Maria Laach 2016
Lecture II:
The EW Standard Model

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$$\mathcal{L}_{\text{classical}} = -\frac{1}{4}\sum_i W^{i\ \mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}\ ,$$

- $W^i_{\mu\nu}$ and $B_{\mu\nu}$ are the field strength tensors of the U(1) gauge field B and the SU(2) gauge fields W^i, (g_W is SU(2) gauge coupling.)

$$\begin{aligned} W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_W \epsilon^{ijk} W^j_\mu W^k_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\ , \end{aligned}$$

- The Lagrangian evidently describes four massless vector bosons forming a singlet (B) and a triplet (W^1, W^2, W^3) under weak isospin.
- The coupling of the gauge fields to fermionic matter fields is implemented using the covariant derivative, which is

$$D^\mu = \delta_{ij}\partial^\mu + i g_W (T\cdot W^\mu)_{ij} + iY\delta_{ij} g'_W B^\mu$$

where g'_W is the U(1) gauge coupling.

- The matrices T are a representation of the SU(2) *weak isospin* algebra and the U(1) charge Y is called the *weak hypercharge*. In order to specify the coupling to matter we therefore have to choose the SU(2) representation, T, and the U(1) gauge charge, Y, for the matter fields.

$$[T^i, T^j] = i\epsilon^{ijk}T^k, \ \epsilon^{123} = 1 \ .$$

- Defining $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ and $T^\pm = T^1 \pm iT^2$ we have

$$W_\mu \cdot T = W_\mu^3 T_3 + \frac{1}{\sqrt{2}} W_\mu^+ T^+ + \frac{1}{\sqrt{2}} W_\mu^- T^-$$

where the matrices $T^\pm$and T^3 satisfy the relations

$$[T^+, T^-] = 2T^3, \quad [T^3, T^\pm] = \pm T^\pm \ .$$

T^+ and T^- are the weak isospin raising and lowering operators. For example, in the doublet representation of SU(2) we have

$$T^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}, \ T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \ T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} .$$

- Inserting a mass term for the W and B fields breaks gauge invariance.

- Inserting a mass term for the W and B fields violates gauge invariance, but adopt a practical approach and add one anyway

$$\mathcal{L}_{\rm mass} = \frac{1}{2}\left[M^2 \sum_i W^i_\mu W^{i\,\mu} + M_0^2 B^\mu B_\mu + 2M_{03}^2 W^3_\mu B^\mu\right]$$

- including the mass term, the terms bilinear in the boson fields become,

$$\begin{aligned}\mathcal{L}_{\rm gauge} &= -\frac{1}{2} W^+_{\mu\nu} W^{-\,\mu\nu} + M^2 W^+_\mu W^{-\,\mu} \\ &- \frac{1}{4}\left[W^3_{\mu\nu} W^{3\,\mu\nu} + B_{\mu\nu} B^{\mu\nu}\right. \\ &- \frac{1}{2}\left[M^2 W^3_\mu (W^3)^\mu + M_0^2 B_\mu B^\mu + 2M_{03}^2 W^3_\mu B^\mu\right]\end{aligned}$$

- First line defines 2 electrically charged spin one bosons with mass $M = M_W$.
- The mass matrix for the electrically neutral fields is

$$\frac{1}{2}(W^3_\mu, B_\mu)\mathcal{M}\left(\begin{array}{c} W^{3\,\mu} \\ B^\mu \end{array}\right), \mathcal{M} = \begin{pmatrix} M^2 & M_{03}^2 \\ M_{03}^2 & M_0^2 \end{pmatrix}$$

- Matrix is not arbitrary, should have zero eigenvalue corresponding to the zero photon mass, $\det\mathcal{M} = 0 \quad \Rightarrow \quad M M_0 = M_{03}^2$

- We therefore redefine the electrically neutral fields by introducing rotated fields A_μ and Z_μ which propagate independently, $c_W = \cos\theta_W$, $s_W = \sin\theta_W$

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix},$$

- Since the massless field must correspond to the massless eigenvalue, we have that

$$\begin{aligned} M_A^2 = (s_W, c_W)\mathcal{M}\begin{pmatrix} s_W \\ c_W \end{pmatrix} &= M^2 s_W^2 + 2MM_0 s_W c_W + M_0^2 c_W \\ &= M^2(s_W + \frac{M_0}{M}c_W)^2 = 0, \rightarrow \frac{M_0}{M} = -\frac{s_W}{c_W} \end{aligned}$$

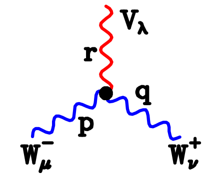
- Correspondingly the mass of the Z is,

$$\begin{aligned} M_Z^2 = (c_W, -s_W)\mathcal{M}\begin{pmatrix} c_W \\ -s_W \end{pmatrix} &= M^2 c_W^2 - 2MM_0 s_W c_W + M_0^2 s_W \\ &= M^2(c_W - \frac{M_0}{M}s_W)^2 \\ &= \frac{M^2}{c_W^2} \end{aligned}$$

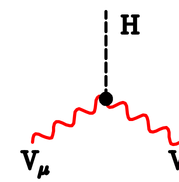
Boson rules

Electroweak Lagrangian
Glashow model
Boson rules
Fermion couplings
Abelian Higgs model
The Higgs mechanism
Higgs production
Higgs Lagrangian
Running of the Higgs coupling
Stability bounds
Higgs pair production
recap
Bibliography

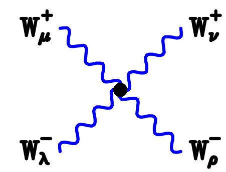
- 3 and 4 point vertices determined by the non-abelian term in the field strength.



