Particle Cosmology

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences, Moscow
and
Moscow State University

Topics

- Basics of Hot Big Bang cosmology
- Dark matter:
  - WIMPs
  - Axions
  - Warm dark matter, gravitinos
- Baryon asymmetry
  - Electroweak mechanism
  - Leptogenesis

Lecture 1

Outline of Lecture 1

- Basics of Hot Big Bang cosmology
- Dark matter: evidence
- WIMPs

Basics of Hot Big Bang cosmology

- The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean (observational fact!)
  All this is encoded in space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$\mathbf{x}$: comoving coordinates, label distant galaxies.

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

$$z(t) = \frac{a_0}{a(t)} - 1 : \quad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength $\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$.

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

$$H_0 = (70.4 \pm 1.4)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

$1\ \text{Mpc} = 3 \cdot 10^6\ \text{light yrs} = 3 \cdot 10^{24}\ \text{cm}$

- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Fig.

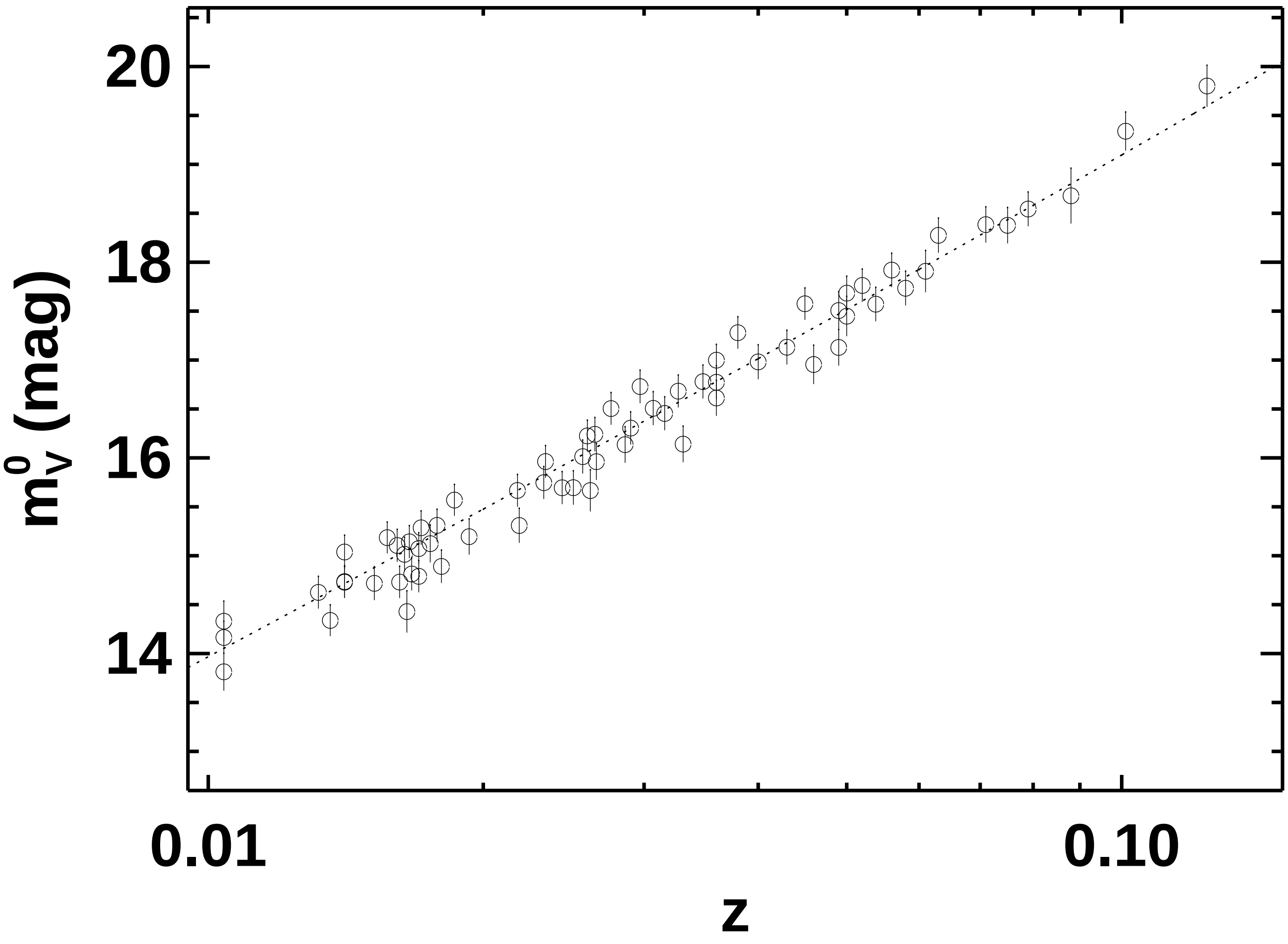
- The Universe is warm: CMB temperature today

$$T_0 = 2.726\ \text{K}$$

Fig.

