Maria Laach 2016
Lecture III:
The QCD parton model: Partons and Vector Bosons

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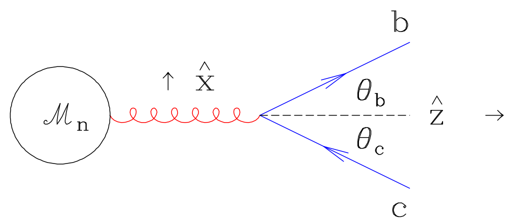


$$
\begin{aligned}
p_a &= (E_a + \frac{p_a^2}{4E_a}, 0, 0, E_a - \frac{p_a^2}{4E_a}) \\
p_b &= (E_b, +E_b \sin\theta_b, 0, +E_b \cos\theta_b) \\
p_c &= (E_c, -E_c \sin\theta_c, 0, +E_c \cos\theta_c)
\end{aligned}
$$

- the kinematics and notation for the branching of parton a into $b + c$. We assume that

$$
p_b^2,\, p_c^2 \ll p_a^2 \equiv t
$$

- a is an outgoing parton, which is called timelike branching since $t > 0$.
- The opening angle is $\theta = \theta_b + \theta_c$. Defining the energy fraction as

$$
z = E_b/E_a = 1 - E_c/E_a \ ,
$$

  we have for small angles, $t = 2E_bE_c(1 - \cos\theta) = z(1-z)E_a^2\theta^2$
- using transverse momentum conservation, $(E_b\theta_b = E_c\theta_c)$,

$$
\theta = \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}} = \frac{\theta_b}{1-z} = \frac{\theta_c}{z} \ .
$$

- The fermions involved in high energy processes can often be taken to be massless.
- We choose an explicit representation for the gamma matrices. The Bjorken and Drell representation is,

$$\gamma^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \gamma^5 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix},$$

The Weyl representation is more suitable at high energy

$$\gamma^0 = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}, \gamma^i = \begin{pmatrix} \mathbf{0} & -\sigma^i \\ \sigma^i & \mathbf{0} \end{pmatrix}, \gamma^5 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix},$$

In the Weyl representation upper and lower components have different helicities.

- Both representations satisfy the same commutation relations (West coast metric!)

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}, \quad \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

- in the Weyl representation $\gamma^0 \gamma^i = \begin{pmatrix} \sigma^i & \mathbf{0} \\ \mathbf{0} & -\sigma^i \end{pmatrix}$. σ are the Pauli matrices.

- In the Weyl representation

$$p_\mu \gamma^\mu = \sqrt{p^+ p_-} \begin{pmatrix} 0 & 0 & \sqrt{\frac{p^+}{p^-}} & e^{-i\varphi} \\ 0 & 0 & e^{+i\varphi} & \sqrt{\frac{p^-}{p^+}} \\ \sqrt{\frac{p^-}{p^+}} & -e^{-i\varphi} & 0 & 0 \\ -e^{+i\varphi} & \sqrt{\frac{p^+}{p^-}} & 0 & 0 \end{pmatrix}$$

$$e^{\pm i\varphi_p} \equiv \frac{p^1 \pm ip^2}{\sqrt{(p^1)^2 + (p^2)^2}} = \frac{p^1 \pm ip^2}{\sqrt{p^+ p^-}}, \qquad p^\pm = p^0 \pm p^3.$$

- The massless spinors solns of Dirac eqn, $\not{p} u_+(p) = \not{p} u_-(p) = 0$ are

$$u_+(p) = \begin{bmatrix} \sqrt{p^+} \\ \sqrt{p^-} e^{i\varphi_p} \\ 0 \\ 0 \end{bmatrix}, \quad u_-(p) = \begin{bmatrix} 0 \\ 0 \\ \sqrt{p^-} e^{-i\varphi_p} \\ -\sqrt{p^+} \end{bmatrix},$$

- In this representation the Dirac conjugate spinors are

$$\overline{u}_+(p) \equiv u_+^\dagger(p)\gamma^0 = \left[0, 0, \sqrt{p^+}, \sqrt{p^-} e^{-i\varphi_p}\right], \quad \overline{u}_-(p) = \left[\sqrt{p^-} e^{i\varphi_p}, -\sqrt{p^+}, 0, 0\right]$$

Normalization $u_\pm^\dagger u_\pm = 2p^0$

- Consider the case where

$$
\begin{aligned}
p_a &= (E_a + \frac{p_a^2}{4E_a}, 0, 0, E_a - \frac{p_a^2}{4E_a}) \\
p_b &\sim (E_b, +E_b\theta_b, 0, +E_b) \\
p_c &\sim (E_c, -E_c\theta_c, 0, +E_c)
\end{aligned}
$$

- Thus for example, $(\theta_b = (1-z)\theta, \theta_c = z\theta)$

$$
u_+^\dagger(p) = \sqrt{2E_b}\left[1, \frac{\theta_b}{2}, 0, 0\right], \quad u_+(p_c) \equiv v_-(p_c) = \sqrt{2E_c}\begin{bmatrix} 1 \\ -\frac{\theta_c}{2} \\ 0 \\ 0 \end{bmatrix}
$$

Hence for polarization vectors $\varepsilon_{in} = (0,1,0,0), \varepsilon_{out} = (0,0,1,0)$

$$
g\bar{u}_+^b\, \gamma^0\gamma^1\, v_-^c = g\sqrt{4E_bE_c}\left(1, \frac{\theta_b}{2}\right)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -\frac{\theta_c}{2} \end{pmatrix} = -g\sqrt{E_bE_c}(\theta_b - \theta_c)
$$

$$-g\bar{u}^b_+\gamma_\mu\varepsilon^{\text{in}\,\mu}_a v^c_- = g\sqrt{E_bE_c}(\theta_b-\theta_c) = g\sqrt{z(1-z)}(1-2z)E_a\theta \approx g(1-2z)\sqrt{t},$$

$$-g\bar{u}^b_+\gamma_\mu\varepsilon^{\text{out}\,\mu}_a v^c_- = ig\sqrt{E_bE_c}(\theta_b+\theta_c) = ig\sqrt{z(1-z)}E_a\theta \approx ig\sqrt{t}.$$