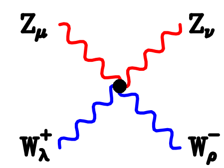
$+ig_V\ [(p-q)_\lambda g_{\mu\nu}+(q-r)_\mu g_{\nu\lambda}+(r-p)_\nu g_{\lambda\mu}]$
(all momenta incoming,
$g_A=e$, $g_Z=g_W\ \cos\theta_W$)



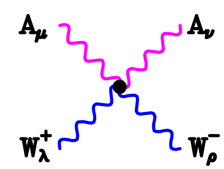
$+ig_{VH}M_W g_{\mu\nu}$
($g_{WH}=g_W$, $g_{ZH}=g_W/\cos^2\theta_W$)



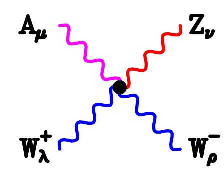
$+ig_W^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$



$-ig_W^2\cos^2\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$



$-ie^2[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$



$-ieg_W\cos\theta_W[2g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}]$

$$\mathcal{L} = \bar{\psi}_R i(\partial\!\!\!/ + i g'_W Y_R B\!\!\!/)\psi_R + \bar{\psi}_L i(\partial\!\!\!/ + i g_W T \cdot W\!\!\!\!/ + i g'_W Y_L B\!\!\!/)\psi_L \; .$$

- The U(1) charges of the left- and right-handed fermions, Y_L and Y_R, are chosen to satisfy the relation $Q = T^3 + Y$,

$$\psi_L = \gamma_L \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \; \gamma_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \; \gamma_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

- The right-handed fields are all SU(2) singlets:

$$\psi_R = \gamma_R e^-, \; \gamma_R \mu^-, \; \gamma_R \tau^- \; .$$

Fermion			T^3_L	Y_L	T^3_R	Y_R	Q_f
u	c	t	$+\frac{1}{2}$	$+\frac{1}{6}$	0	$+\frac{2}{3}$	$+\frac{2}{3}$
d	s	b	$-\frac{1}{2}$	$+\frac{1}{6}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
ν_e	ν_μ	ν_τ	$+\frac{1}{2}$	$-\frac{1}{2}$	-	-	0
e^-	μ^-	τ^-	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1	-1

The interaction Lagrangian can be expressed in terms of physical fields by substituting for B and W^3

$$\begin{aligned}\mathcal{L} &= \sum_f \bar{\psi}_f \left(i\partial\!\!\!/ - m_f - g_W \frac{m_f H}{2M_W} \right) \psi_f - e \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f \, A^\mu \\ &- \frac{g_W}{\cos\theta_W} \sum_f \bar{\psi}_f \gamma^\mu (R_f \gamma_R + L_f \gamma_L) \psi_f Z_\mu \\ &- \frac{g_W}{\sqrt{2}} \sum_f \bar{\psi}_f (T^+ W^+_\mu \gamma^\mu \gamma_L + T^- W^-_\mu \gamma^\mu \gamma_L) \psi_f\end{aligned}$$

The couplings of the fermions to the Z boson are, ($\gamma_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$)

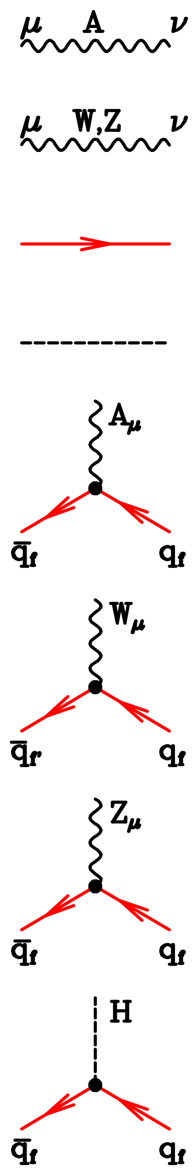
$$R_f = -Q_f \sin^2\theta_W, \quad L_f = T_f^3 - Q_f \sin^2\theta_W \ ,$$

where Q_f is the charge of the fermion in units of the positron electric charge e. The values of e and the weak SU(2) charge g_W are related by

$$e = g_W \sin\theta_W = g'_W \cos\theta_W \ .$$

Fermion rules

- The propagators are shown in the Unitary gauge.
- This gauge eliminates fields that do not correspond to physical particles.
- In this gauge the propagators have worse ultra-violet behaviour.
- The Weinberg angle fixes the coupling to the Z boson.
- Measurements of the Weinberg angle fix the ratio of the Z and W masses



$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}]\frac{i}{q^2}$$

$$[-g^{\mu\nu}+\frac{q^\mu q^\nu}{M^2}]\frac{i}{(q^2-M^2)}$$

$$\frac{i}{\not{q}-m}$$

$$\frac{i}{(q^2-M_H^2)}$$

$$-ieQ_f\gamma_\mu$$

$$\frac{-ig_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)(T^+)_{f'f} \qquad g_W=\frac{e}{\sin\theta_W}$$

$$\frac{-ig_W}{2\cos\theta_W}\gamma^\mu(V_f-A_f\gamma_5)=\frac{-ig_W}{\cos\theta_W}\gamma^\mu(R_f\gamma_R+L_f\gamma_L$$

$(V_f=T_f^3-2Q_f\sin^2\theta_W,\ A_f=T_f^3),\ T_f^3=\pm1/2$

$(R_f=-Q_f\sin^2\theta_W,\ L_f=T_f^3-Q_f\sin^2\theta_W)$

$$\frac{-ig_W m_f}{2M_W}$$

- Add a complex scalar field ϕ to a U(1) gauge theory

$$\mathcal{L} = -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + |D^{\alpha}\phi|^2 - V(\phi), \quad D_{\alpha} = \partial_{\alpha} - ieA_{\alpha}.$$

- Choose a potential with the "wrong" sign for the mass, $V(\phi) = -\mu^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2$
- L is invariant under a local $U(1)$ gauge transformation

$$\phi(x) \to \phi'(x) = e^{i\Lambda(x)}\,\phi(x), \quad A_{\alpha}(x) \to A'_{\alpha}(x) = A_{\alpha}(x) - \frac{1}{e}\partial_{\alpha}\Lambda(x)$$

