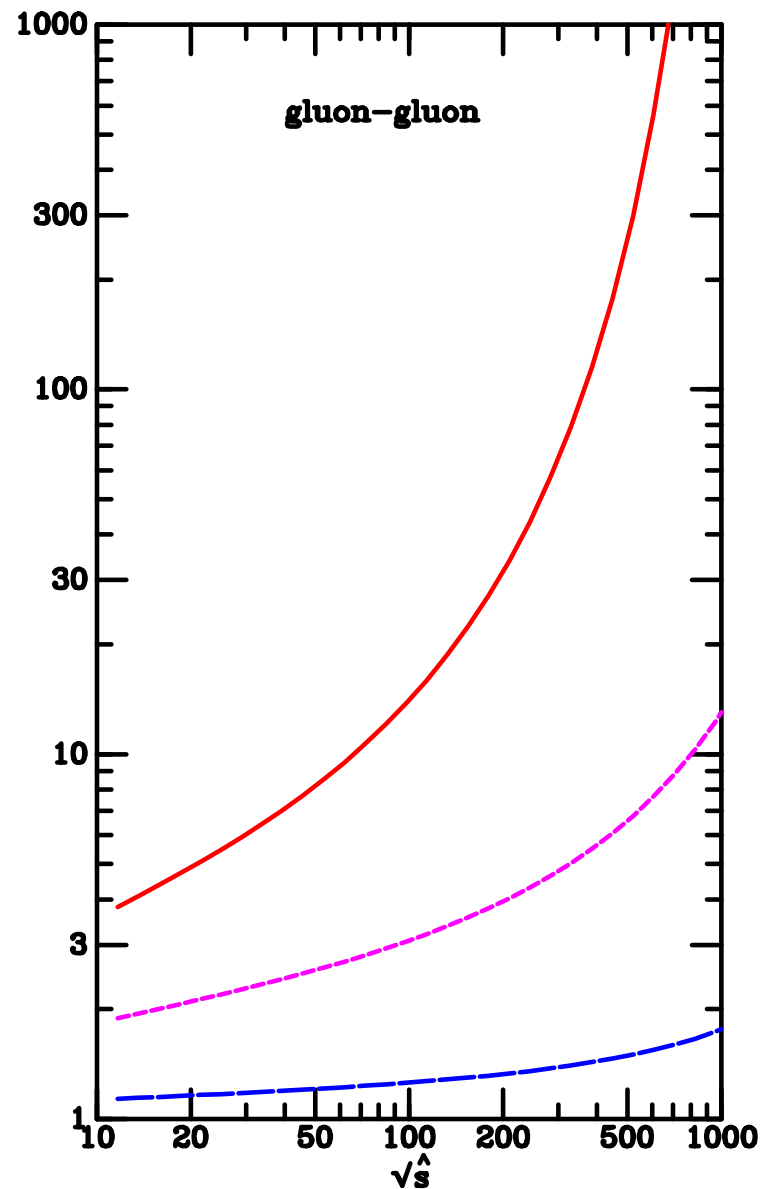
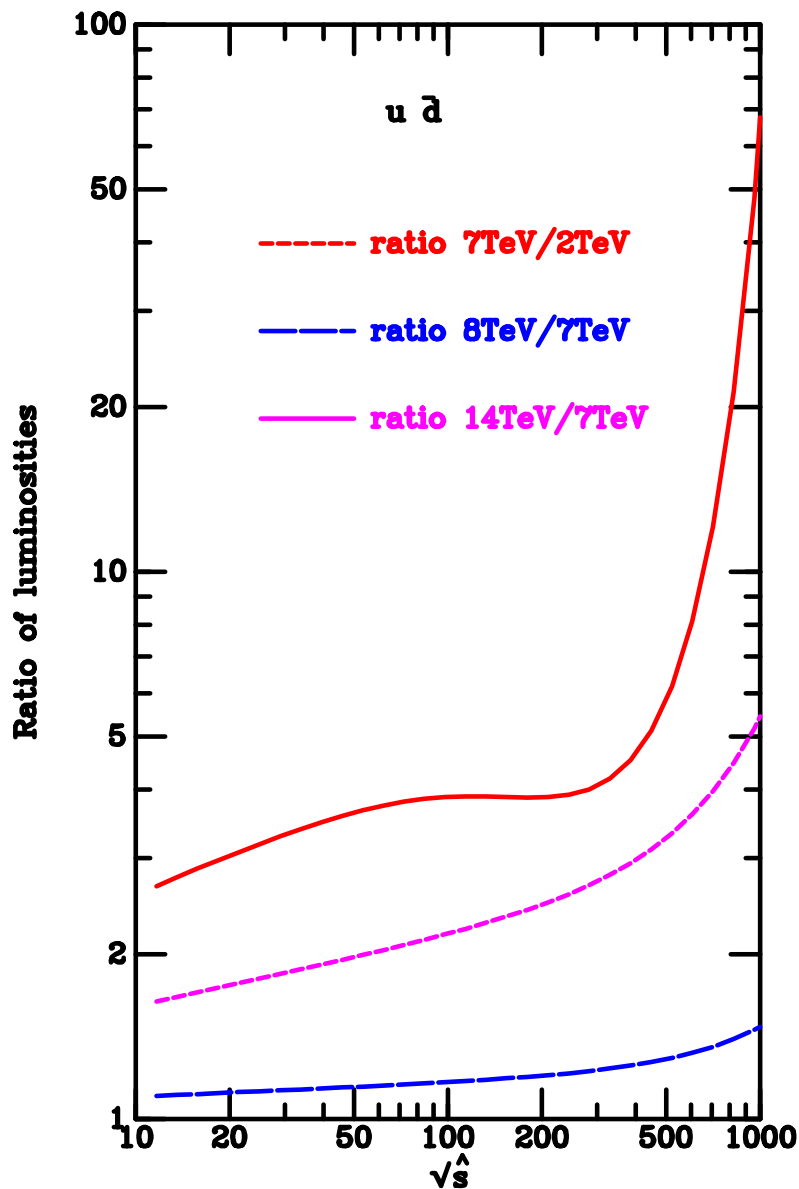
- The collider luminosity is quite distinct from the parton luminosity. The former is a property of a machine, whereas the latter is a property of the proton.
- We now assume that  $\hat{\sigma}$  depends only on  $\hat{s}$ .

$$\sigma(s) = \sum_{\{ij\}} \int_{\tau_0}^1 \frac{d\tau}{\tau} \left[ \frac{1}{s} \frac{dL_{ij}}{d\tau} \right] \left[ \hat{s} \hat{\sigma}_{ij} \right],$$