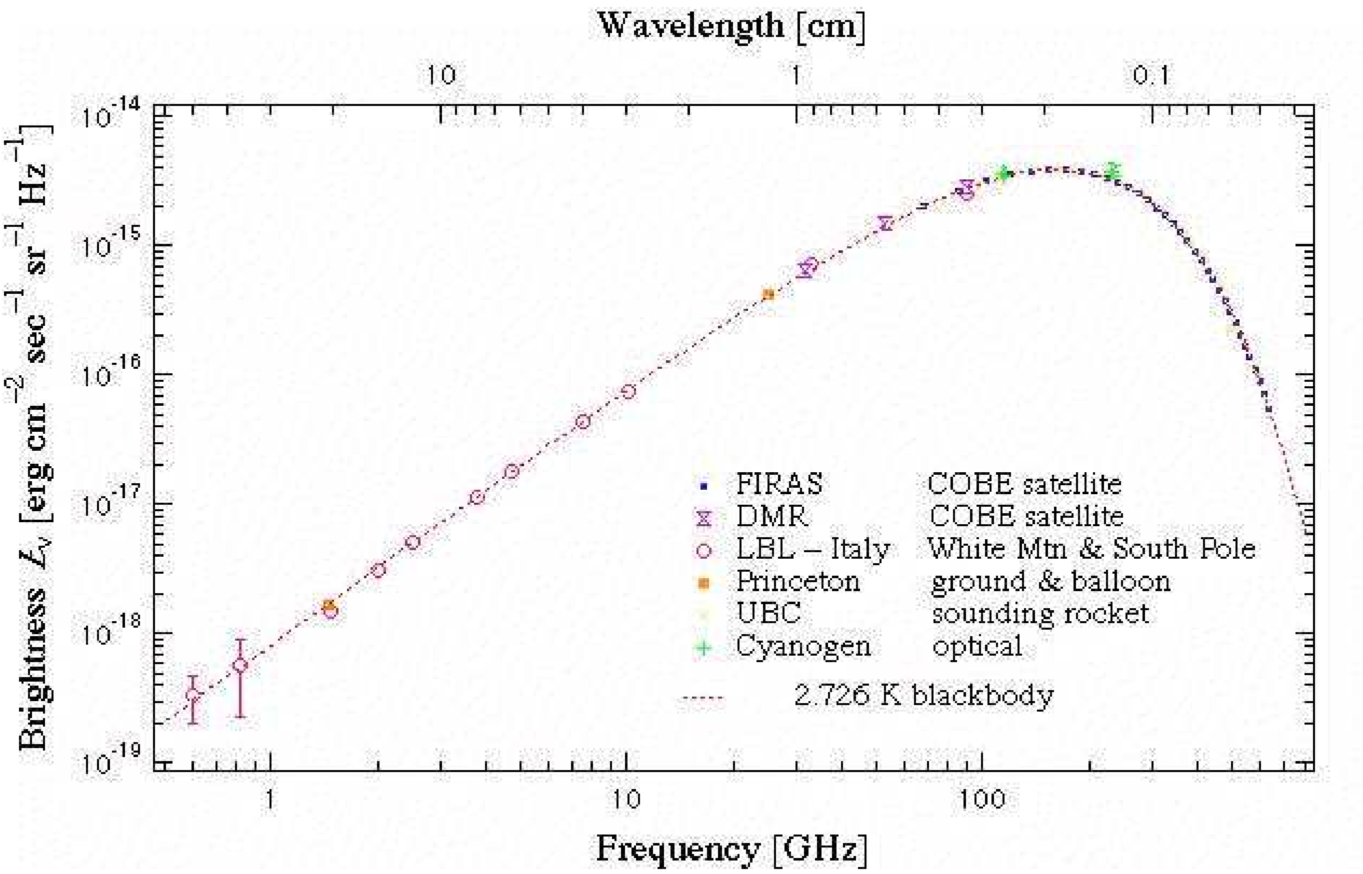
It was denser and warmer at early times.

Hubble diagram for SNe1a



$$\text{mag} = 5\log_{10} r + \text{const}$$

CMB spectrum



$T = 2.726$ K

- Present number density of photons

$$n_\gamma = \# T^3 = 410 \frac{1}{\text{cm}^3}$$

- Present entropy density

$$s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\text{cm}^3}$$

In early Universe

$$s = \frac{2\pi^2}{45} g_* T^3$$

g_*: number of relativistic degrees of freedom with $m \lesssim T$; fermions contribute with factor $7/8$.

Entropy density scales exactly as a^{-3}

Temperature scales approximately as a^{-1}.

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

  Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature.

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi}H_0^2 = 5\cdot 10^{-6}\ \frac{\text{GeV}}{\text{cm}^3}$$

Present composition of the Universe

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c}$$

present fractional energy density of i-th type of matter.

$$\sum_i \Omega_i = 1$$

- Dark energy: $\Omega_\Lambda = 0.72$
  ρ_Λ stays (almost?) constant in time
- Non-relativistic matter: $\Omega_M = 0.28$
  $\rho_M = mn(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^3$
  - Dark matter: $\Omega_{DM} = 0.23$
  - Usual matter (baryons): $\Omega_B = 0.046$
- Relativistic matter (radiation): $\Omega_{rad} = 8.4 \cdot 10^{-5}$ (for massless neutrinos)
  $\rho_{rad} = \omega(t)n(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^4$