- The squared branching probabilities both vanish in the forward direction
- the matrix element relation for the branching is

$$|\mathcal{M}_{n+1}|^2 \sim \frac{g^2}{t}T_R F(z;\varepsilon_a,\lambda_b,\lambda_c)|\mathcal{M}_n|^2$$

where the colour factor is now $\text{Tr}(t^At^A)/8 = T_R = 1/2$. The non-vanishing functions $F(z;\varepsilon_a,\lambda_b,\lambda_c)$ for quark and antiquark helicities λ_b and λ_c are

ε_a	λ_b	λ_c	$F(z;\varepsilon_a,\lambda_b,\lambda_c)$
in	$\pm$	$\mp$	$(1-2z)^2$
out	$\pm$	$\mp$	1

Summing over the polarizations we get

$$2\left[(1-2z)^2+1\right] = 4(z^2+(1-z)^2).$$

This is the branching probability for gluon into a quark, $P_{qg} = T_R(z^2+(1-z)^2)$

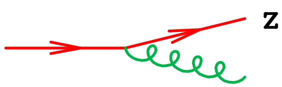
$$d\sigma_{n+1} = d\sigma_n \frac{dt}{t}\, dz \,\frac{d\phi}{2\pi}\, \frac{\alpha_S}{2\pi}\, CF, \quad \int \frac{d\phi}{2\pi} CF = \hat{P}_{ba}(z)$$

where $\hat{P}_{ba}(z)$ is the appropriate splitting function, (C=colour factor,F=polarization dependent splitting function)

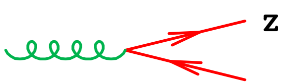
$$d\sigma_{n+1} = d\sigma_n \frac{dt}{t}\, dz\, \frac{\alpha_S}{2\pi}\, \hat{P}_{ba}(z)\ .$$

- Including all the color factors we find the results for the unregulated branching probabilities.

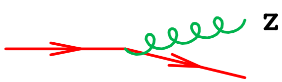
$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)}\right],$$



$$\hat{P}_{qg}(z) = T_R \left[z^2 + (1-z)^2\right], \quad T_R = \frac{1}{2},$$

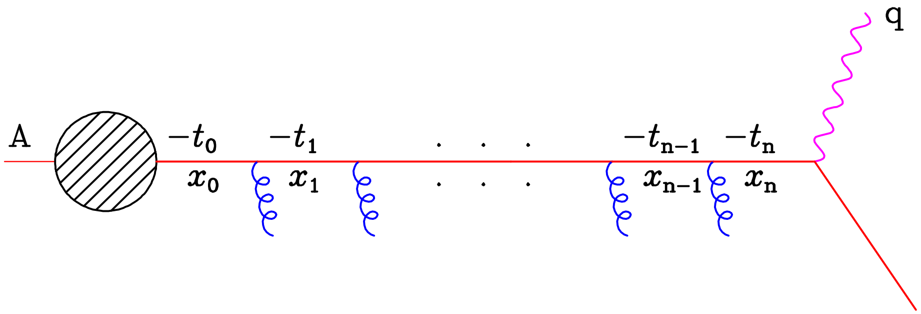


$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right],$$



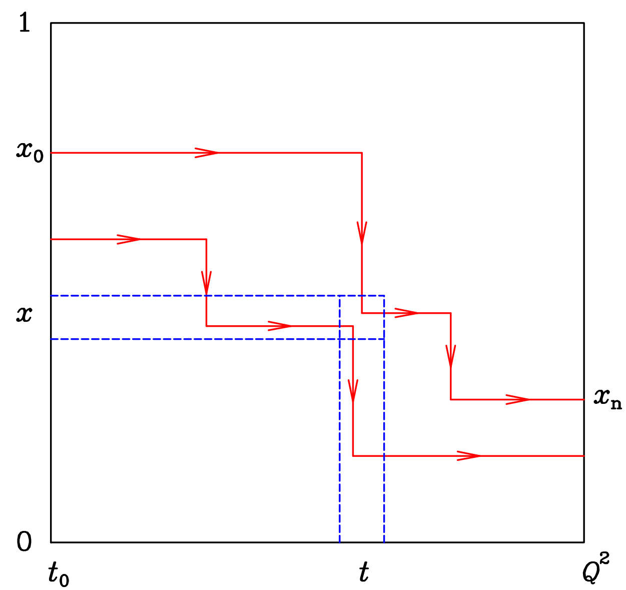
$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z\,(1-z)\right]$$

- Consider enhancement of higher-order contributions due to multiple small-angle parton emission, for example in deep inelastic scattering (DIS)



- Incoming quark from target hadron, initially with low virtual mass-squared $-t_0$ and carrying a fraction x_0 of hadron's momentum, moves to more virtual masses and lower momentum fractions by successive small-angle emissions, and is finally struck by photon of virtual mass-squared $q^2 = -Q^2$.
- Cross section will depend on Q^2 and on momentum fraction distribution of partons seen by virtual photon at this scale, $D(x, Q^2)$.

- To derive evolution equation for Q^2-dependence of $D(x, Q^2)$, first introduce pictorial representation of evolution, also useful later for Monte Carlo simulation.



- Represent sequence of branchings by path in (t, x)-space. Each branching is a step downwards in x, at a value of t equal to (minus) the virtual mass-squared after the branching.
- At $t = t_0$, paths have distribution of starting points $D(x_0, t_0)$ characteristic of target hadron at that scale. Then distribution $D(x, t)$ of partons at scale t is just the x-distribution of paths at that scale.

- Consider change in the parton distribution $D(x,t)$ when t is increased to $t+\delta t$. This is number of paths arriving in element $(\delta t, \delta x)$ minus number leaving that element, divided by δx.
- Number arriving is branching probability times parton density integrated over all higher momenta $x' = x/z$,

$$\begin{aligned}\delta D_{\text{in}}(x,t) &= \frac{\delta t}{t}\int_x^1 dx'\,dz\,\frac{\alpha_{\text{S}}}{2\pi}\hat{P}(z)D(x',t)\,\delta(x-zx') \\ &= \frac{\delta t}{t}\int_0^1 \frac{dz}{z}\frac{\alpha_{\text{S}}}{2\pi}\hat{P}(z)D(x/z,t)\end{aligned}$$

- For the number leaving element, must integrate over lower momenta $x' = zx$:

$$\begin{aligned}\delta D_{\text{out}}(x,t) &= \frac{\delta t}{t}D(x,t)\int_0^x dx'\,dz\,\frac{\alpha_{\text{S}}}{2\pi}\hat{P}(z)\,\delta(x'-zx) \\ &= \frac{\delta t}{t}D(x,t)\int_0^1 dz\,\frac{\alpha_{\text{S}}}{2\pi}\hat{P}(z)\end{aligned}$$