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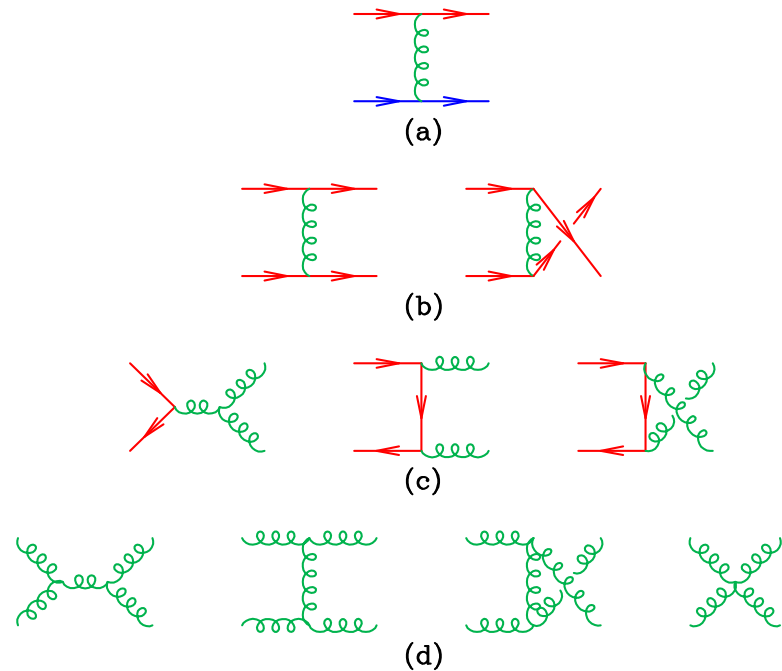
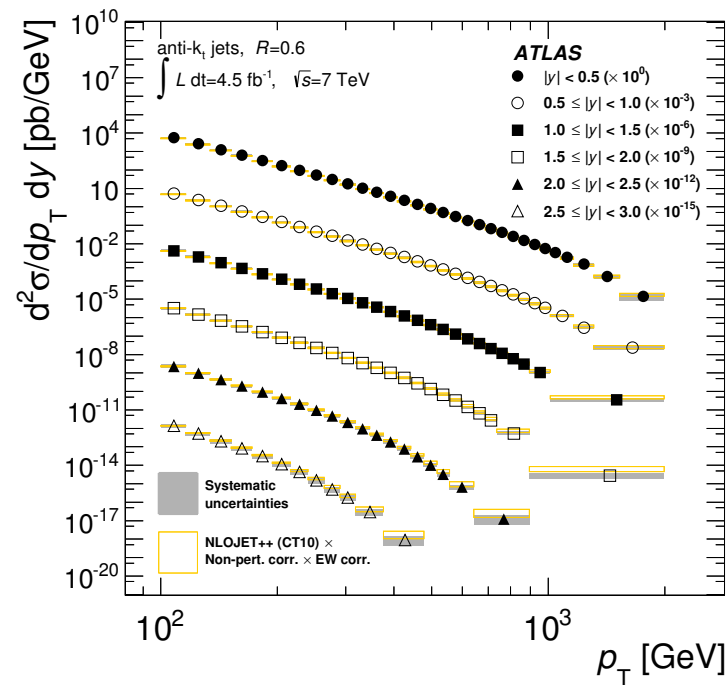
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- There is little doubt that jet physics displays fundamental scattering of constituents.
- Probes pointlike behaviour on shortest distance scales.
- But what is a jet?



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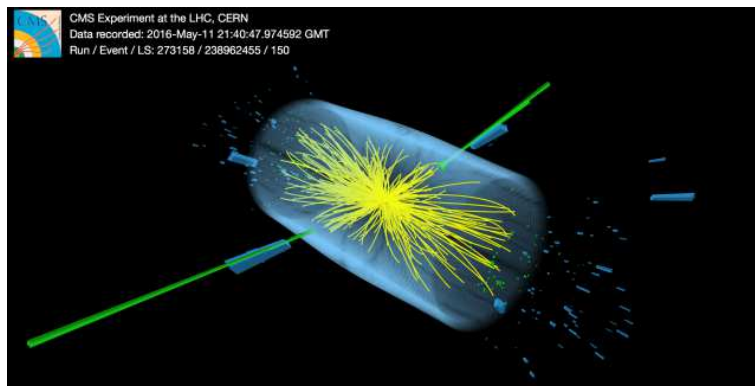
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- Jet structure is obvious to the naked eye;
- A jet definition is like a legal contract between theorists and experimenters; many contracts are possible, different contracts are useful in different circumstances, but all contracts must be defined precisely
- a jet definition requires:-
  - a jet algorithm
  - jet parameters, e.g. a cone size  $R$
  - a recombination scheme for combining entities
- Desirable properties; infrared safety, speed, defined jet area for subtraction of pile up and underlying event.
- Two types of jet algorithms.
  - Sequential recombination algorithm
  - Cone algorithms

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## ■ Sequential recombination algorithms

- Combine entities (particles) starting with the closest ones
- Requires definition of close, (distance measure)
- Iterate recombination until there are few entities (jets) left
- Examples, Jade,  $k_t$ , Cambridge/Aachen, anti- $k_t$

## ■ Cone algorithms

- Identify regions with large energy flow
- Cone algorithms give rise to regular jets which are easier to calibrate, and to remove underlying event.
- Examples, ATLAS cone, CMS cone, SIScone

I shall only talk about sequential recombination algorithms in the following.



# Sequential recombination jet algorithms

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- introduce distances ( $d_{ij}$ ) between entities (particles, pseudojets)  $i$  and  $j$  and between an entity and the beam ( $d_{iB}$ )

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^{2p}$$

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- $k_{ti}, y_i, \phi_i$  are the transverse momentum, rapidity and azimuth of entity  $i$
- Clustering proceeds by identifying the smallest of the distances;
  - if it is  $d_{ij}$  recombine entities  $i$  and  $j$ ;
  - if it is  $d_{iB}$ , call  $i$  a jet and remove it from the list of entities to be clustered.
  - iterate
- Note that entities separated in angle, such that  $\Delta R_{ij}^2 > R^2$  will never be clustered.
- Sequential recombination algorithms return not only a list of jets, but also a clustering sequence, which contains valuable information about the morphology of the event.

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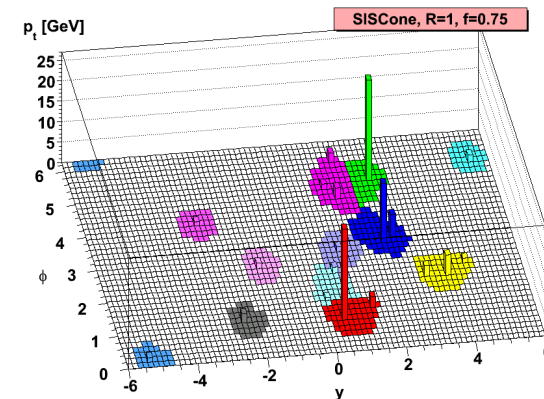
- introduce distances between entities (particles, pseudojets)  $i$  and  $j$  ( $d_{ij}$ ) and between an entity and the beam ( $d_{iB}$ )

$$d_{ij} = \min(k_{ti}^{2p}, k_{tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = k_{ti}^{2p}$$

- Establish IR safety by asking how clustering sequence would change, with the addition of soft or collinear radiation.
  - emission of a collinear particle,  $\Delta y^2 + \Delta\phi^2 \rightarrow 0$  in all cases means that the jet measure  $d_{ij} \rightarrow 0$ . Hence collinearly emitted particles are clustered first, leaving resultant jets unchanged.
    - for  $p=1$ , a new soft particle,  $k_{ti} \rightarrow 0$  gives the smallest  $d_{ij}$ , hence clustered first leaving jets unchanged.
    - for  $p=0$ , a new soft particle can be a new jet of zero momentum, leaving hard jets unchanged
    - for  $p=-1$ , a new soft particle  $k_{ti} \rightarrow 0$  gives the largest  $d_{ij} \rightarrow \infty$ , clustered last or new zero-momentum jet, leaving hard jets unchanged.

Jet algorithm	distance measure	Authors	Scaling
SIScone	Seedless iterative cone with split-merge	Salam et al, 0704.0292	$N^2 \ln N$
$k_t$	$d_{ij} = \min(k_{ti}^2, k_{tj}^2) \Delta_{ij} / R^2$ , Ordered in $k_t$	Catani et al, NPB406 (1993) S.Ellis et al, 9305266	$N \ln N$
Cambridge/Aachen	$d_{ij} = \Delta_{ij} / R^2$ , Ordered in angle	Dokshitzer et al, 9907280 Wengler et al, 9907280	$N \ln N$
anti- $k_t$	$d_{ij} = \min(k_{ti}^{-2}, k_{tj}^{-2}) \Delta_{ij} / R^2$ , Gives conical hard jets	Cacciari et al, 0802.1189	$N^{3/2}$

- Siscone, Seedless infra-red safe algorithm
- Infrared safety is both a theoretical necessity and an experimental imperative.