Friedmann equation

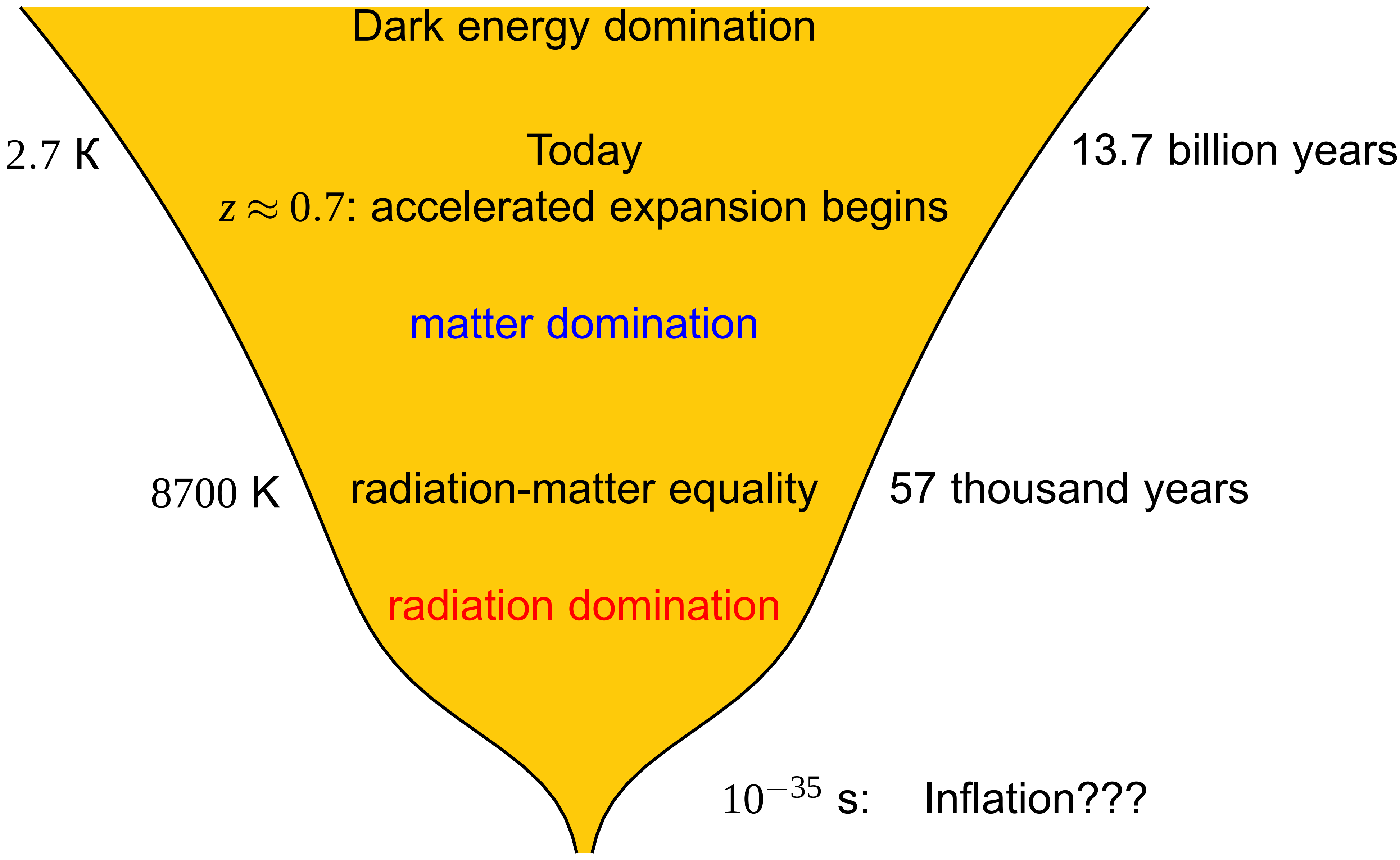
$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}\left[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)\right] = H_0^2\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a(t)}\right)^4\right]$$

$\ldots \Longrightarrow$ Radiation domination $\Longrightarrow$ Matter domination $\Longrightarrow$ Λ–domination

$z_{eq} = 3200$ now

$T_{eq} = 8700$ K $= 0.75$ eV

$t_{eq} = 57 \cdot 10^3$ yrs



Dark energy domination
2.7 K
Today
13.7 billion years
$z \approx 0.7$: accelerated expansion begins
matter domination
8700 K
radiation-matter equality
57 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}}\rho$$

- Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M^*_{Pl}}$$

with $M^*_{Pl} = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \quad \Longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

- Decelerated expansion: $\ddot{a} < 0$.
  - NB: Λ-domination

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const} \Longrightarrow a(t) = \mathrm{e}^{H_\Lambda t}$$

accelerated expansion.

Cornerstones of thermal history

- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$\begin{aligned} p+n &\rightarrow {}^2H \\ {}^2H+p &\rightarrow {}^3He \\ {}^3He+n &\rightarrow {}^4He \\ \text{up to} & \quad {}^7Li \end{aligned}$$

  Abundances of light elements: measurements vs theory

  $T = 10^{10} \rightarrow 10^9$ K, $t = 1 \rightarrow 300$ s

  Earliest time in thermal history probed so far Fig.