- Covariant derivative transforms in the same way as the field itself under a gauge transformation

$$D_{\alpha}\phi(x) \to D'_{\alpha}\phi'(x) = e^{i\Lambda(x)}\,D_{\alpha}\phi(x)$$

- Case 1: $\mu^2 < 0$
  - QED with $m_A = 0$ and and a charged scalar particle of mass, $m_{\phi} = \mu$.
  - Unique minimum at $\phi = 0$.
- Case 2: $\mu^2 > 0$
  - Minimum energy state at $\langle\phi\rangle = |\phi_0| = \sqrt{\frac{\mu^2}{\lambda}} = \frac{v}{\sqrt{2}}$
  - Vacuum breaks $U(1)$ symmetry

$$V(\phi) = -\mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2, \quad |\phi_0| = \sqrt{\frac{\mu^2}{\lambda}} = \frac{v}{\sqrt{2}}$$

- Take ϕ_0 to be real and positive and define shifted fields, $\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}\big(\phi_1(x) + i\phi_2(x)\big)$
- In terms of the shifted fields,

$$V(\phi) = -\frac{1}{2\lambda}\mu^4 + \mu^2 \phi_1^2 + O(\phi_i^3)$$

- Potential describes a massive particle ϕ_1 of mass $\sqrt{2}\mu$ and a massless particle (Goldstone boson) ϕ_2.
- Perform the shift for the covariant derivative term,

$$|D^\alpha \phi|^2 = \frac{1}{2}(\partial_\alpha \phi_1)^2 + \frac{1}{2}(\partial_\alpha \phi_2)^2 + evA_\alpha \partial^\alpha \phi_2 + e^2 \phi_0^2 A_\alpha A^\alpha + \ldots;$$

- Photon has acquired a mass $m_A = ev$.

$$|D^\alpha \phi|^2 = \frac{1}{2}(\partial_\alpha \phi_1)^2 + \frac{1}{2}(\partial_\alpha \phi_2)^2 + evA_\alpha \partial^\alpha \phi_2 + \frac{1}{2}e^2 v^2 A_\alpha A^\alpha + \ldots;$$

- Weird mixing term $evA_\alpha \partial^\alpha \phi_2 = m_A A_\alpha \partial^\alpha \phi_2$
- To interpret this it is convenient to go to a Unitary gauge.
- Perform a gauge tranformation $\Lambda(x)$, such that at every point $\phi_2(x) = 0$.
- Before symmetry breaking, the theory had 4 degrees of freedom, two polarizations for the massless photon and two scalar degrees of ϕ_1, ϕ_2
- After symmetry breaking the theory has 4 degrees of freedom, a "photon" of mass $m_A = ev$, (3 degrees of freedom) and a scalar field with mass $m_H^2 = 2\mu^2$
- Full Lagrangian in this gauge, after symmetry breaking,

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{2}m_A^2 A^\alpha A_\alpha + \frac{1}{4}\mu^2 v^2 \\ &+ \frac{1}{2}(\partial^\alpha \phi_1)^2 - \mu^2 \phi_1^2 + em_A A_\alpha A^\alpha \phi_1 + \frac{1}{2}e^2 A_\alpha A^\alpha \phi_1^2 - \frac{1}{2}\lambda v \phi_1^3 - \frac{1}{8}\lambda \phi_1^4\end{aligned}$$

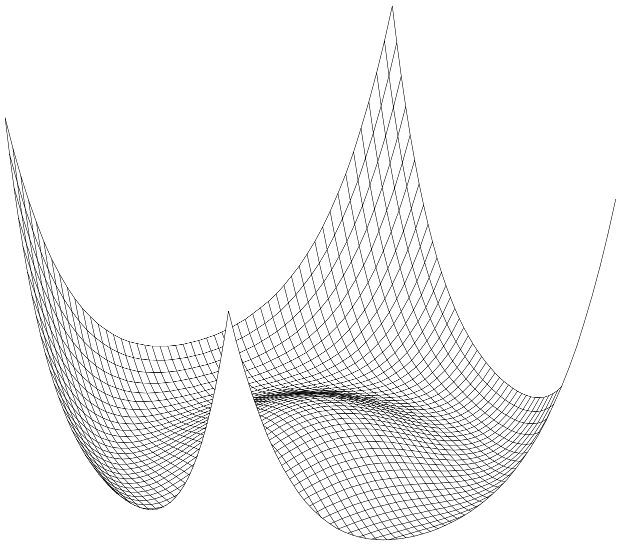
The Higgs potential $\mathcal{V}$ is chosen to be of the form

$$\mathcal{V}(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda\,(\phi^\dagger\phi)^2 \,.$$

Note that this potential has a different sign for the mass term from that usually chosen. As a result, with the parameters λ, $\mu^2 > 0$, this potential has a classical minimum which is not at $\phi = 0$.
To illustrate this phenomenon it is helpful to replace the complex fields in the potential by four real components

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_3 \\ \phi_2 + i\phi_4 \end{pmatrix} \,.$$

- Since the potential $\mathcal{V}$ is a function of $\phi^\dagger\phi = \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$ it is clearly invariant under four-dimensional rotations.
- in order to represent a four-dimensional rotation in a two-dimensional plot, we arbitrarily set ϕ_3 and ϕ_4 equal to zero.
- The figure then shows the potential $\mathcal{V}(\phi^\dagger\phi)$ in the ϕ_1, ϕ_2 plane. There is evidently a circle of degenerate minima, corresponding to



$$|\phi| = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}} \,.$$

- Transitions between field configurations on this circle cost no energy and correspond to massless excitations.

We next choose a particular direction in the internal SU(2) space for the minimum of ϕ,

$$\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} .$$

The direction chosen is of no consequence; the important feature is the residual U(1) invariance. Transformations generated by $T^3 + Y$ leave the vacuum expectation value invariant, *i.e.*

$$(T^3 + Y) \langle\phi\rangle = 0 .$$

This combination is the single unbroken generator which is identified with the electric charge,

$$Q \equiv T^3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} .$$

- This justifies *a posteriori* the labels $+$ and 0 on the field ϕ.