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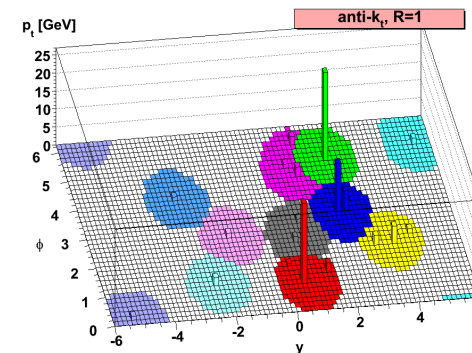
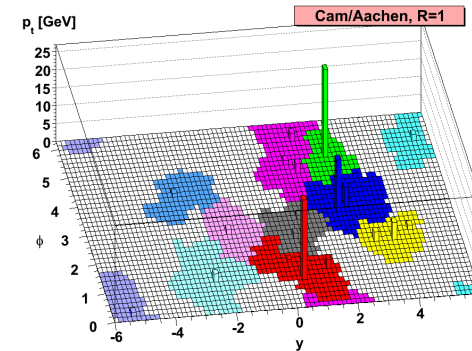
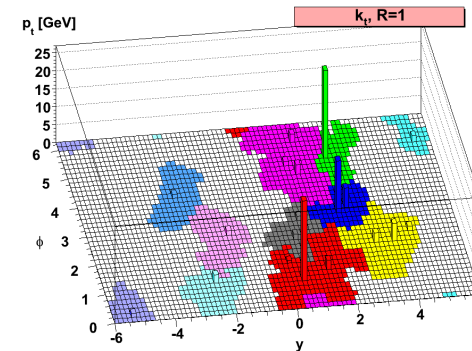
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- $k_t$  algorithm ( $p = 1$ );  $d_{ij}$  distance measure is the inverse of the branching probability, (the pair which is recombined first is the one with the largest probability to have branched last). The clustering sequence has a physical meaning. Helpful for theoretical resummation.
- C/A algorithm ( $p = 0$ ); still contains features of the parton shower because of the angular ordering property of QCD radiation; it is a compromise between the structure of the parton shower and limiting the sensitivity to soft radiation.
- the anti- $k_t$  algorithm ( $p = -1$ ); soft radiation is always clustered last; gives rise to approximately conical jets. A bizarre choice, which nevertheless gives very useful jets for pileup and underlying event subtraction.



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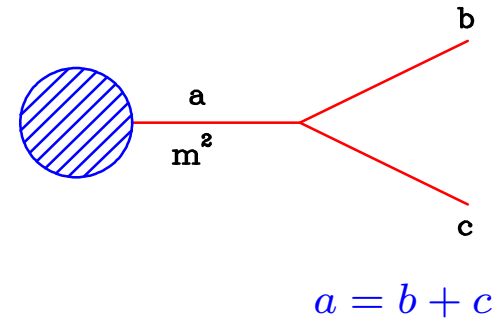
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$$a^\mu = p^\mu + \frac{\mathbf{p}_T^2 + m^2}{2} n^\mu + \mathbf{p}_T^\mu$$

$$b^\mu = zp^\mu + \frac{\mathbf{b}_T^2}{2z} n^\mu + \mathbf{b}_T^\mu$$

$$c^\mu = (1-z)p^\mu + \frac{\mathbf{c}_T^2}{2(1-z)} n^\mu + \mathbf{c}_T^\mu$$



where  $n \cdot p = 1, p \cdot p = n \cdot n = 0,$   
 $p \cdot \mathbf{x}_T = n \cdot \mathbf{x}_T = 0.$  Setting

$$\mathbf{b}_T = z\mathbf{p}_T + \mathbf{k}_T, \quad \mathbf{c}_T = (1-z)\mathbf{p}_T - \mathbf{k}_T$$

we find in the small  $\mathbf{k}_T$  approximation that

$$(y_b - y_c)^2 = \frac{(\mathbf{p}_T \cdot \mathbf{k}_T)^2}{\mathbf{p}_T^4 z^2 (1-z)^2}, \quad (\phi_b - \phi_c)^2 = \frac{\mathbf{p}_T^2 \mathbf{k}_T^2 - (\mathbf{p}_T \cdot \mathbf{k}_T)^2}{\mathbf{p}_T^4 z^2 (1-z)^2}$$

$$a^2 = m^2 \approx \frac{\mathbf{k}_T^2}{z(1-z)}, \quad \Delta R_{bc}^2 = (y_b - y_c)^2 + (\phi_b - \phi_c)^2 \approx \frac{m^2}{z(1-z)\mathbf{p}_T^2}$$

$$\text{Rescaled distance measure } \Delta y_{bc}^2 = \frac{\min\{\mathbf{b}_T^2, \mathbf{c}_T^2\}}{m^2} \times \Delta R_{bc}^2 \simeq \frac{z}{1-z}, \quad \text{for } z < \frac{1}{2}$$

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- In the collinear approximation the amplitude to radiate an extra parton can be written as

$$d\sigma_{n+1} \approx d\sigma_n dz \frac{dt}{t} \frac{\alpha_s}{2\pi} P(z),$$

- $m^2 = t$  is the jet mass, and  $z$  is the longitudinal momentum branching fraction. The splitting function  $P(z)$  for  $q \rightarrow gq$  is

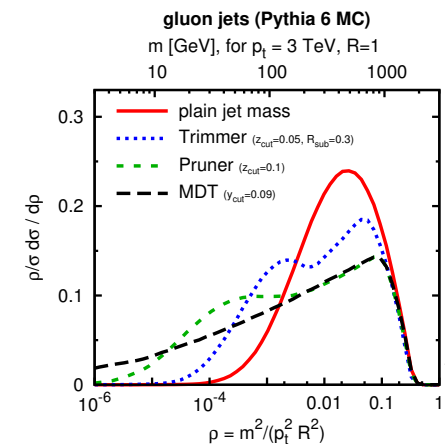
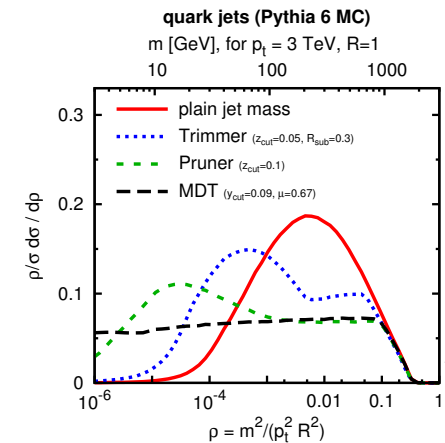
$$P_{gq}(z) = C_F \frac{1 + (1-z)^2}{z}.$$

$$\langle m^2 \rangle = \langle t \rangle \approx \int_0^1 dz \int_0^{t_{\max}} \frac{dt}{t} t \frac{\alpha_s}{2\pi} P_{gq}(z).$$

The jet algorithm imposes that  $\Delta R_{bc} < R$ , and hence that  $t_{\max} = \mathbf{p}_T^2 R^2 z(1-z)$ .