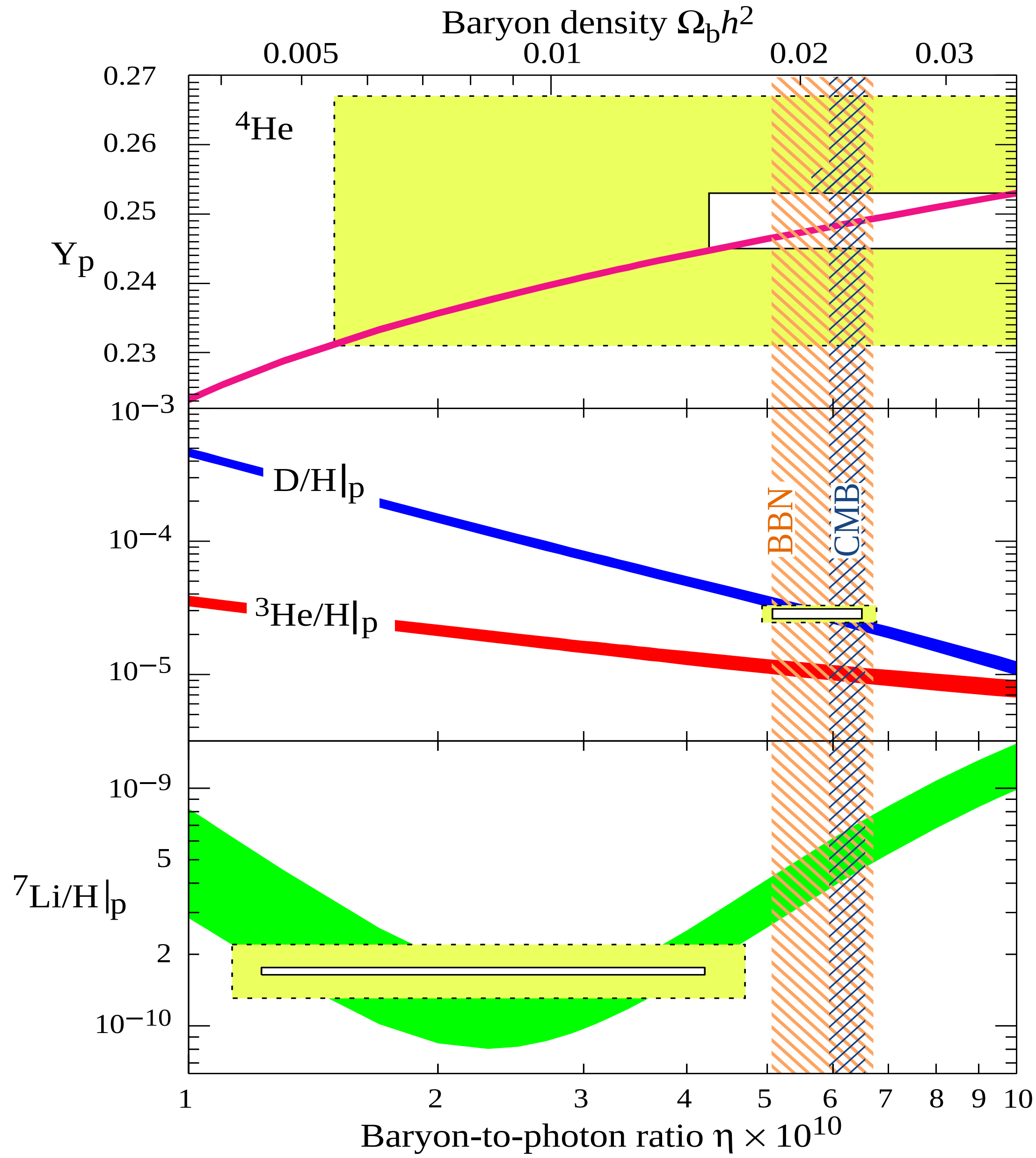
- Recombination, transition from plasma to gas.

  $z = 1090$, $T = 3000$ K, $t = 370\,000$ years

  Last scattering of CMB photons Fig.