The choice of the vacuum expectation value breaks the SU(2)⊗U(1) symmetry, since it identifies a particular direction in the internal group space. We shall need to consider the fluctuations around this new minimum, so we introduce a reparametrization of the Higgs field,

$$\phi = U^{-1}(\xi) \begin{pmatrix} 0 \\ (H+v)/\sqrt{2} \end{pmatrix}$$

$$U(\xi) = \exp(-iT \cdot \xi / v) \; .$$

Note that we still have four real degrees of freedom, (three ξs and one H), equivalent to the two complex fields. We now make a gauge transformation of the form

$$\begin{aligned} \phi &\rightarrow U(\xi)\phi \\ T \cdot W^{\mu} &\rightarrow UT \cdot W^{\mu} U^{-1} + \frac{i}{g_W}(\partial^{\mu} U) U^{-1} \; . \end{aligned}$$

The ξ degrees of freedom no longer appear in the Higgs Lagrangian. They will reappear as the longitudinal modes of the massive gauge bosons. This gauge, in which there are no unphysical degrees of freedom, is called the *unitary gauge*. The Higgs boson H is the only remaining dynamical field.

The Higgs Lagrangian now becomes

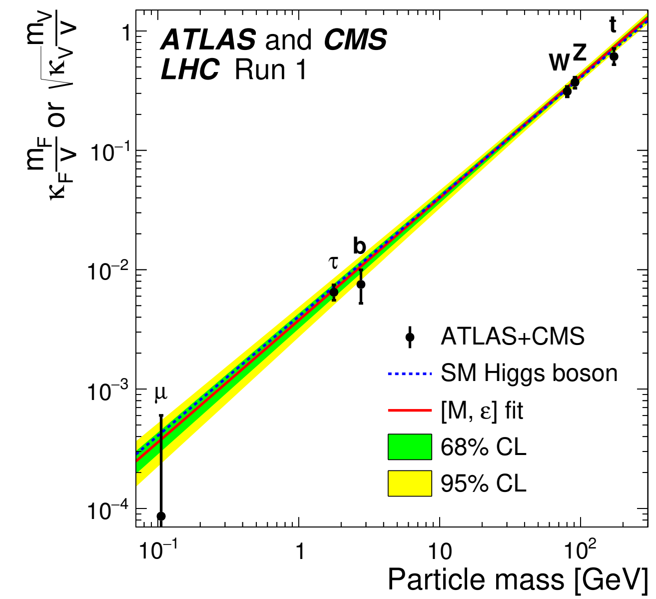
$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu H \partial^\mu H - \mathcal{V}\left(\frac{(v+H)^2}{2}\right) \\ &+ \frac{(v+H)^2}{8}\chi^\dagger (2g_W T\cdot W_\mu + g'_W B_\mu)(2g_W T\cdot W^\mu + g'_W B^\mu)\chi \,,\end{aligned}$$

where χ is a unit vector along the direction of the vacuum expectation value (14)

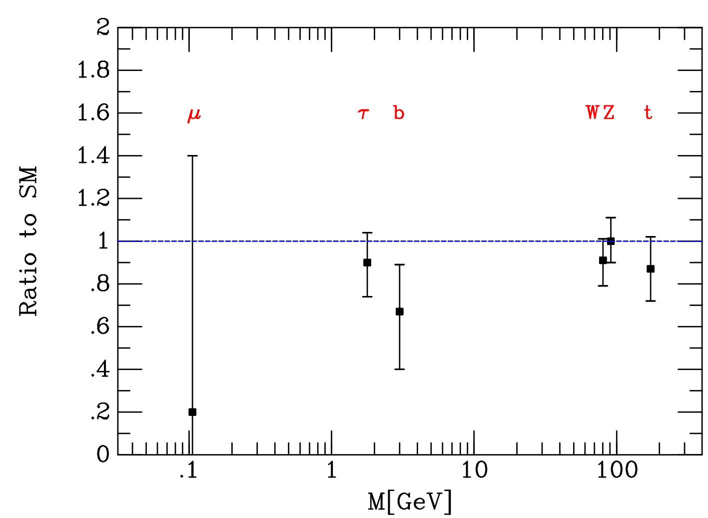
$$\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

We therefore have three generators of the SU(2)⊗U(1) symmetry which are spontaneously broken. Goldstone's theorem would lead us to expect three massless bosons. However, this theorem does not apply if the theory contains long-range vector fields with the same quantum numbers as the would-be Goldstone bosons. When such fields are present, the massless Goldstone modes provide the extra longitudinal degrees of freedom necessary to change the quanta of the vector fields from massless to massive bosons.

- Higgs couplings is indeed observed to be proportional to mass . . .



- . . . but the errors are still large, $> \pm 10\%$

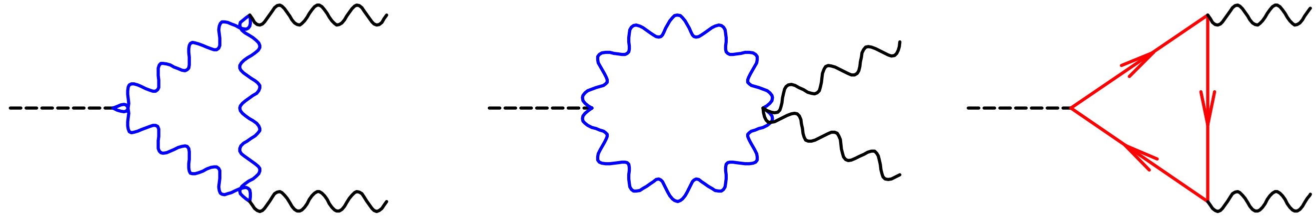


- The leading-order partial decay widths are readily obtained using the Feynman rules