- Change in population of element is

$$\begin{aligned}\delta D(x,t) &= \delta D_{\text{in}} - \delta D_{\text{out}} \\ &= \frac{\delta t}{t}\int_0^1 dz\, \frac{\alpha_{\text{S}}}{2\pi}\hat{P}(z)\left[\frac{1}{z}D(x/z,t) - D(x,t)\right] .\end{aligned}$$

- Introduce plus-prescription with definition

$$\int_0^1 dx\; f(x)\, g(x)_+ = \int_0^1 dx\; [f(x) - f(1)]\, g(x) .$$

- Using this we can define regularized splitting function

$$P(z) = \hat{P}(z)_+$$

- Plus-prescription, like the Dirac-delta function, is only defined under integral sign.
- Plus-prescription includes some of the effects of virtual diagrams.

We obtain the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation:

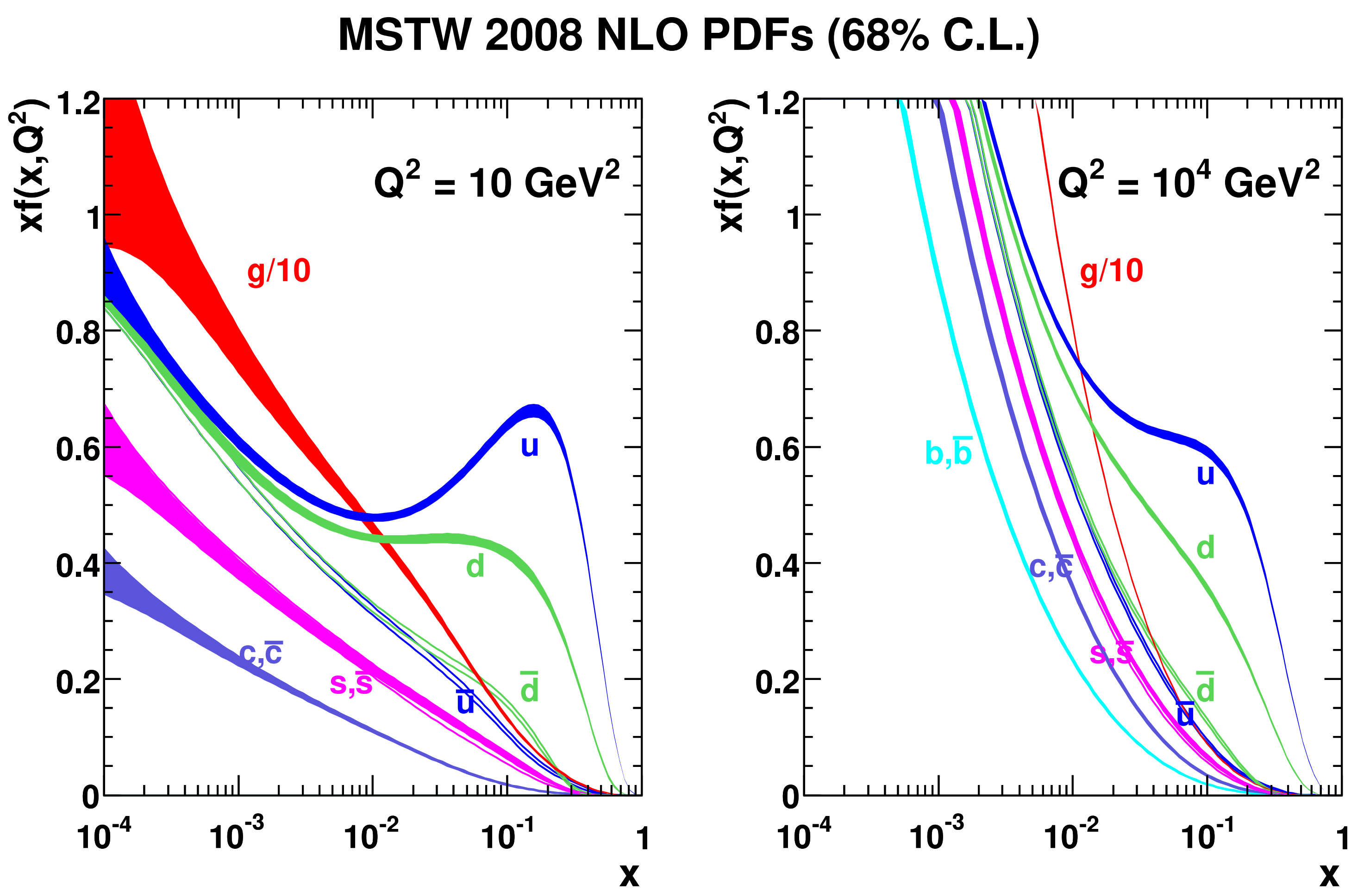
$$t\frac{\partial}{\partial t}D(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P(z)D(x/z,t)\ .$$

- Here $D(x,t)$ represents parton momentum fraction distribution inside incoming hadron probed at scale t.
- In timelike branching, it represents instead hadron momentum fraction distribution produced by an outgoing parton. Boundary conditions and direction of evolution are different, but evolution equation remains the same.
- For several different types of partons, must take into account different processes by which parton of type i can enter or leave the element $(\delta t, \delta x)$. This leads to coupled DGLAP evolution equations of form

$$t\frac{\partial}{\partial t}D_i(x,t) = \sum_j \int_x^1 \frac{dz}{z}\frac{\alpha_S}{2\pi}P_{ij}(z)D_j(x/z,t)\ .$$

- Quark ($i = q$) can enter element via either $q \to qg$ or $g \to q\bar{q}$, but can only leave via $q \to qg$. Thus plus-prescription applies only to $q \to qg$ part, giving

$$P_{qg}(z) = \hat{P}_{qg}(z) = T_R\,[z^2 + (1-z)^2], \quad P_{qq}(z) = \hat{P}_{qq}(z)_+ = C_F\left(\frac{1+z^2}{1-z}\right)_+$$



- Scale dependent parton distributions determined from experiment.
- Their behaviour with Q^2 (large x:shrinkage, small x:growth), determined by DGLAP eqn.
- N^2LO terms (and partial N^3LO terms) in DGLAP equation now known.