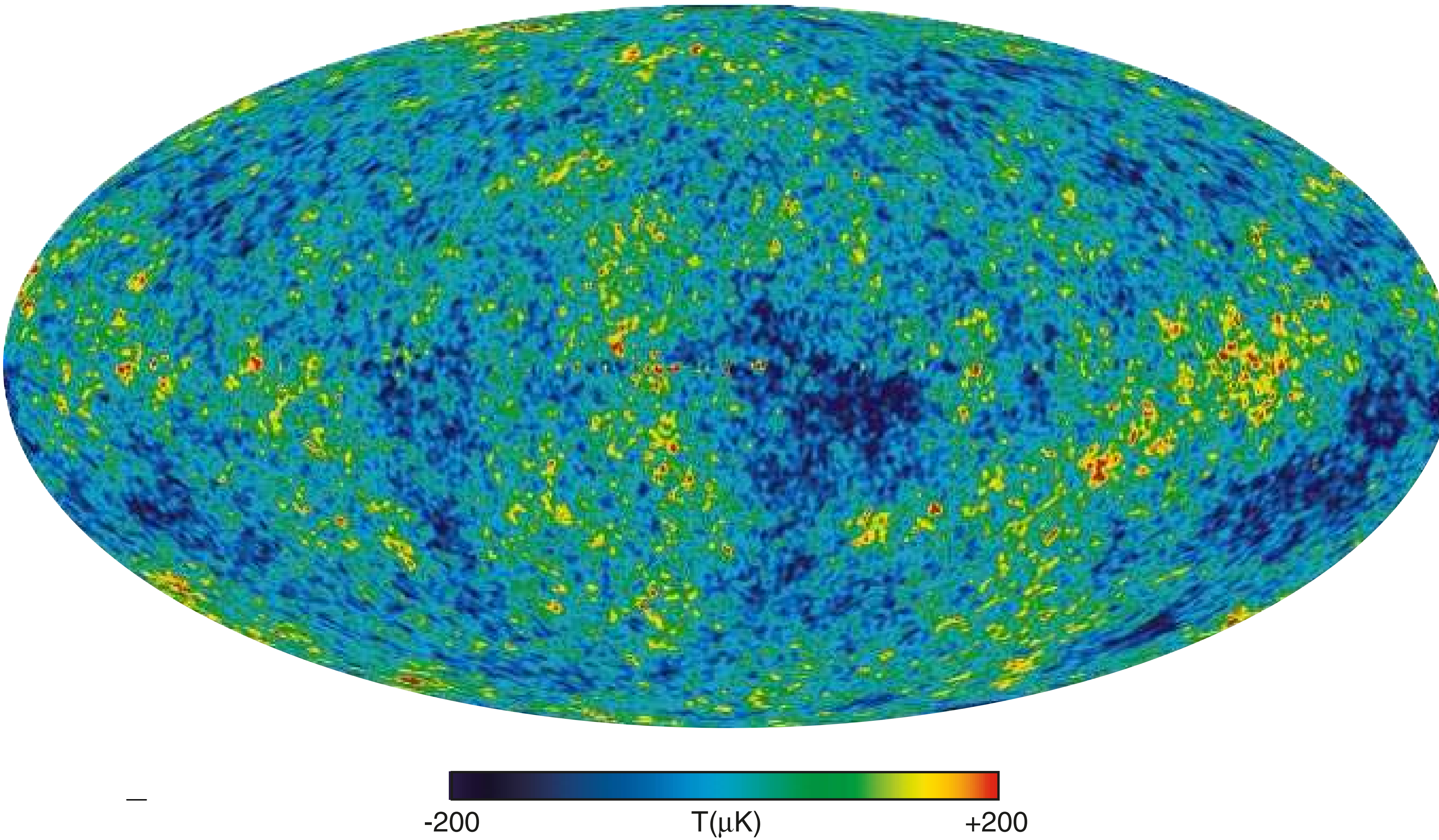
- Neutrino decoupling: $T = 2-3$ MeV $\sim 3 \cdot 10^{10}$K, $t \sim 0.1-1$s
- Generation of dark matter*
- Generation of matter-antimatter asymmetry*