- taking a fixed  $\alpha_s$  for the moment we see that

$$\langle m^2 \rangle \approx \mathbf{p}_T^2 R^2 \frac{\alpha_s}{2\pi} \int dz z(1-z) P_{bc}(z).$$



- leading to the conclusion  $\langle m^2 \rangle \propto \mathbf{p}_T^2 R^2 \frac{\alpha_s}{2\pi} C_F / C_A$ .
- Plots display Casimir broadening

# Higgs and QCD branchings compared: angular separation

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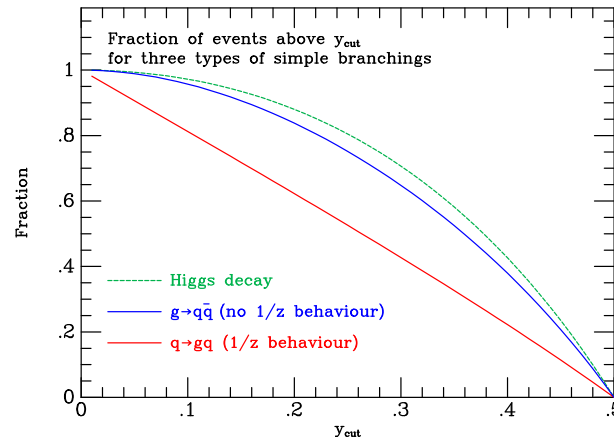
- From the kinematics of branching alone, we have established that

$$\sqrt{\Delta R_{ij}^2} \sim \frac{m}{p_T} \frac{1}{\sqrt{z(1-z)}} \sim \frac{2m}{p_T}.$$

- Evaluate the distance measure  $y_{bc}$

$$y_{bc} = \frac{\min(\mathbf{b}_T^2, \mathbf{c}_T^2)}{m^2} \times \Delta R_{bc}^2 \approx \frac{p_T^2 z^2}{m^2} \times \frac{m^2}{p_T^2 z(1-z)} \approx \frac{z}{1-z}.$$

- weight with splitting function  $P \rightarrow bc$  (if appropriate)
- At fixed jet mass this is the result for the decay of a Higgs boson with  $P(z) = 1$ . QCD jets give rise to a different result, especially because of the  $1/z$ -behaviour.



$$P_{h \rightarrow b\bar{b}} \propto 1$$

$$P_{q \rightarrow gq} \propto \frac{1 + (1-z)^2}{z}$$

$$P_{g \rightarrow q\bar{q}} \propto z^2 + (1-z)^2$$

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- A boosted Higgs with mass  $m_H$ , decays to (essentially) massless daughters in one step;
- QCD splittings favour slower degradation in the virtuality.
- the Sudakov form factor expresses the probability of evolving from an initial virtuality  $t_0$  to a final virtuality  $t$  without branching:

$$\Delta(t) = \exp \left[ - \int_{t_0}^t \frac{dt'}{t'} dz \frac{\alpha_s}{2\pi} P(z) \right].$$

- In the fixed coupling constant approximation we find that,

$$\Delta(t) \propto \left( \frac{t_0}{t} \right)^p$$

- for  $p > 0$ ,  $\Delta(t) \rightarrow 0$  for large  $t$ . The probability of a QCD jet arriving at mass squared  $t_0$ , from a large mass squared  $t$  falls like a positive power of  $(t_0/t)$



# Separating signal and background in $VH(\rightarrow b\bar{b})$

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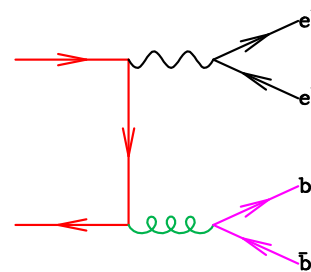
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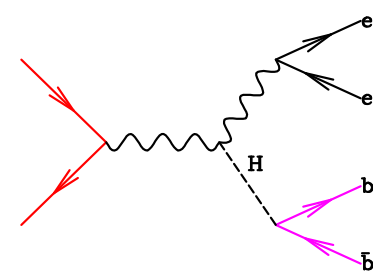
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- A Higgs boson  $H$  decaying to  $b\bar{b}$  differs in two ways from a QCD branching

- the splitting will be symmetric,
- the jet mass fall drop faster in the branching



Background

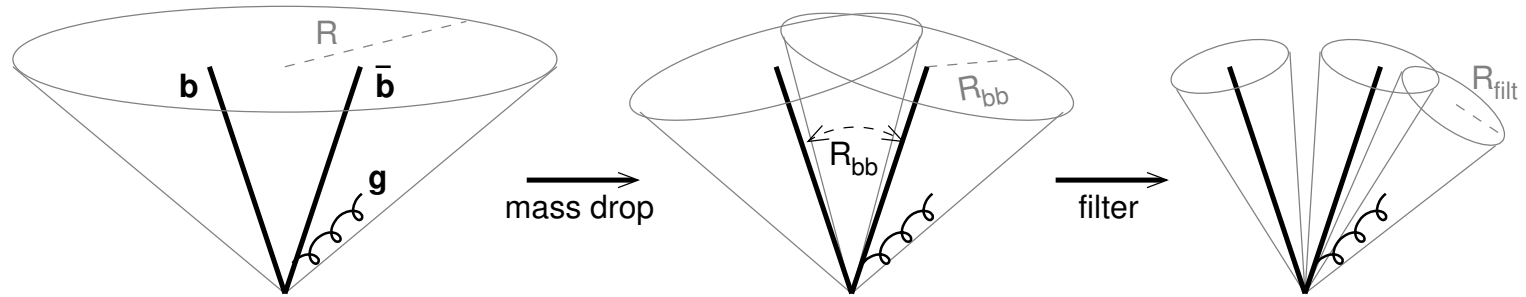


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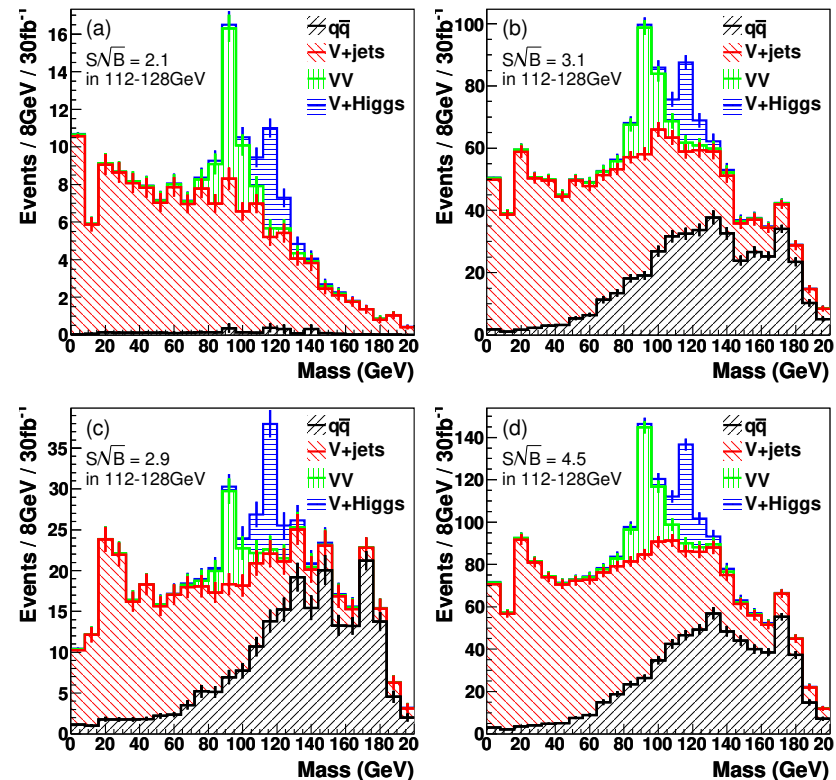
- the BDRS procedure (arXiv:0802.2470) exploits these two features
- First cluster the event on a large angular scale,  $R_{b\bar{b}}$ , using the Cambridge-Aachen jet definition. (The clustering scale of  $R_{b\bar{b}}$  is set by  $2m/|\mathbf{p}_T|$ ).
- Undo the cluster sequence one branching at a time and check on the mass drop and symmetry of the branching, to identify whether the branching belongs in the Higgs neighbourhood. The declustering involves two dimensionless parameters,  $\mu$  (0.67) and  $y_{cut}$  (0.09):

$$\max(m_i, m_j) < \mu \times m_P(\text{massdrop}), \quad y_{ij} > y_{cut}(\text{symmetric})$$

- Continue until either a Higgs-like branching has been identified or no jets remain.