$$
\begin{aligned}
\Gamma(H \to f\bar{f}) &= \frac{C G_F m_f^2 M_H}{4\pi\sqrt{2}} \left(1 - \frac{4m_f^2}{M_H^2}\right)^{\frac{3}{2}}, \\
\Gamma(H \to W^+W^-) &= \frac{G_F M_H^3}{8\pi\sqrt{2}} \left(1 - \frac{4M_W^2}{M_H^2}\right)^{\frac{1}{2}} \left(1 - \frac{4M_W^2}{M_H^2} + \frac{12 M_W^4}{M_H^4}\right), \\
\Gamma(H \to ZZ) &= \frac{G_F M_H^3}{16\pi\sqrt{2}} \left(1 - \frac{4M_Z^2}{M_H^2}\right)^{\frac{1}{2}} \left(1 - \frac{4M_Z^2}{M_H^2} + \frac{12 M_Z^4}{M_H^4}\right).
\end{aligned}
$$

  where C is a colour multiplicity factor: $C = 3$ for quarks and $C = 1$ for leptons.
- Note that the results are only valid *above* the two-particle threshold in each case. The W^+W^- and ZZ, decays occur *below threshold* and these formulae are no longer valid.
- Partial width to bottom quarks is 2.2 GeV, using $m_b = 3$ GeV.
- This dominates the total width, which in the standard model is 4.2 MeV.

- An example is the $H \to \gamma\gamma$ decay, which is mediated by intermediate W and quark triangle loops:



- That decay is induced by quantum loop corrections involving the W boson and fermion (the top quark), The gauge invariant decay amplitude is given by

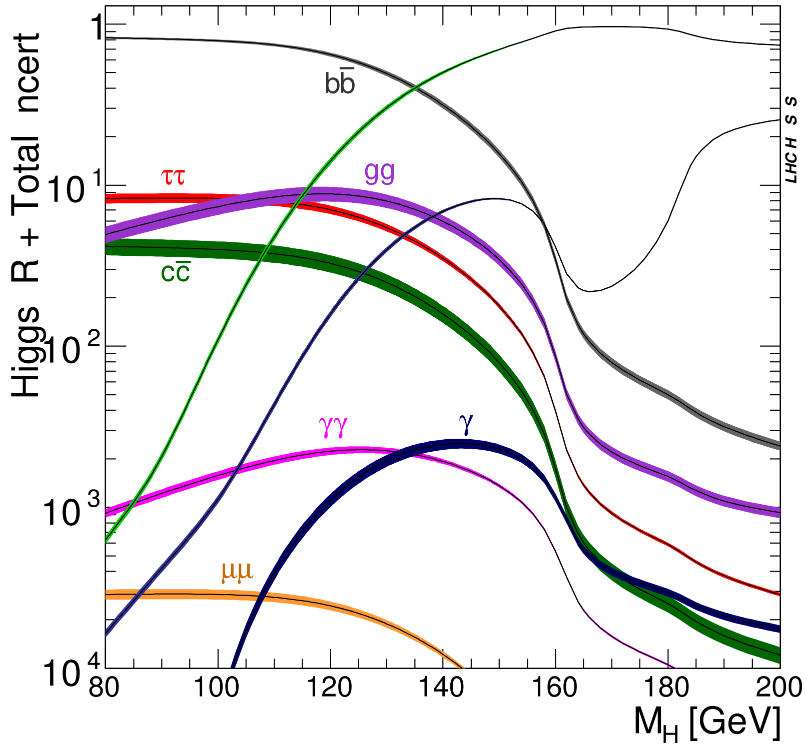
$$\mathcal{M} = \frac{e^2 g}{(4\pi)^2 m_W} F(k_1 \cdot k_2 g^{\mu\nu} - k_2^\mu k_1^\nu)\epsilon_\mu(k_1)\epsilon_\nu(k_2)$$

where F includes contributions from W loops and quark loops:

$$\begin{aligned} F &= F_W(\tau_W) + \sum_f 3Q_f^2 F_f(\tau_f), \quad \tau_W = \frac{4m_W^2}{m_H^2}, \quad \tau_f = \frac{4m_f^2}{m_H^2}. \\ &= 8.34 \qquad\qquad - 1.37 \text{ (top only)} \end{aligned}$$