*may have happend before the hot Big Bang epoch

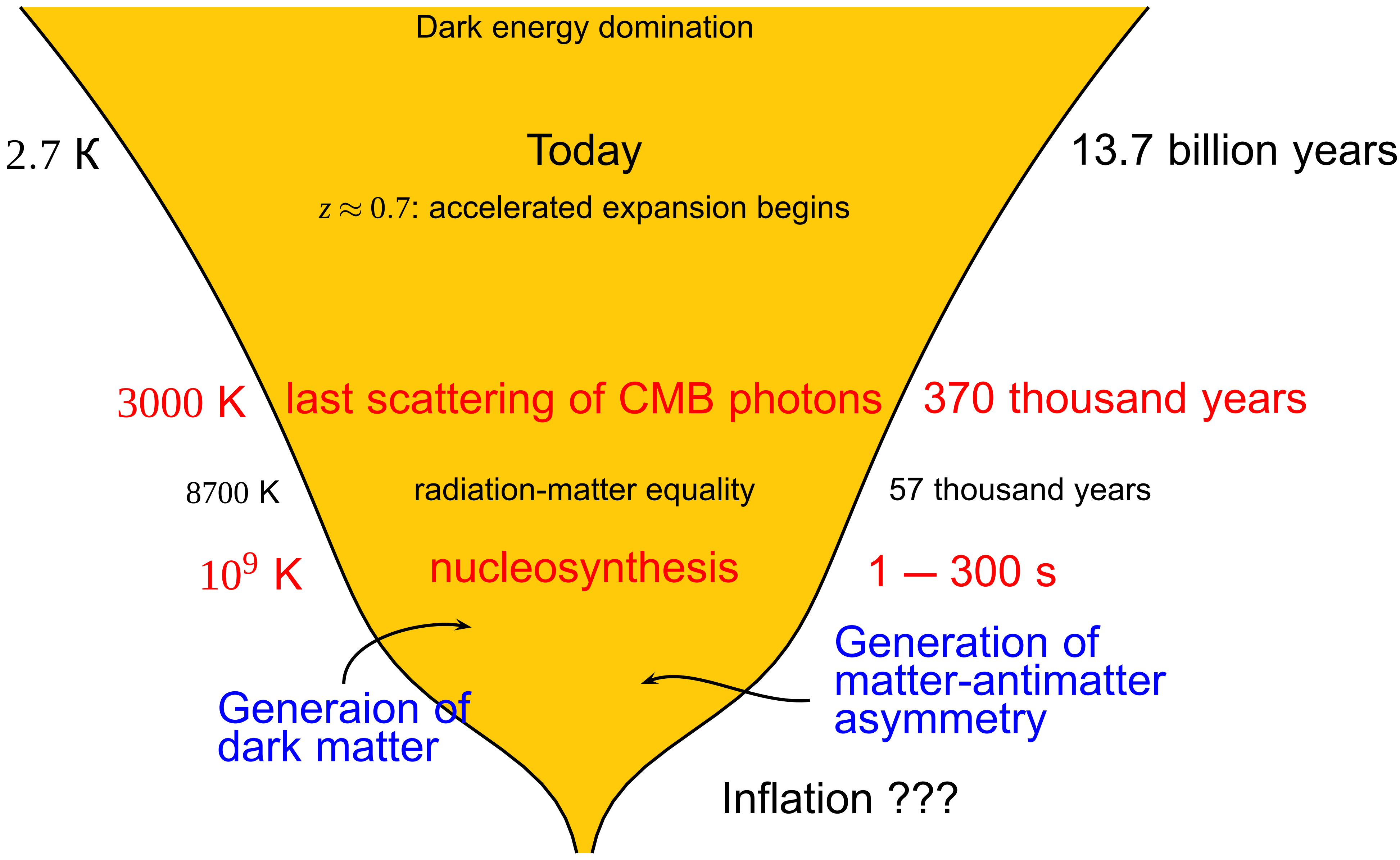


$\eta_{10} = \eta \cdot 10^{-10} =$ baryon-to-photon ratio. Consistent with CMB determination of η

$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-5}$$



WMAP



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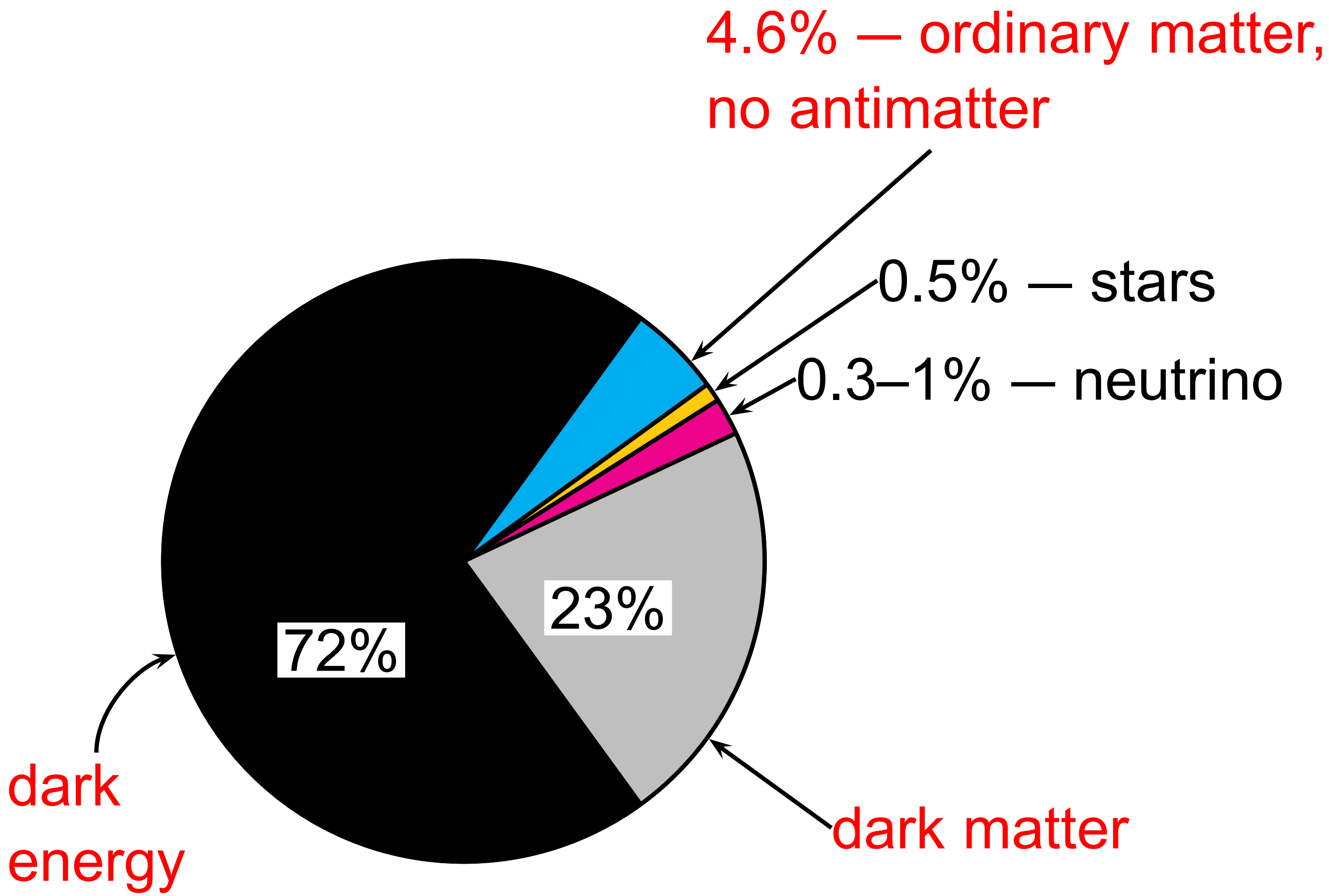
57 thousand years
10^9 K
nucleosynthesis
1 — 300 s
Generation of matter-antimatter asymmetry
Generaion of dark matter
Inflation ???

Unknowns


4.6% — ordinary matter,
no antimatter
0.5% — stars
0.3–1% — neutrino
23%
72%
dark
energy
dark matter

Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
  - Velocity curves of galaxies

    Fig. Backup slide 1
  - Velocities of galaxies in clusters

    Original Zwicky's argument, 1930's

$$v^2 = G\frac{M(r)}{r}$$

  - Temperature of gas in X-ray clusters of galaxies
  - Gravitational lensing of clusters

    Fig. Backup slide 2
  - Etc.