- Filter the Higgs neighbourhood, by clustering the events on a smaller angular scale  $R_{\text{filt}} < R_{b\bar{b}}$  and keep only the three hardest subjets (to allow for  $b, \bar{b}$  and possible parton radiation).
- This step helps to remove pile-up.
- MC results for  $30 \text{ fb}^{-1}$  (a)  $e^+e^-$  or  $\mu^+\mu^-$  ( $ZH$ ) (b) lepton + Missing transverse momentum ( $WH$ ) (c) Missing transverse momentum ( $Z \rightarrow \nu\bar{\nu}H$ ) (d) All channels



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- The top quark cross section is large at LHC energies, one event in  $10^6$
- Since  $m_t > M_W + m_b$  a top quark decays predominantly into a  $b$  quark and an on-shell  $W$  boson

$$t \rightarrow W^+ + b$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad \rightarrow l^+ + \nu$$

$$t \rightarrow W^+ + b$$

$$\quad \quad \quad \downarrow$$

$$\quad \quad \quad \rightarrow q + \bar{q}$$

- In the limit  $m_t \gg M_W$  the result for the total width is

$$\Gamma(t \rightarrow bW) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \approx 1.76 \text{ GeV} \left( \frac{m_t}{175 \text{ GeV}} \right)^3.$$

$V_{tb} \approx 1$  as suggested by the unitarity relation  $|V_{tb}|^2 + |V_{cb}|^2 + |V_{ub}|^2 = 1$ .

- The top quark decays before it has time to hadronize.
- The top is a copious source of  $b$ 's and  $W$ 's

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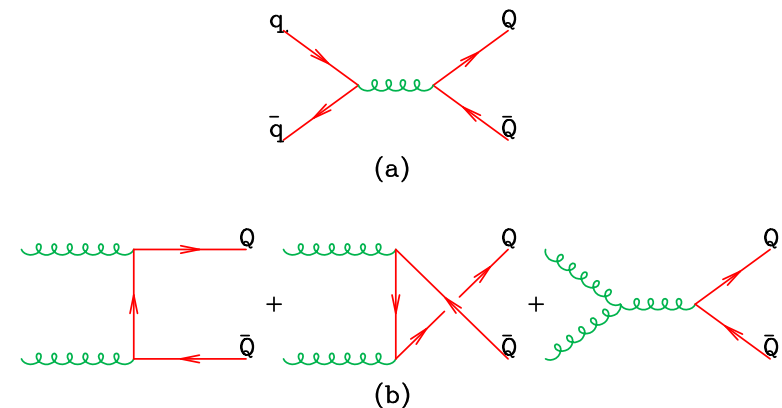
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- The leading-order processes for the production of a heavy quark  $Q$  of mass  $m$  in hadron-hadron collisions

$$(a) \quad q(p_1) + \bar{q}(p_2) \rightarrow Q(p_3) + \bar{Q}(p_4)$$

$$(b) \quad g(p_1) + g(p_2) \rightarrow Q(p_3) + \bar{Q}(p_4)$$

where the four-momenta of the partons are given in brackets ( $\rho = 4m^2/s$ ).



Process	$\overline{\sum}  \mathcal{M} ^2 / g^4$
$q \bar{q} \rightarrow Q \bar{Q}$	$\frac{4}{9} \left( \tau_1^2 + \tau_2^2 + \frac{\rho}{2} \right)$
$g g \rightarrow Q \bar{Q}$	$\left( \frac{1}{6\tau_1\tau_2} - \frac{3}{8} \right) \left( \tau_1^2 + \tau_2^2 + \rho - \frac{\rho^2}{4\tau_1\tau_2} \right)$

- The matrix elements squared have been averaged (summed) over initial (final) colours and spins, as indicated by  $\overline{\sum}$ .
- We have introduced the following notation for the ratios of scalar products:

$$\tau_1 = \frac{2p_1 \cdot p_3}{\hat{s}}, \quad \tau_2 = \frac{2p_2 \cdot p_3}{\hat{s}}, \quad \rho = \frac{4m^2}{\hat{s}}, \quad \hat{s} = (p_1 + p_2)^2.$$

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- The short-distance cross section is obtained from the invariant matrix element in the usual way:

$$d\hat{\sigma}_{ij} = \frac{1}{2\hat{s}} \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

The first factor is the flux factor for massless incoming particles. The other terms come from the phase space for  $2 \rightarrow 2$  scattering.

- In terms of the rapidity  $y = \frac{1}{2} \ln((E + p_z)/(E - p_z))$  and transverse momentum,  $p_T$ , the relativistically invariant phase space volume element of the final-state heavy quarks is

$$\frac{d^3p}{E} = dy d^2p_T .$$

The result for the invariant cross section may be written as

$$\frac{d\sigma}{dy_3 dy_4 d^2p_T} = \frac{1}{16\pi^2 \hat{s}^2} \sum_{ij} x_1 f_i(x_1, \mu^2) x_2 f_j(x_2, \mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

$x_1$  and  $x_2$  are fixed if we know the transverse momenta and rapidity of the outgoing heavy quarks.

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- In the centre-of-mass system of the incoming hadrons we may write

$$p_1 = \frac{1}{2}\sqrt{s}(x_1, 0, 0, x_1)$$

$$p_2 = \frac{1}{2}\sqrt{s}(x_2, 0, 0, -x_2)$$

$$p_3 = (m_T \cosh y_3, p_T, 0, m_T \sinh y_3)$$

$$p_4 = (m_T \cosh y_4, -p_T, 0, m_T \sinh y_4).$$

Applying energy and momentum conservation, we obtain

$$x_1 = \frac{m_T}{\sqrt{s}}(e^{y_3} + e^{y_4})$$

$$x_2 = \frac{m_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4})$$

$$\hat{s} = 2m_T^2(1 + \cosh \Delta y).$$

The quantity  $m_T = \sqrt{(m^2 + p_T^2)}$  is the transverse mass of the heavy quarks and  $\Delta y = y_3 - y_4$  is the rapidity difference between them.

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- In these variables the leading order cross section is

$$\frac{d\sigma}{dy_3 dy_4 d^2 p_T} = \frac{1}{64\pi^2 m_T^4 (1 + \cosh(\Delta y))^2} \times \sum_{ij} x_1 f_i(x_1, \mu^2) x_2 f_j(x_2, \mu^2) \overline{\sum} |\mathcal{M}_{ij}|^2.$$

Expressed in terms of  $m$ ,  $m_T$  and  $\Delta y$ , the matrix elements for the two processes are

$$\overline{\sum} |\mathcal{M}_{q\bar{q}}|^2 = \frac{4g^4}{9} \left( \frac{1}{1 + \cosh(\Delta y)} \right) \left( \cosh(\Delta y) + \frac{m^2}{m_T^2} \right),$$

$$\overline{\sum} |\mathcal{M}_{gg}|^2 = \frac{g^4}{24} \left( \frac{8 \cosh(\Delta y) - 1}{1 + \cosh(\Delta y)} \right) \left( \cosh(\Delta y) + 2 \frac{m^2}{m_T^2} - 2 \frac{m^4}{m_T^4} \right).$$

- As the rapidity separation  $\Delta y$  between the two heavy quarks becomes large

$$\overline{\sum} |\mathcal{M}_{q\bar{q}}|^2 \sim \text{constant}, \quad \overline{\sum} |\mathcal{M}_{gg}|^2 \sim \exp \Delta y.$$

- The cross section is damped at large  $\Delta y$  and heavy quarks produced by  $q\bar{q}$  annihilation are more closely correlated in rapidity those produced by  $gg$  fusion.