- DGLAP equations convenient for evolution of parton distributions. To study structure of final states, slightly different form is useful. Consider again simplified treatment with only one type of branching. Introduce Sudakov form factor:

$$\Delta(t) \equiv \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \, \frac{\alpha_S}{2\pi} \hat{P}(z)\right] ,$$

- the DGLAP equation derived previously can be written as,

$$\frac{tD(x,t)}{dt} = \int_0^1 dz \, \frac{\alpha_S}{2\pi} \hat{P}(z) \left[\frac{1}{z} D(x/z,t) - D(x,t)\right] .$$

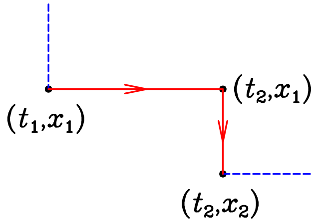
- This can be written in terms of the Sudakov form factor as

$$\begin{aligned} t\frac{\partial}{\partial t} D(x,t) &= \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) D(x/z,t) + \frac{D(x,t)}{\Delta(t)} t \frac{\partial}{\partial t} \Delta(t) , \\ t\frac{\partial}{\partial t}\left(\frac{D}{\Delta}\right) &= \frac{1}{\Delta} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \hat{P}(z) D(x/z,t) . \end{aligned}$$

Integrated form of DGLAP equation and shower Monte Carlo

$$D(x,t) = \Delta(t)D(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \frac{\alpha_S}{2\pi} \int \frac{dz}{z} P(z) D(x/z,t')\ .$$

- the first term on the right-hand side is the contribution from paths that do not branch between scales t_0 and t.
- Thus the Sudakov form factor $\Delta(t)$ is simply the probability of evolving from t_0 to t without branching.
- The second term is the contribution from all paths which have their last branching at scale t'.
- The basic problem that the Monte Carlo branching algorithm has to solve is as follows: given the virtual mass scale and momentum fraction (t_1, x_1) after some step of the evolution, or as initial conditions, generate the values (t_2, x_2) after the next step.

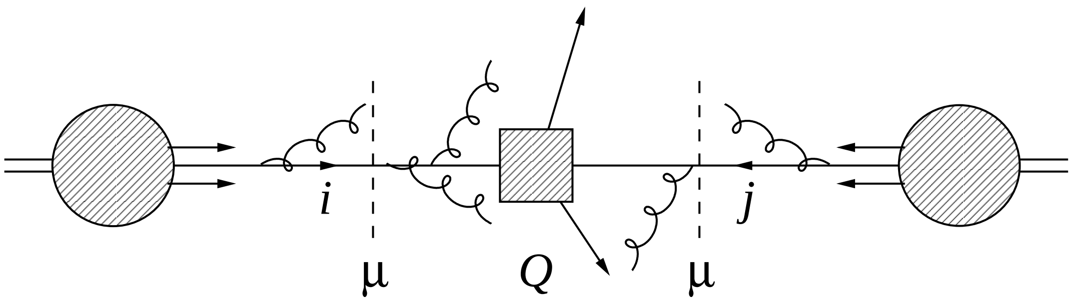


- t_2 and x_2 can be generated with the right distributions with two random numbers by solving the following relations,

$$\frac{\Delta(t_2)}{\Delta(t_1)} = \mathcal{R}$$

$$\int_{\epsilon}^{x_2/x_1} dz \frac{\alpha_S}{2\pi} P(z) = \mathcal{R}' \int_{\epsilon}^{1-\epsilon} dz \frac{\alpha_S}{2\pi} P(z)$$

- In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



- For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu^2), Q^2/\mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

- ☐ Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- ☐ Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to soft underlying event and/or multiple hard scattering.

- Why does the factorization property hold and when it should fail?
- For a heuristic argument, consider the vector boson production, the simplest hard process involving two hadrons

$$H_1(P_1) + H_2(P_2) \to V + X.$$

- Do the partons in hadron H_1, through the influence of their colour fields, change the distribution of partons in hadron H_2 before the vector boson is produced? Soft gluons which are emitted long before the collision are potentially troublesome.
- A simple model from classical electrodynamics. The vector potential due to an electromagnetic current density J is given by

$$A^\mu(t, \vec{x}) = \int dt' d\vec{x}' \, \frac{J^\mu(t', \vec{x}')}{|\vec{x} - \vec{x}'|} \, \delta(t' + |\vec{x} - \vec{x}'| - t) \, ,$$

where the delta function provides the retarded behaviour required by causality.

- Consider a particle with charge e travelling in the positive z direction with constant velocity β. The non-zero components of the current density are

$$
\begin{aligned}
J^t(t',\vec{x}') &= e\delta(\vec{x}' - \vec{r}(t'))\ , \\
J^z(t',\vec{x}') &= e\beta\delta(\vec{x}' - \vec{r}(t')),\ \ \vec{r}(t') = \beta t'\hat{z},
\end{aligned}
$$

$\hat{z}$ is a unit vector in the z direction. At an observation point (the supposed position of hadron H_2) described by coordinates x, y and z, the vector potential (either performing the integrations using the current density given above, or by Lorentz transformation of the scalar potential in the rest frame of the particle) is

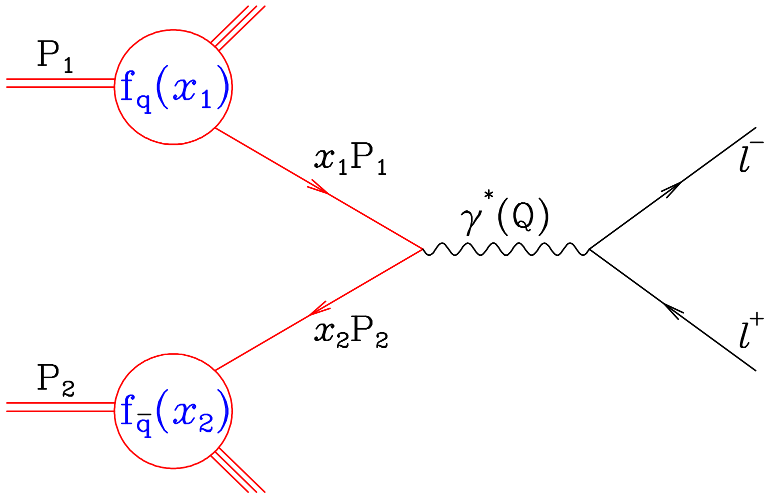
$$
\begin{aligned}
A^t(t,\vec{x}) &= \frac{e\gamma}{\sqrt{[x^2+y^2+\gamma^2(\beta t - z)^2]}} \\
A^x(t,\vec{x}) &= 0 \\
A^y(t,\vec{x}) &= 0 \\
A^z(t,\vec{x}) &= \frac{e\gamma\beta}{\sqrt{[x^2+y^2+\gamma^2(\beta t - z)^2]}},
\end{aligned}
$$

where $\gamma^2 = 1/(1-\beta^2)$. Target hadron H_2 is at rest near the origin, so that $\gamma \approx s/m^2$.