Outcome

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.2 - 0.3$$

Assuming mass-to-light ratio everywhere the same as in clusters
NB: only 10 % of galaxies sit in clusters

Nucleosynthesis, CMB:

$$\Omega_B = 0.046$$

The rest is non-baryonic, $\Omega_{DM} \approx 0.23$.

Physical parameter: mass-to-entropy ratio. Stays constant in time. Its value

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} = \frac{0.2 \cdot 0.5 \cdot 10^{-6}\ \text{GeV cm}^{-3}}{3000\ \text{cm}^{-3}} = 3 \cdot 10^{-10}\ \text{GeV}$$

Cosmological evidence: growth of structure

CMB anisotropies: baryon density perturbations at recombination $\approx$ photon last scattering, $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{z=1100} \simeq \left(\frac{\delta T}{T}\right)_{CMB} \sim 10^{-4}$$

In matter dominated Universe, matter perturbations grow as

$$\frac{\delta\rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only

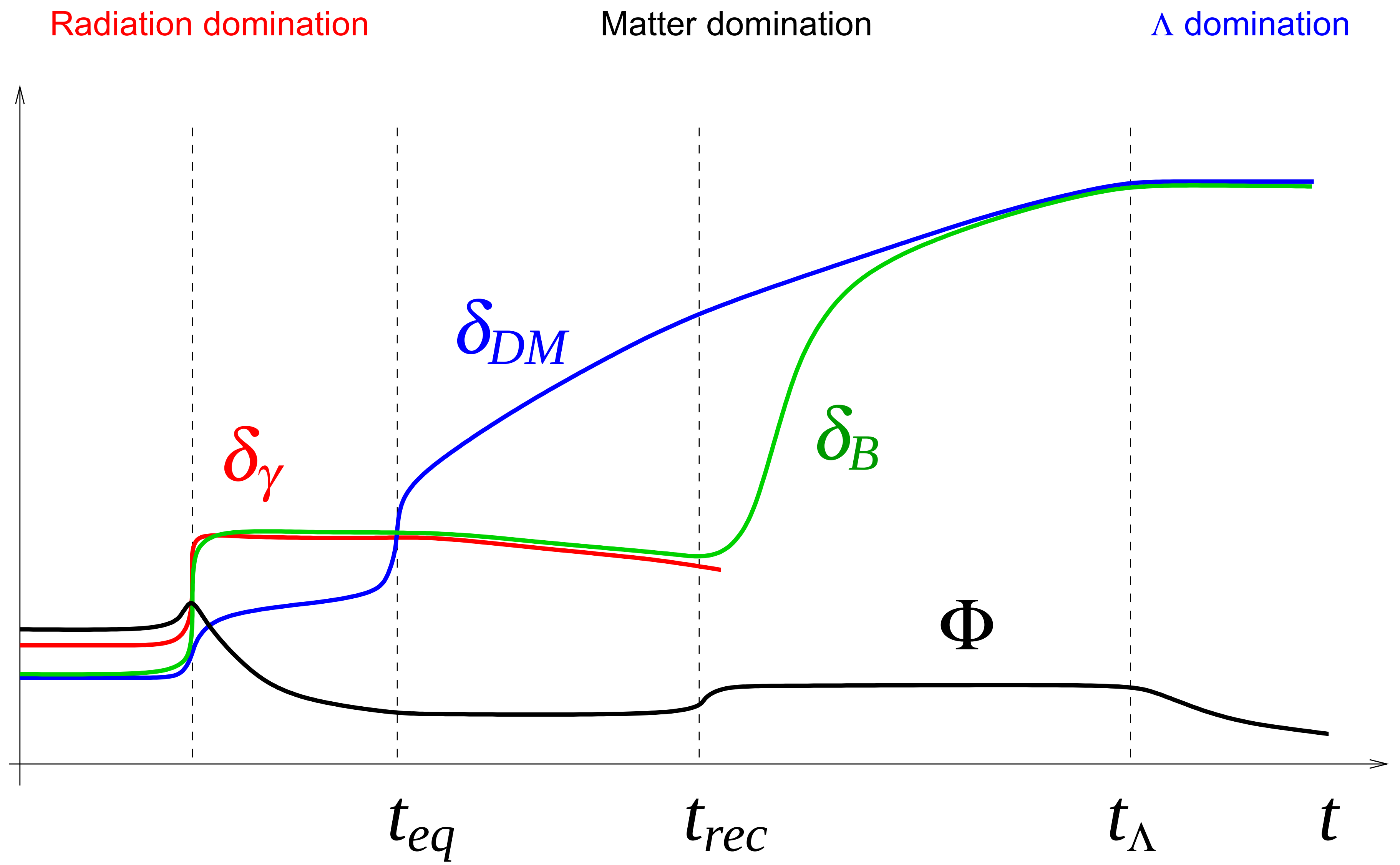
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} = 1100 \times 10^{-4} \sim 0.1$$

No galaxies, no stars...

Perturbations in dark matter start to grow much earlier (already at radiation-dominated stage)

Growth of perturbations (linear regime)



NB: Need dark matter particles non-relativistic early on.