- Consider the propagators in the diagrams.

$$\begin{aligned}(p_1 + p_2)^2 &= 2p_1 \cdot p_2 = 2m_T^2 (1 + \cosh \Delta y) , \\(p_1 - p_3)^2 - m^2 &= -2p_1 \cdot p_3 = -m_T^2 (1 + e^{-\Delta y}) , \\(p_2 - p_3)^2 - m^2 &= -2p_2 \cdot p_3 = -m_T^2 (1 + e^{\Delta y}) .\end{aligned}$$

Note that the propagators are all off-shell by a quantity of least of order  $m^2$ .

- Thus for a sufficiently heavy quark we expect the methods of perturbation theory to be applicable. It is the mass  $m$  (which by supposition is very much larger than the scale of the strong interactions  $\Lambda$ ) which provides the large scale in heavy quark production. We expect corrections of order  $\Lambda/m$
- This does not address the issue of whether the charm or bottom mass is large enough to be adequately described by perturbation theory.

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In NLO heavy quark production  $m$  is the heavy quark mass.

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 \hat{\sigma}_{ij}(x_1 x_2 S, m^2, \mu^2) F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

$$\hat{\sigma}_{i,j}(\hat{s}, m^2, \mu^2) = \sigma_0 c_{ij}(\hat{\rho}, \mu^2)$$

where  $\hat{\rho} = 4m^2/\hat{s}$ ,  $\bar{\mu}^2 = \mu^2/m^2$ ,  $\sigma_0 = \alpha_s^2(\mu^2)/m^2$  and  $\hat{s}$  in the parton total c-of-m energy squared. The coupling satisfies

$$\frac{d\alpha_s}{d \ln \mu^2} = -b_0 \frac{\alpha_s^2}{2\pi} + O(\alpha_s^3), \quad b_0 = \frac{11N - 2n_f}{6}$$

$$c_{ij}\left(\rho, \frac{\mu^2}{m^2}\right) = c_{ij}^{(0)}(\rho) + 4\pi\alpha_s(\mu^2) \left[ c_{ij}^{(1)}(\rho) + \bar{c}_{ij}^{(1)}(\rho) \ln\left(\frac{\mu^2}{m^2}\right) \right] + O(\alpha_s^2)$$

The lowest-order functions  $c_{ij}^{(0)}$  are obtained by integrating the lowest order matrix elements

$$c_{q\bar{q}}^{(0)}(\rho) = \frac{\pi\beta\rho}{27} \left[ (2 + \rho) \right], \quad c_{gq}^{(0)}(\rho) = c_{g\bar{q}}^{(0)}(\rho) = 0,$$

$$c_{gg}^{(0)}(\rho) = \frac{\pi\beta\rho}{192} \left[ \frac{1}{\beta} [\rho^2 + 16\rho + 16] \ln\left(\frac{1+\beta}{1-\beta}\right) - 28 - 31\rho \right],$$

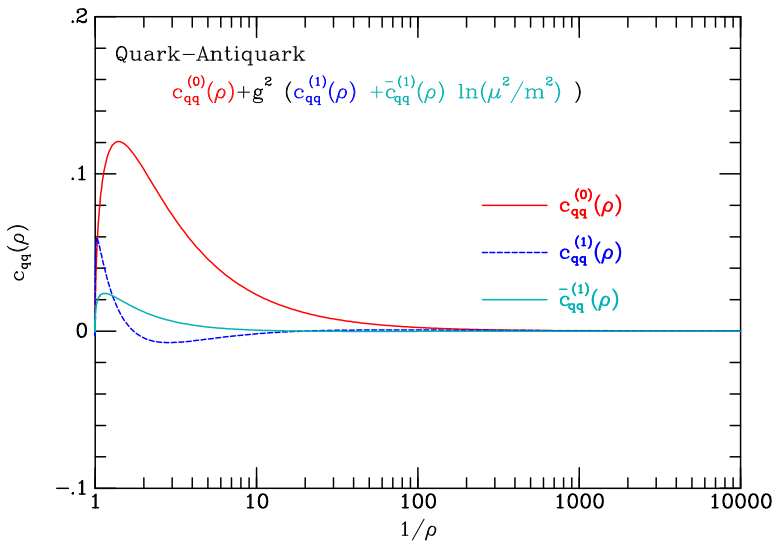
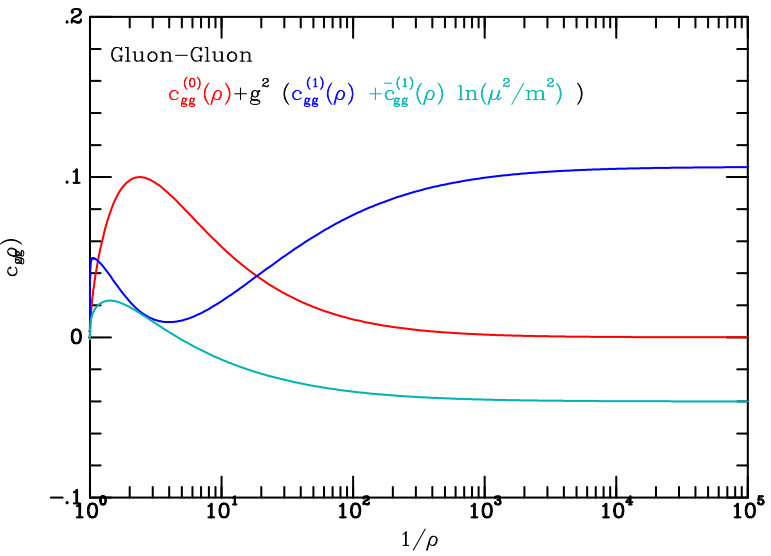
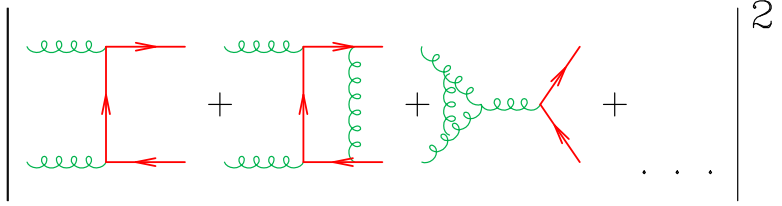
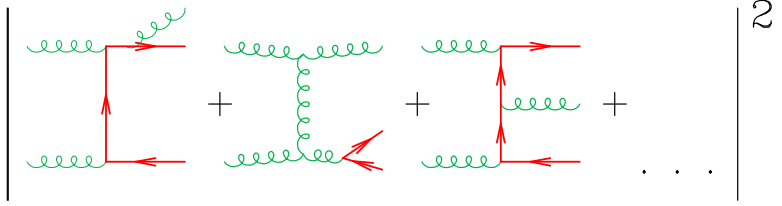
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- The functions  $c_{ij}^{(1)}$  are also known
- In order to calculate the  $c_{ij}$  in perturbation theory we must perform both renormalization and factorization of mass singularities. The subtractions required for renormalization and factorization are done at mass scale  $\mu$ .