- Note that for large γ and fixed non-zero $(\beta t - z)$ some components of the potential tend to a constant independent of γ, suggesting that there will be non-zero fields which are not in coincidence with the arrival of the particle, even at high energy.
- However at large γ the potential is a pure gauge piece, $A^\mu = \partial^\mu \chi$ where χ is a scalar function
- Covariant formulation using the vector potential A has large fields which have no effect.
- For example, the electric field along the z direction is

$$E^z(t, \vec{x}) = F^{tz} \equiv \frac{\partial A^z}{\partial t} + \frac{\partial A^t}{\partial z} = \frac{e\gamma(\beta t - z)}{[x^2 + y^2 + \gamma^2(\beta t - z)^2]^{\frac{3}{2}}}.$$

The leading terms in γ cancel and the field strengths are of order $1/\gamma^2$ and hence of order m^4/s^2. The model suggests the force experienced by a charge in the hadron H_2, at any fixed time before the arrival of the quark, decreases as m^4/s^2.



- Mechanism for Lepton pair production, W-production, Z-production, Vector-boson pairs, . . .
- Collectively known as the Drell-Yan process.
- Colour average $1/N$.

$$\frac{d\hat{\sigma}}{dQ^2} = \frac{\sigma_0}{N}\, Q_q^2\, \delta(\hat{s} - Q^2), \qquad \sigma_0 = \frac{4\pi\alpha^2}{3Q^2}, \quad \text{cf } e^+e^- \text{ annihilation.}$$

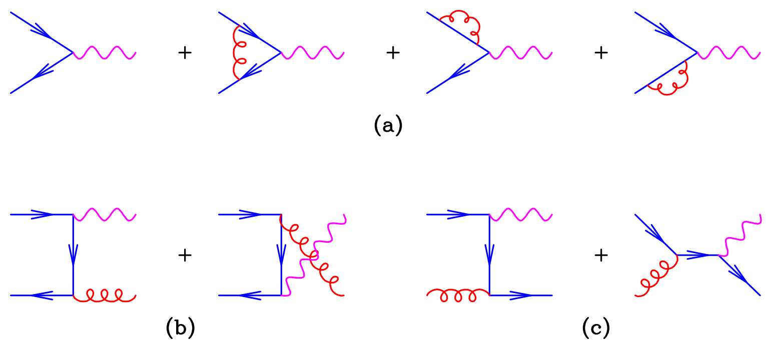
- In the CM frame of the two hadrons, the momenta of the incoming partons are

$$p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1), \quad p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)\ .$$

The square of the $q\bar{q}$ collision energy $\hat{s}$ is related to the overall hadron-hadron collision energy by $\hat{s} = (p_1 + p_2)^2 = x_1 x_2 s$. The parton-model cross section for this process is:

$$\begin{aligned} \frac{d\sigma}{dM^2} &= \int_0^1 dx_1 dx_2 \sum_q \{f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\} \frac{d\hat{\sigma}}{dM^2}(q\bar{q} \to l^+ l^-) \\ &= \frac{\sigma_0}{Ns} \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} \delta(1-z) \left[\sum_q Q_q^2 \{f_q(x_1) f_{\bar{q}}(x_2) + (q \leftrightarrow \bar{q})\} \right] . \end{aligned}$$

- For later convenience we have introduced the variable $z = \frac{Q^2}{\hat{s}} = \frac{Q^2}{x_1 x_2 s}$.
- The sum here is over quarks only and the $\bar{q}q$ contributions are indicated explicitly.



- The contribution of the real diagrams (in four dimensions) is

$$|M|^2 \sim g^2 C_F \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut} \right] = g^2 C_F \left[\left(\frac{1+z^2}{1-z} \right) \left(\frac{-s}{t} + \frac{-s}{u} \right) - 2 \right]$$

  where $z = Q^2/s, s+t+u = Q^2$.
- Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.
- The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.
- Ignore for simplicity the diagrams with incoming gluons.

- Control the divergences by continuing the dimensionality of space-time, $d = 4 - 2\epsilon$, (technically this is dimensional reduction).

$$PS = \frac{c_\Gamma}{8\pi}\left(\frac{1}{Q^2}\right)^\epsilon z^\epsilon (1-z)^{1-2\epsilon}\int_0^1 dy\,(y(1-y))^{-\epsilon}$$

where

$$s = \frac{Q^2}{z},\quad t = -\frac{Q^2}{z}(1-z)(1-y)\quad u = -\frac{Q^2}{z}(1-z)y,\quad y = \frac{1}{2}(1+\cos\theta).$$

- Performing the phase space integration, the total contribution of the real diagrams is

$$\begin{aligned}\sigma_R &= \frac{\alpha_S}{2\pi}C_F\left(\frac{\mu^2}{Q^2}\right)^\epsilon c_\Gamma\Bigg[\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\frac{\pi^2}{3}\right)\delta(1-z)-\frac{2}{\epsilon}P_{qq}(z)\\ &- 2(1-z)+4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z\Bigg]\end{aligned}$$

with $c_\Gamma = (4\pi)^\epsilon/\Gamma(1-\epsilon)$.