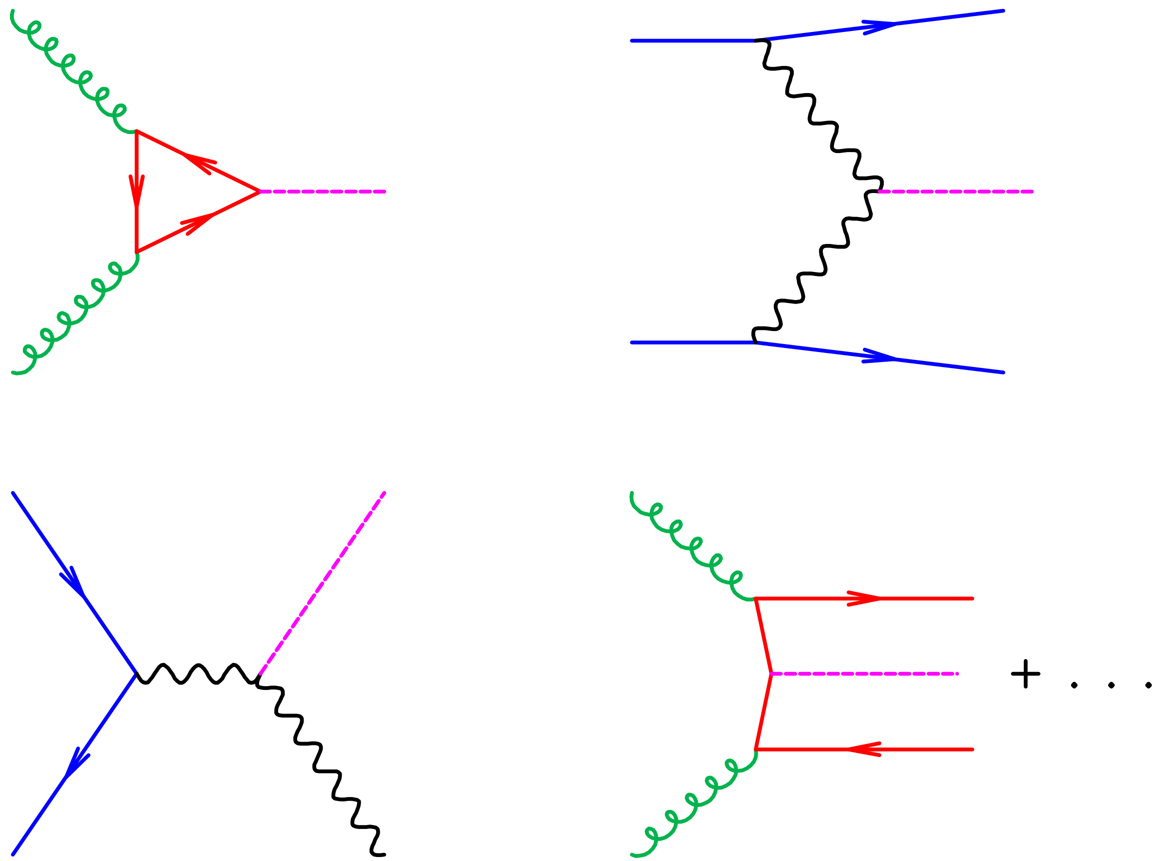
$$F_W(\tau) = [2 + 3\tau + 3\tau(2-\tau)f(\tau)], \qquad F_f(\tau) = -2\tau\,[1 + (1-\tau)f(\tau)]$$

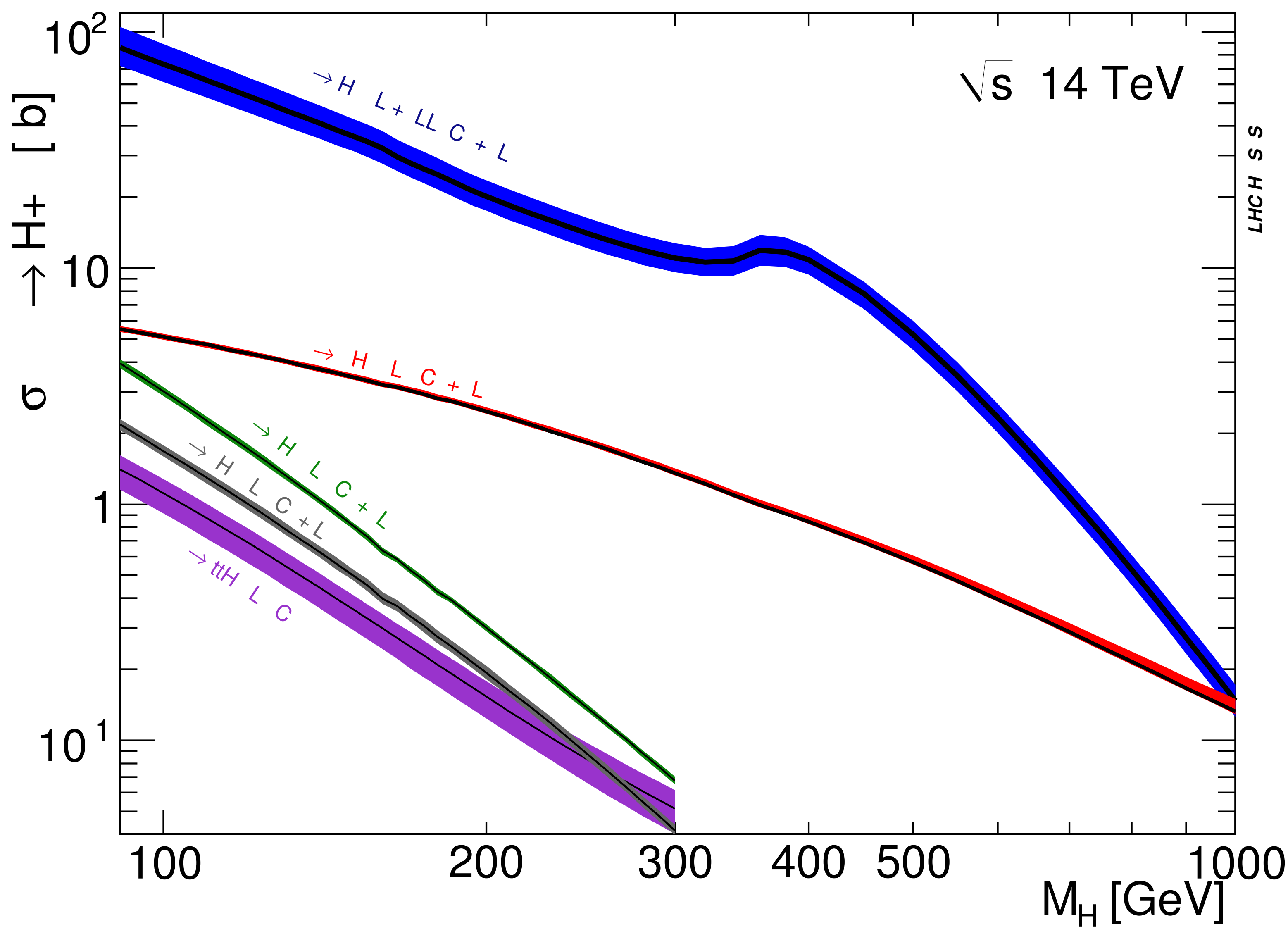
$$f(\tau) = \begin{cases} \arcsin^2(\tau^{-\frac{1}{2}}) & \text{for } \tau \geq 1 \\ -\frac{1}{4}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \text{for } \tau < 1 \end{cases}.$$



- At 125 GeV we are blessed with many channels with large branching ratios, $b\bar{b}, WW^*, gg, \tau^+\tau^-, c\bar{c}, ZZ^*, \gamma\gamma, Z\gamma, \mu^+\mu^-$.
- Purely hadronic ones are difficult to observe
- Labels in the plot are somewhat garbled.