$\mu$  is an unphysical parameter. The physical predictions should be invariant under changes of  $\mu$  at the appropriate order in perturbation theory. If we have performed a calculation to  $O(\alpha_S^3)$ , variations of the scale  $\mu$  will lead to corrections of  $O(\alpha_S^4)$ ,

$$\mu^2 \frac{d}{d\mu^2} \sigma = O(\alpha_S^4).$$

- The term  $\bar{c}^{(1)}$ , which controls the  $\mu$  dependence of the higher-order perturbative contributions, is fixed in terms of the lower-order result  $c^{(0)}$ :

$$\begin{aligned} \bar{c}_{ij}^{(1)}(\rho) &= \frac{1}{8\pi^2} \left[ 4\pi b c_{ij}^{(0)}(\rho) - \int_{\rho}^1 dz_1 \sum_k c_{kj}^{(0)}\left(\frac{\rho}{z_1}\right) P_{ki}^{(0)}(z_1) \right. \\ &\quad \left. - \int_{\rho}^1 dz_2 \sum_k c_{ik}^{(0)}\left(\frac{\rho}{z_2}\right) P_{kj}^{(0)}(z_2) \right]. \end{aligned}$$

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- In obtaining this result we have used the renormalization group equation for the running coupling

$$\mu^2 \frac{d}{d\mu^2} \alpha_s(\mu^2) = -b\alpha_s^2 + \dots$$

and the lowest-order form of the DGLAP equation

$$\mu^2 \frac{d}{d\mu^2} f_i(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_k \int_x^1 \frac{dz}{z} P_{ik}^{(0)}(z) f_k\left(\frac{x}{z}, \mu^2\right) + \dots$$

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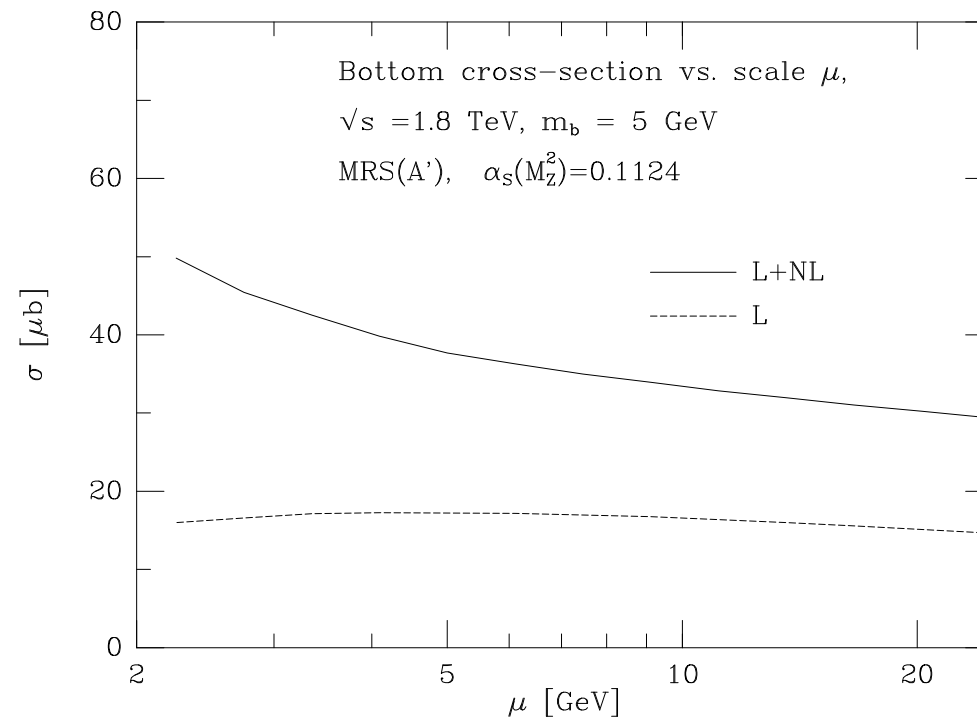
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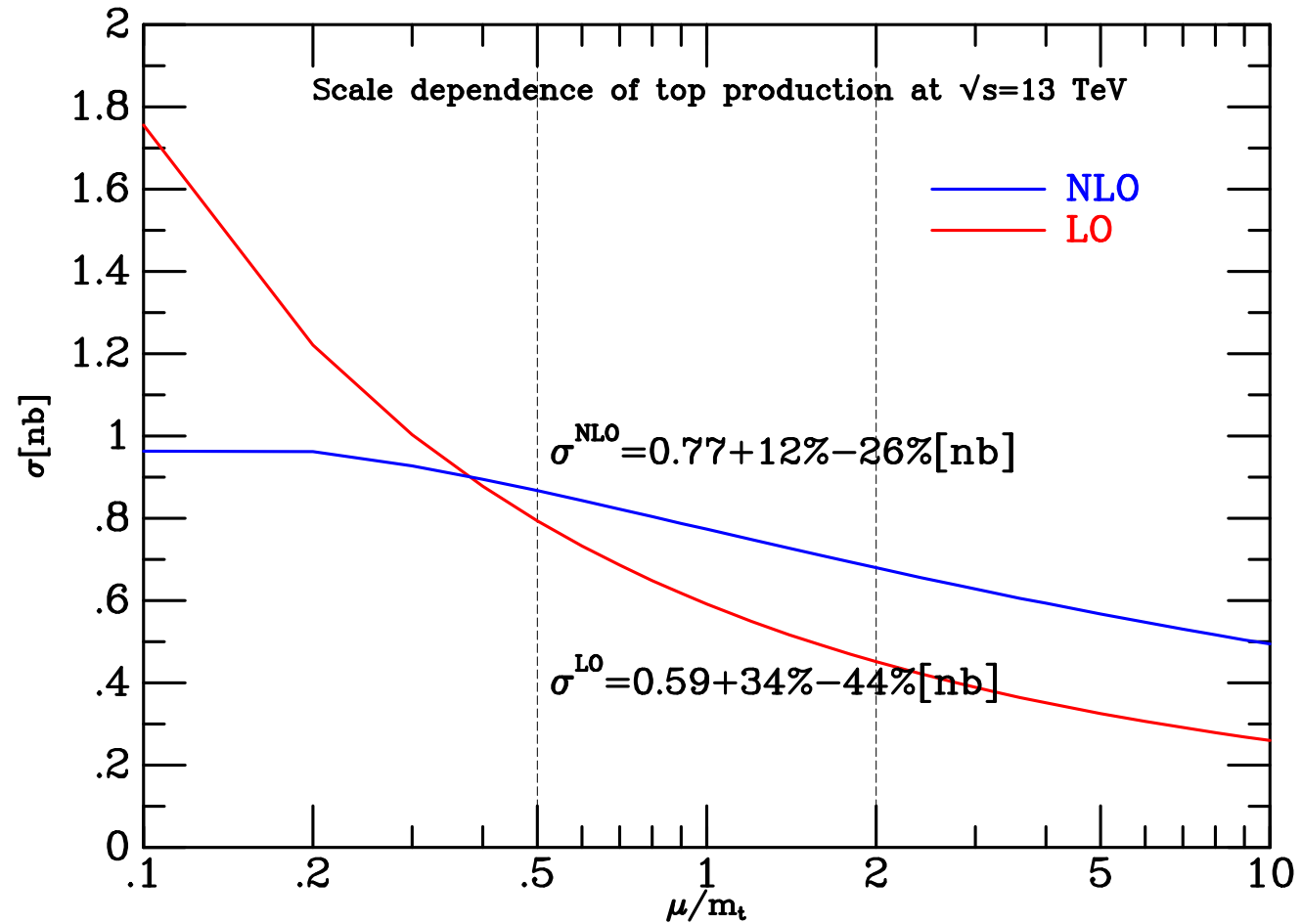
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- This illustrates an important point which is a general feature of renormalization group improved perturbation series in QCD.
- The coefficient of the perturbative correction depends on the choice made for the scale  $\mu$ , but the scale dependence changes the result in such a way that the physical result is independent of that choice.
- Thus the scale dependence is formally small because it is of higher order in  $\alpha_S$ .
- This does not assure us that the scale dependence is actually *numerically* small for all series.
- A pronounced dependence on the scale  $\mu$  is a signal of an untrustworthy perturbation series.



# Scale dependence of top cross section



- Note that despite the fact that  $\alpha_S$  is of order 10%, we do not obtain 10% predictions at NLO.
- This is 'feature' of renormalization group improved perturbation theory.