- The contribution of the virtual diagrams is (neglecting terms of order ϵ)

$$\sigma_V = \delta(1-z)\left[1+\frac{\alpha_S}{2\pi}C_F\left(\frac{\mu^2}{Q^2}\right)^\epsilon c'_\Gamma\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-6+\pi^2\right)\right]$$

- Adding it up we get in dim-reduction

$$\sigma_{R+V} = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \Bigg[\left(\frac{2\pi^2}{3} - 6\right)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) - 2(1-z)$$
$$+ \quad 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z\Bigg]$$

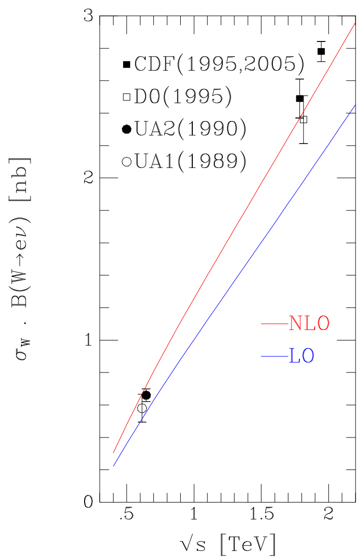
- The divergences, proportional to the branching probability , are universal.
- We will factorize them into the parton distributions. We perform the mass factorization by subtracting the counterterm, (The finite terms are necessary to get us to the $\overline{MS}$-scheme).

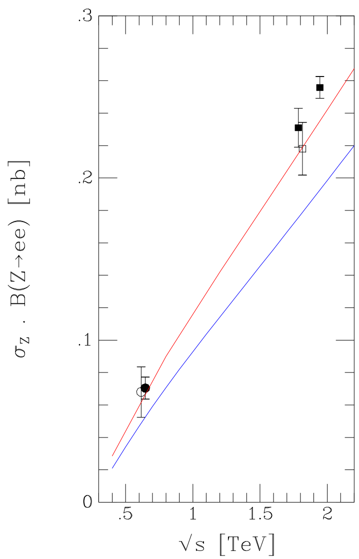
$$2\frac{\alpha_S}{2\pi} C_F \left[\frac{-c_\Gamma}{\epsilon} P_{qq}(z) - (1-z) + \delta(1-z)\right]$$

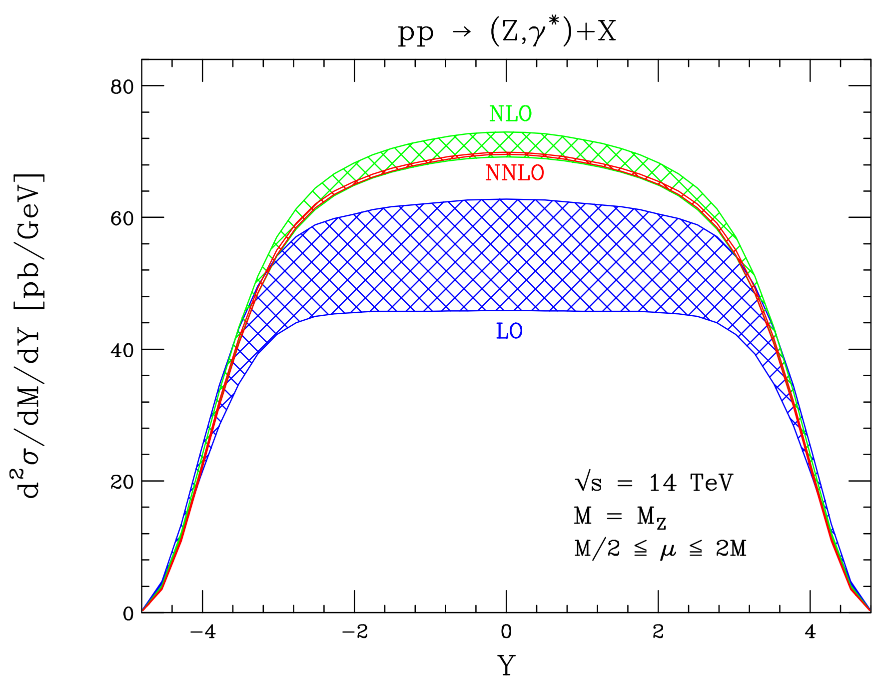
$$\hat{\sigma} = \frac{\alpha_S}{2\pi} C_F \Bigg[\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z$$
$$+ \quad 2P_{qq}(z)\ln\frac{Q^2}{\mu^2}\Bigg]$$

- Similar correction for incoming gluons.

- Agreement with NLO theory is good.
- LO curves lie about 25% too low.
- NNLO results are also known and lead to a further modest (4%) increase at the Tevatron.
- NLO corrections for Z and W production at $\sqrt{s} = 13$ TeV remain a 22% effect.
- NNLO corrections are small at the LHC







- We would like to go beyond the results for the total cross section to get results for distributions.
- We have two separate divergent integrals which must be combined before numerical integration

$$\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

- Note that the jet definition can be arbitratrily complicated.

$$d\sigma^R = PS_{m+1}|\mathcal{M}_{m+1}|^2 F^J_{m+1}(p_1, \ldots p_{m+1})$$

  We need to combine the two pieces, which reside in phase-spaces of different dimensionality, without knowledge of F^J.
- Divergences regularized in $d = 4 - 2\epsilon$ dimensions.
- Two solutions: phase space slicing and subtraction.
- Illustrate with a simple one-dimensional example.

$$|\mathcal{M}_{m+1}|^2 \equiv \frac{1}{x}\mathcal{M}(x), \;\; |\mathcal{M}_m|^2 \equiv \frac{1}{\epsilon}\nu + k$$

  x is the energy of an emitted gluon.

Subtraction - one dimensional model

- Divergences regularized in $d = 4 - 2\epsilon$ dimensions. Two solutions: phase space slicing and subtraction.
- Thus the full cross section in d dimensions is

$$\sigma = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + (\frac{1}{\epsilon}\nu + k) F_0^J$$

- Infrared safety: $F_1^J(0) = F_0^J$, KLN cancellation theorem, $\mathcal{M}(0) = \nu$
- Exact identity

$$\begin{aligned} \sigma &= \int_0^1 \frac{dx}{x^{1+\epsilon}} \Big[\mathcal{M}(x) F_1^J(x) - \mathcal{M}(0) F_1^J(0)\Big] + \int_0^1 \frac{dx}{x^{1+\epsilon}} \nu F_0^J + (\frac{1}{\epsilon}\nu + k) F_0^J \\ &= \int_0^1 \frac{dx}{x} \Big[\mathcal{M}(x) F_1^J(x) - \mathcal{M}(0) F_1^J(0)\Big] + k F_0^J \end{aligned}$$