Higgs production




10^2
10
1
10 1
σ → H+ [b]
→H L + LL C + L
→ H L C + L
→ H L C + L
→ H L C + L
→ ttH L C
√s 14 TeV
LHC H S S
100
200
300
400
500
1000
M_H [GeV]

- We can write an effective Lagrangian for Higgs production

$$\mathcal{L} = \frac{1}{v} g_{ggH} H G^{\alpha\beta} G_{\alpha\beta}$$

- the coefficient in the effective lagrangian is ($\tau = \frac{4m_t^2}{m_H^2}$)

$$\frac{1}{v} g_{ggH} = -i \frac{\alpha_s}{12\pi v} I(\tau), \qquad I(\tau) = \frac{3}{2}\tau[1 + (1-\tau) f(\tau)]$$

- $I(\tau)$ involves the same triangle function $f(\tau)$ as before.

$$f(\tau) = \begin{cases} \arcsin^2(\tau^{-\frac{1}{2}}) & \text{for} \quad \tau \geq 1 \\ -\frac{1}{4}\left[\ln \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \text{for} \quad \tau < 1 \end{cases} .$$

- In the large m_t, $I(\tau)$ tends to 1 (no decoupling of heavy quark loop).

$$\begin{aligned} f(\tau) &\rightarrow \frac{1}{\tau} + \frac{1}{3\tau^2} + \frac{8}{45\tau^3} + \dots \\ I(\tau) &\rightarrow 1 + \frac{7}{30}\frac{1}{\tau} \end{aligned}$$

Higgs Lagrangian

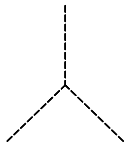
- The self-couplings of the Higgs boson are

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\partial_\mu H \partial^\mu H - \mu^2 H^2 - \lambda v H^3 - \tfrac{1}{4}\lambda H^4 \,,$$

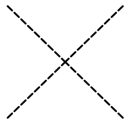
  and its mass M_H is therefore

$$M_H = \sqrt{2}\mu \equiv \sqrt{2\lambda} v \,.$$

  One combination of the parameters in the Higgs potential, $\mu^2/\lambda = v^2$ is fixed by the measured parameters of the electroweak theory.
- The other parameter, which is related to the physical Higgs mass, is essentially arbitrary.
- With the measured $M_H = 125$ GeV, we get $\lambda = 0.129$
- This is the ‘minimal model’. Many extensions of the Standard Model (supersymmetry, grand unification, . . .) retain the Higgs mechanism as the primary method for mass generation for gauge bosons, but with more complicated Higgs sectors and more Higgs bosons.



$-6i\lambda v = -3i m_H^2/v$



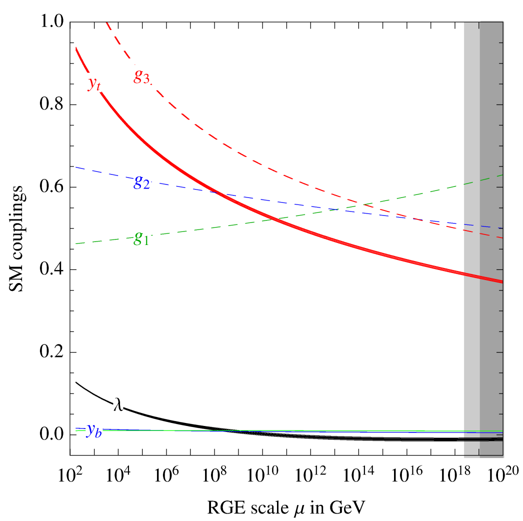
$-6i\lambda = -3i(m_H/v)^2$

- At one loop self-coupling of the Higgs

$$\frac{d\lambda}{d\ln Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6y_t^2\lambda - 3y_t^4 - \frac{3}{2}(\frac{3}{5}g_1^2 + 3g_2^2)\lambda + \frac{3}{16}((\frac{3}{5}g_1^2 + g_2^2)^2 + 2g_2^4)\right]$$
$$= \frac{1}{16\pi^2}\left[0.20 + 0.78 - 2.9 - 0.28 + 0.13\right] \text{ (values at scale } v\text{)}$$

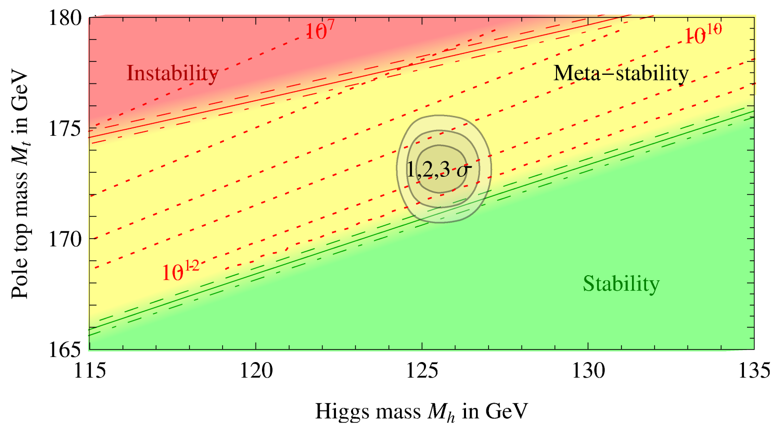
- $g_2^2 = g_w^2, g_1^2 = \frac{5}{3}g_w'^{\,2}$
- The dominant effect is the top Yukawa coupling which drives λ towards zero.
- All the other couplings run too.