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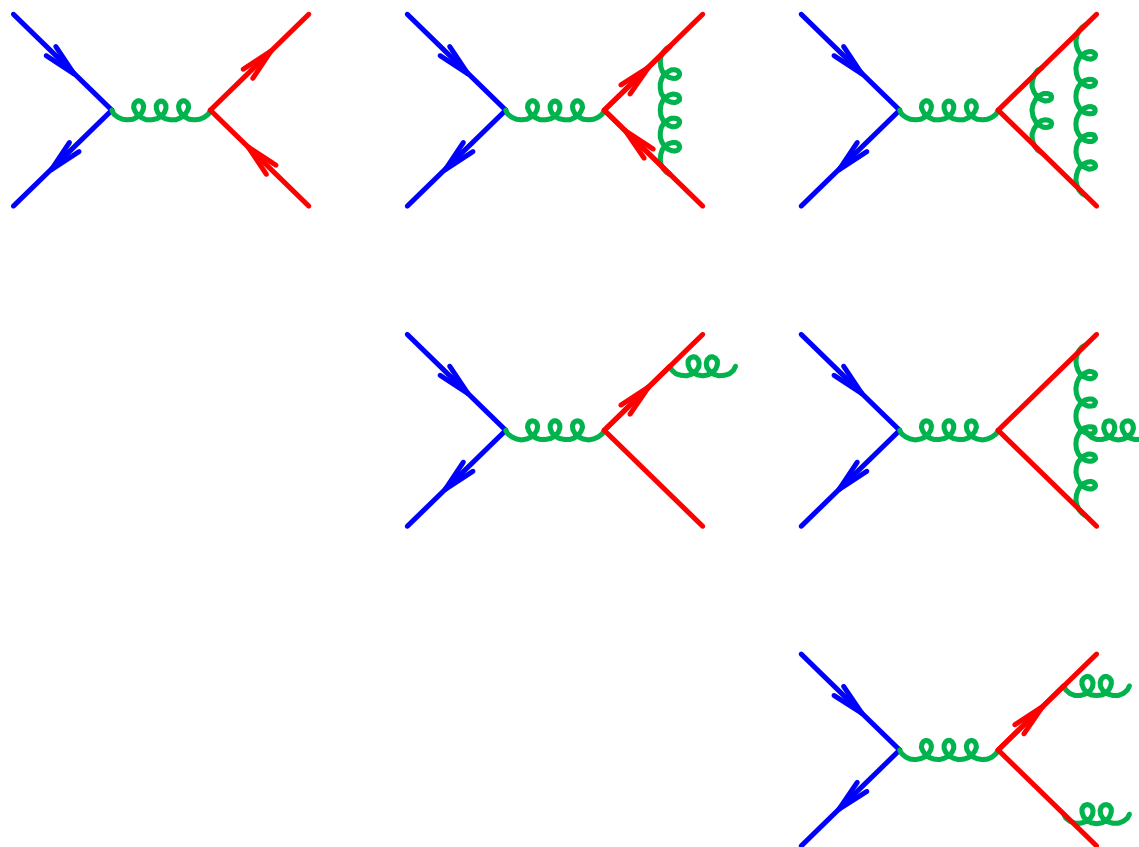
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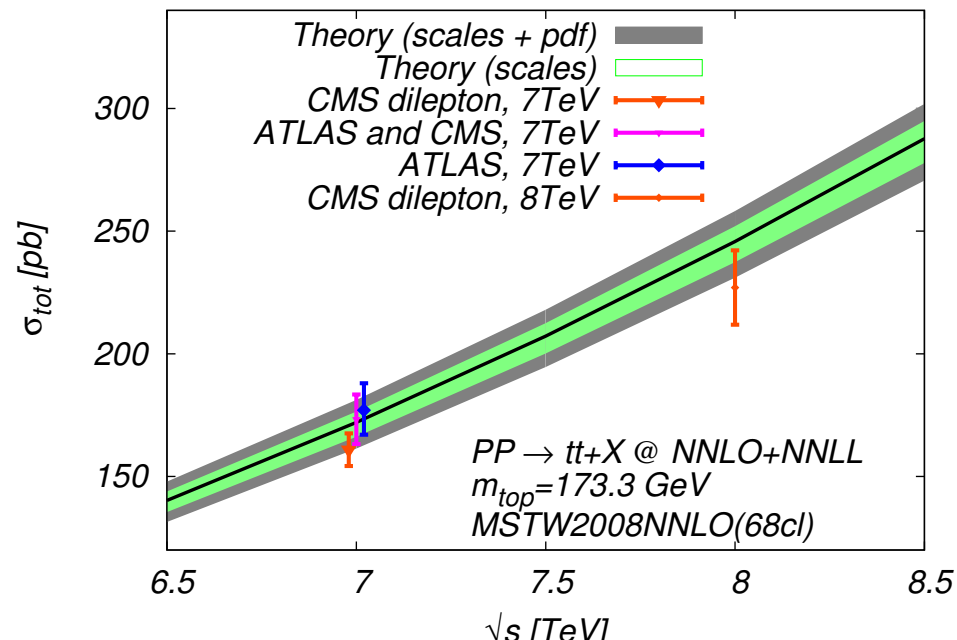


- Challenge is not the calculation of the individual diagrams, but rather the assembly of pieces that individually contain infrared divergences
- as before in the NLO calculations, tension between the need to cancel infra-red divergences, which for the higher multiplicity processes are only manifest after integration and the desire to have a fully differential prediction.

Collider	$\sigma_{\text{tot}}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	+0.110(1.5%) -0.200(2.8%)	+0.169(2.4%) -0.122(1.7%)
LHC 7 TeV	172.0	+4.4(2.6%) -5.8(3.4%)	+4.7(2.7%) -4.8(2.8%)
LHC 8 TeV	245.8	+6.2(2.5%) -8.4(3.4%)	+6.2(2.5%) -6.4(2.6%)
LHC 14 TeV	953.6	+22.7(2.4%) -33.9(3.6%)	+16.2(1.7%) -17.8(1.9%)

Table 1: Best NNLO+NNLL theoretical predictions for various colliders and c.m. energies.

- c.f. scale uncertainty at NLO +12% - 26%



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## ■ Current status of NNLO calculations

dijets	gluon-gluon	PDFs,strong couplings,BSM	1407.5558
H+0jet	fully inclusive N3LO	Higgs couplings	1503.06056
H+1jet	fully exclusive	Higgs couplings,probing GGH vertex	1408.5325,1504.07922,1505.03893
tt pair	fully exclusive, stable tops	mass,pt, FB asymmetry,PDFs BSM	1601.05375
single top	fully exclusive, stable tops, t-channel	$V_{tb}$ ,width, PDFs	1404.7116
WBF	exclusive VBF cuts	Higgs couplings	1506.02660
$W + j$	fully exclusive, decays	PDFs	1504.02131
$Z + j$	decay, off-shell effects	PDFs	1601.04569,1507.20850,1507.02850
ZH	decays to bb at NLO	Higgs couplings	1407.4747,1601.00658
WH	fully exclusive	Higgs couplings	1312.1669, 1601.00658
ZZ	fully exclusive, off-shell	trilinear gauge couplings,BSM	1405.2219, 1507.06257
WW	fully inclusive	trilinear gauge couplings,BSM	1408.5243
$W\gamma, Z\gamma$	fully exclusive	trilinear gauge couplings,BSM	1601.06751
$\gamma\gamma$	fully differential	Background studies	1110.2375,1603.02663
top decay	exclusive	Top couplings	1301.7133
$H - bb$	exclusive, massless	Higgs couplings boosted	1110.2368

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NNLO calculations of top production are needed to undertake a number of high-calibre phenomenological LHC analyses. Some examples are:

- validation of different implementations of higher-order effects in MC event generators,
- extraction of NNLO PDFs from LHC data, (especially the gluon distribution).
- improved determination of the top-quark mass
- direct measurement of the running of  $\alpha_S$  at high scales.
- better control over background for BSM searches



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- Jets are visible to the naked eye, but to use them we need a jet definition.
- Two classes of jet algorithms, (cone and sequential recombination).
- QCD jets are different than jets from the decay of a resonance.
- Top cross section is big; it is important to understand both as a background and as a signal
- NNLO corrections to a few processes (including top production) are becoming known.

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