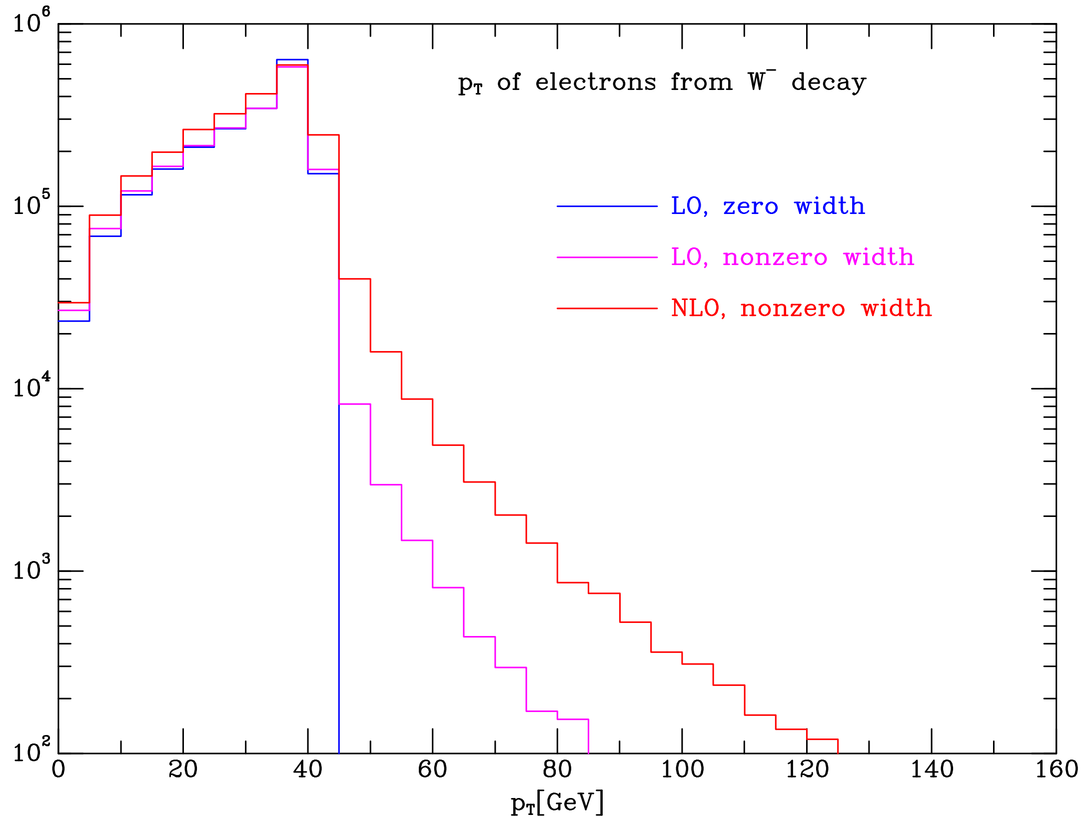
- In practice we have to introduce a cutoff to protect from numerical overflow.

$$\sigma = \int_\delta^1 \frac{dx}{x} \Big[\mathcal{M}(x) F_1^J(x) - \mathcal{M}(0) F_1^J(0)\Big] + k F_0^J$$

Thus the full cross section in d dimensions is

$$
\begin{aligned}
\sigma &= \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + (\frac{1}{\epsilon}\nu + k) F_0^J \\
&= \int_\delta^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + \int_0^\delta \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_1^J(x) + (\frac{1}{\epsilon}\nu + k) F_0^J \\
&\approx \int_\delta^1 \frac{dx}{x} \mathcal{M}(x) F_1^J(x) + \mathcal{M}(0) F_1^J(0) \int_0^\delta \frac{dx}{x^{1+\epsilon}} + (\frac{1}{\epsilon}\nu + k) F_0^J \\
&= \int_\delta^1 \frac{dx}{x} \mathcal{M}(x) F_1^J(x) + \ln(\delta)\nu F_0^J + k F_0^J
\end{aligned}
$$

- δ must be chosen small enough that the power corrections of order δ can be neglected.
- Important to establish that the final result is independent of the slicing parameter δ.
- large numerical cancellations at $\delta \to 0$.



- Calculation of NLO corrections, give a better prediction for the rate.
- At NLO new parton processes can contribute.
- Extra radiation can modify kinematic distributions.

Slicing methods

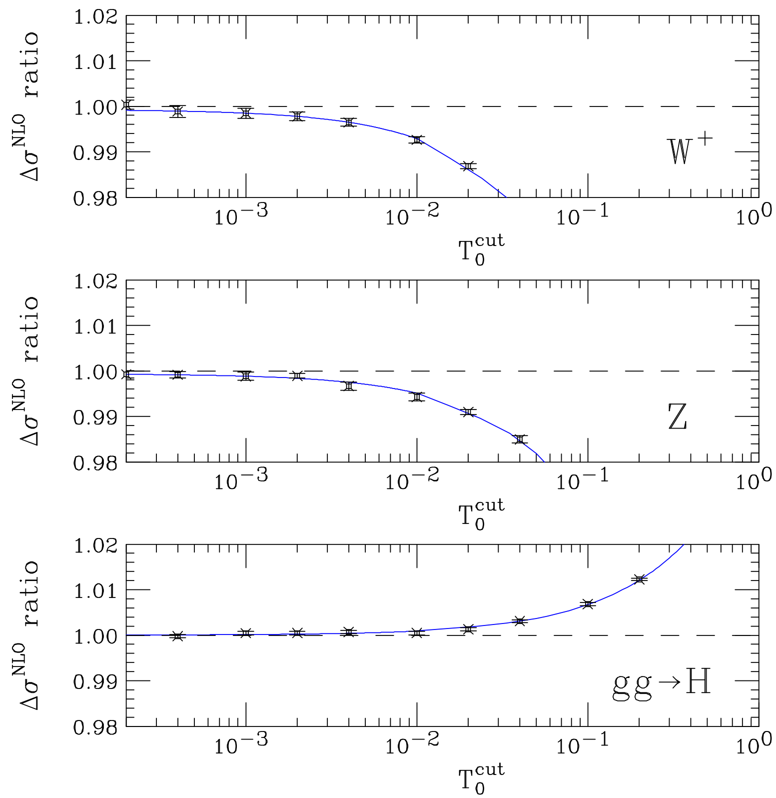
- Slicing methods have recently been applied in NNLO calculations.
- Here for illustration we show results obtained at NLO.
- The resolved region of phase space corresponds to a calculation of the process with one additional final state parton, in this case one gluon emission.
- if a suitable resolution parameter is chosen, the unresolved region can be directly calculated.
- The jettiness of parton j with momentum p_j is defined as

$$\tau(p_j) = \min_{i=a,b,1,\ldots,N} \left\{ \frac{2\, q_i \cdot p_j}{Q_i} \right\} ,$$

- The resolution parameter in the attached plots is the jettiness, and the behaviour below the cut is theoretically calculable.
- The resolution parameter should be chosen small enough, that power corrections are negligible, but not so small that numerical errors in the cancellation dominate.