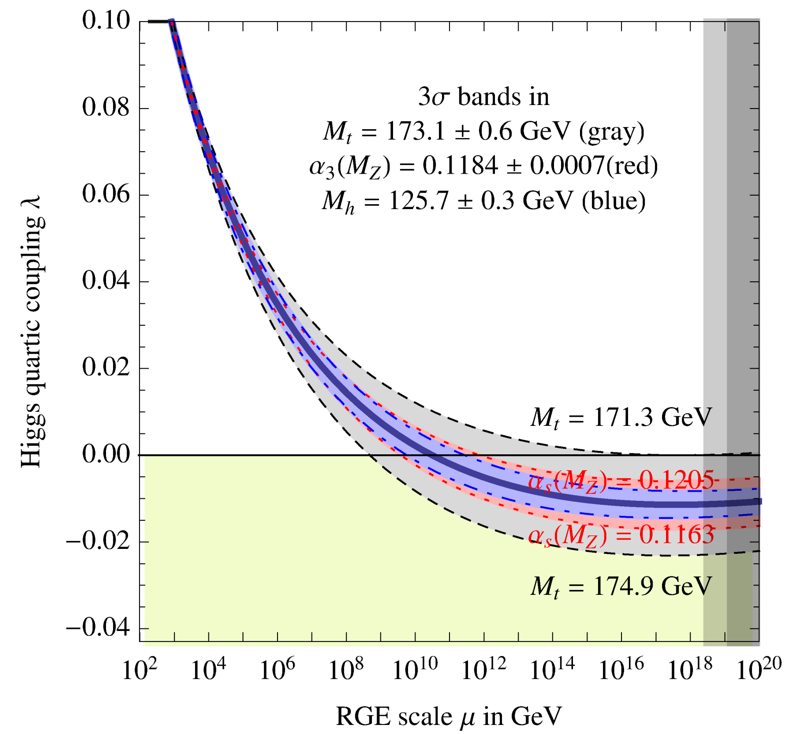
$$\frac{dg_1}{d\ln Q^2} = \frac{g_1}{16\pi^2}(\frac{41}{20}g_1^2)$$
$$\frac{dg_2}{d\ln Q^2} = \frac{g_2}{16\pi^2}(-\frac{19}{12}g_2^2)$$
$$\frac{dg_s}{d\ln Q^2} = \frac{g_s}{16\pi^2}(-\frac{7}{2}g_s^2)$$
$$\frac{dy_t}{d\ln Q^2} = \frac{y_t}{16\pi^2}(\frac{9}{4}y_t^2 - \frac{17}{40}g_1^2 - \frac{9}{8}g_2^2 - 4g_s^3)$$



- No approximation in which we can capture all effects
- Higher loop effects are known.

- SM RG evolution of the gauge couplings $g_1 = \sqrt{5/3}g',\ g_2 = g,\ g_3 = g_s$, of the top and bottom Yukawa couplings (y_t, y_b), and of the Higgs quartic coupling λ. All couplings are defined in the $\overline{\text{MS}}$ scheme. The thickness indicates the $\pm 1\sigma$ uncertainty.

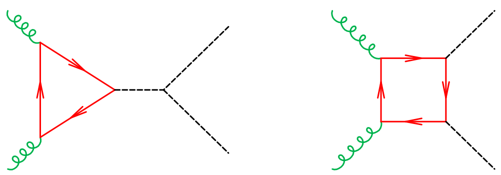




- RG evolution of λ varying M_t, M_h and α_s by $\pm 3\sigma$.
- shows the sensitivity to the Top quark mass

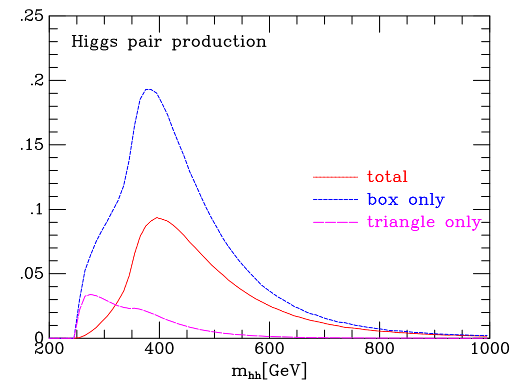
Effective coupling approximation for single and double Higgs production

$$L = \frac{1}{v} g_{ggH} H G^{\mu\nu} G_{\mu\nu}$$
$$L = \frac{1}{v^2} g_{ggHH} H H G^{\mu\nu} G_{\mu\nu}$$



- Insertion of a soft Higgs boson

$$m_t \frac{d}{dm_t} \frac{1}{\not{q} - m_t} = m_t \frac{d}{dm_t} \frac{\not{q} + m_t}{q^2 - m_t^2}$$
$$= \frac{m_t}{q^2 - m_t^2} + 2m_t^2 \frac{\not{q} + m_t}{(q^2 - m_t^2)^2}$$
$$= \frac{m_t(q^2 - m_t^2) + 2m_t^2(\not{q} + m_t)}{(q^2 - m_t^2)}$$
$$= \frac{1}{\not{q} - m_t}(m_t)\frac{1}{\not{q} - m_t}$$



So that we find that

$$m_t^2 \frac{d}{dm_t} \frac{g_{ggH}}{m_t} = g_{ggHH} \Rightarrow g_{ggHH} = -g_{ggH}$$

- The Higgs mechanism reproduces the results of the Glashow model.
- It preserves renormalizabilty and predicts the Higgs boson coupled to mass.
- Higgs self coupling λ tends to zero at high scale
- Higgs pair production can give us information about the shape of the potential.

- T. Plehn, "Higgs Physics," Lect. Notes Phys. **886**, 1 (2015), doi:10.1007/978-3-319-05942-6_1 [arXiv:0910.4182]
- G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, "Higgs mass and vacuum stability in the Standard Model at NNLO," JHEP **1208**, 098 (2012) doi:10.1007/JHEP08(2012)098 [arXiv:1205.6497 [hep-ph]].