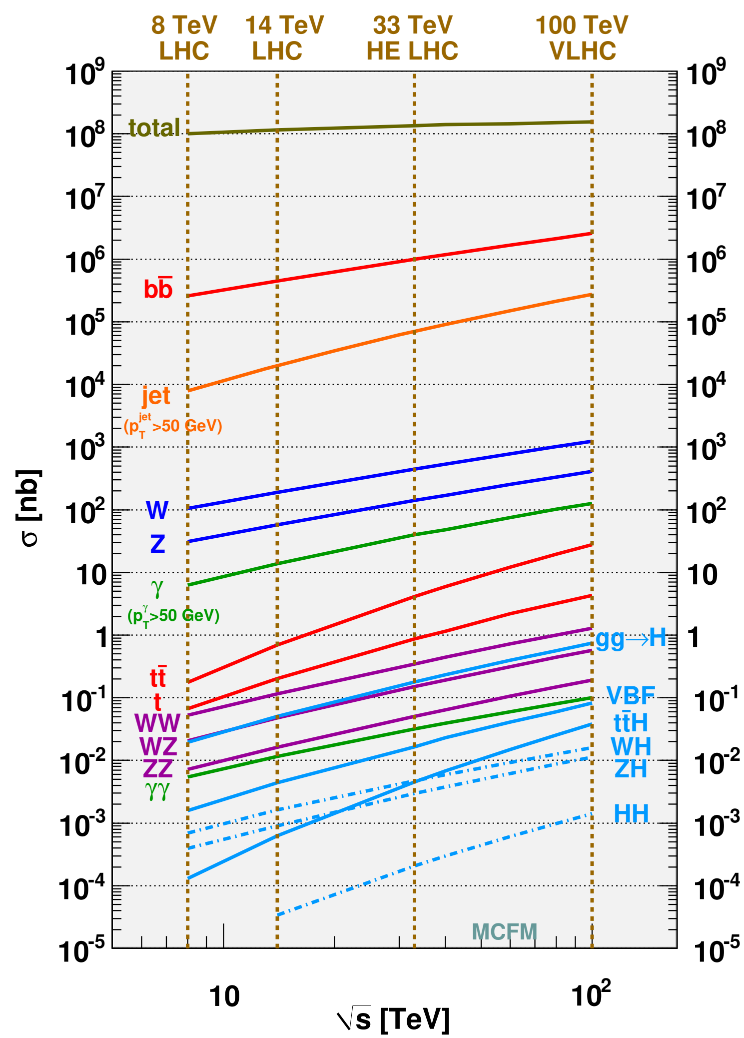
Boughezal et al, 1605.08011

- Comparison of results calculated with MCFM, using subtraction and slicing.



Parton branching - kinematics
Massless Dirac equation
Branching probabilities
DGLAP equation
Evolution of Parton distributions
Sudakov form factor
Hadron-hadron processes
Factorization of the cross section
Lepton-pair production
NLO QCD: Parton level integrators
Why NLO?
Slicing methods
NLO results
Recap
Bibliography

- Fully automatic procedures
  - Madgraph5_aMC@NLO 1405.0301
  - Helac-1Loop 1502.01521
  - Go-Sam 1404.7096
- Approaches for greater number of legs of a less automatic nature.
  - Blackhat-Sherpa 1310.2808
  - Njet 1312.7140
- Libraries of simple processes
  - MCFM, hep-ph/9905386,arXiv:1503.06182
  - VBFNLO, arXiv:1404.3940



Process	μ	n_{lf}	Cross section (pb)	
			LO	NLO
$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
$pp \to t\bar{b}j$	$m_{top}/4$	4	31.37 ± 0.03	32.86 ± 0.04
$pp \to t\bar{b}jj$	$m_{top}/4$	4	11.91 ± 0.006	7.299 ± 0.05
$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	289.7 ± 0.3
$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	54.1 ± 0.6
$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.08241 ± 0.0004
$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.333 ± 0.002

- Parton branching gives rise to universal branching probabilities, independent of the process.
- The DGLAP equation predicts the change with scale of the parton distributions: shrinkage at large x and growth at small x.
- The branching probabilities are the basis of shower Monte Carlo programs
- The master formula predicts that hadron-hadron processes are factorized. Parton distributions measured, e.g. in deep inelastic scattering, can be used at LHC
- NLO corrections can be used to give exclusive predictions using subtraction or slicing methods.
- If the number of partons is not too large, fully automatic procedures can be used to calculate NLO corrections.

- R. K. Ellis, W.J. Stirling and B.R. Webber, *QCD and Collider Physics* (Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology)
- G. Altarelli, R. K. Ellis and G. Martinelli, “Large Perturbative Corrections to the Drell-Yan Process in QCD,” Nucl. Phys. B **157**, 461 (1979). doi:10.1016/0550-3213(79)90116-0
- S. Catani and M. H. Seymour, *The Dipole Formalism for the Calculation of QCD Jet Cross Sections at Next-to-Leading Order*, Phys. Lett. B **378** (1996) 287 [arXiv:hep-ph/9602277].
- R. K. Ellis, Z. Kunszt, K. Melnikov and G. Zanderighi, “One-loop calculations in quantum field theory: from Feynman diagrams to unitarity cuts,” Phys. Rept. **518**, 141 (2012) doi:10.1016/j.physrep.2012.01.008 [arXiv:1105.4319 [hep-ph]].
- G. Ossola, C. G. Papadopoulos and R. Pittau, “Reducing full one-loop amplitudes to scalar integrals at the integrand level,” Nucl. Phys. B **763**, 147 (2007) doi:10.1016/j.nuclphysb.2006.11.012 [hep-ph/0609007].