Neutrinos are not considerable part of dark matter
(way to set cosmological bound on neutrino mass,
$m_\nu < 0.2$ eV for every type of neutrino)

UNKNOWN DARK MATTER PARTICLES ARE
CRUCIAL FOR OUR EXISTENCE

If thermal relic:

Cold dark matter, CDM

$$m_{DM} \gtrsim 100 \text{ keV}$$

Warm dark matter

$$m_{DM} \simeq 1 - 30 \text{ keV}$$

WIMPs

Simple but very suggestive scenario

- Assume there is a new heavy stable particle X
  - Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q}\,,\text{etc}$$

  - Parameters: mass M_X; annihilation cross section at non-relativistic velocity $\sigma(v)$
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density (somewhat oversimplified)
  - Recall

$$H(T) = \frac{T^2}{M_{Pl}^*}$$

with $M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Number density of X-particles in equilibrium at $T < M_X$: Maxwell–Boltzmann

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3} \mathrm{e}^{-\frac{\sqrt{M_X^2+p^2}}{T}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

- Mean free time wrt annihilation: travel distance $\tau_{ann} v$, meet one X particle to annihilate with in volume $\sigma \tau_{ann} v \Longrightarrow$

$$\sigma \tau_{ann} v n_X = 1 \quad \Longrightarrow \quad \tau_{ann} = \frac{1}{n_X \langle \sigma v \rangle}$$

- Freeze-out: $\tau_{ann}^{-1}(T_f) \sim H(T_f) \Longrightarrow n_X(T_f)\langle\sigma v\rangle \sim T_f^2/M_{Pl}^* \Longrightarrow$

$$T_f \simeq \frac{M_X}{\ln(M_X M_{Pl}^* \langle \sigma v \rangle)}$$

NB: large log $\Longleftrightarrow T_f \sim M_X/20$

Define $\langle \sigma v \rangle \equiv \sigma_0$ (constant for s-wave annihilation)

- Number density at freeze-out

$$n_X(T_f) = \frac{T_f^2}{\sigma_0 M_{Pl}^*}$$

- Number-to-entropy ratio at freeze-out and later on

$$\frac{n_X(T_f)}{s(T_f)} = \#\frac{n_X(T_f)}{g_* T_f^3} = \#\frac{\ln(M_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

where $\# = 45/(2\pi^2)$.

- Mass-to-enropy ratio

Confirmed by honest calculation, backup slides 3-5

$$\frac{M_X n_X}{s} = \#\frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Most relevant parameter: annihilation cross section $\sigma_0 \equiv \langle \sigma v \rangle$ at freeze-out

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Correct value, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV, for

$$\sigma_0 \equiv \langle \sigma v \rangle = (1 \div 1.3) \cdot 10^{-36} \text{ cm}^2 = (1 \div 1.3) \text{ pb}$$

  - Weak scale cross section.

    Gravitational physics and EW scale physics combine into

$$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W} \right)^2 \simeq 10^{-10} \text{ GeV}$$

  - Mass M_X should not be much higher than 100 GeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates, $T_f \sim M_X/20$, and in kinetic equilibrium long after.

WIMP candidates

SUSY: neutralinos, $X = \chi$

But situation is often tense: annihilation cross section is often too low

Important supperssion factor: $\langle \sigma v \rangle \propto v^2 \propto T/M_\chi$ because of p-wave annihilation in case $\chi\chi \to Z^* \to f\bar{f}$:

Relativistic $f\bar{f} \Longrightarrow$ total angular momentum $J = 1$

$\chi\chi$: identical fermions $\Longrightarrow L = 0$, parallel spins impossible $\Longrightarrow$ p-wave

Many other possibilities

Example: Higgs portal

Just to have a WIMP, introduce scalar singlet S.

Renormalizable interaction with Higgs only. Impose symmetry $S \to -S \implies$ stable S .

$$L_S = \frac{1}{2}(\partial_\mu S)^2 - \left(\frac{\mu_s^2}{2} S^2 + \frac{\lambda_{SH}}{4} S^2 H^\dagger H + \frac{\lambda_S}{4} S^4 \right)$$

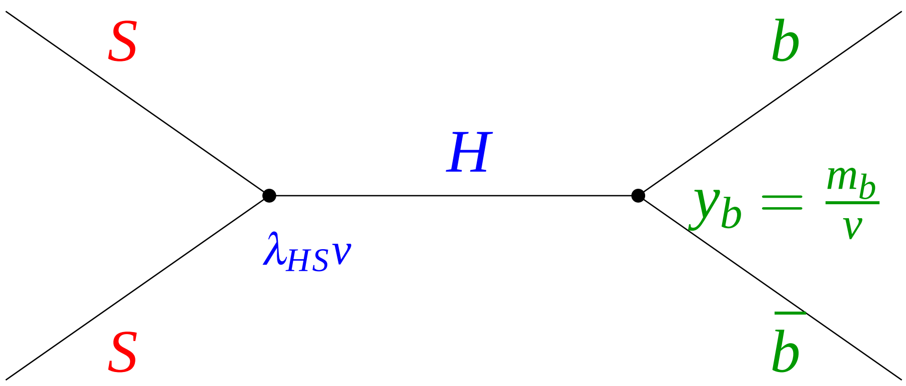
In vacuo $H = v/\sqrt{2} + h/\sqrt{2}$: vertices

$$\frac{\lambda_{SH}}{4} v h S^2 + \frac{\lambda_{SH}}{8} h^2 S^2$$

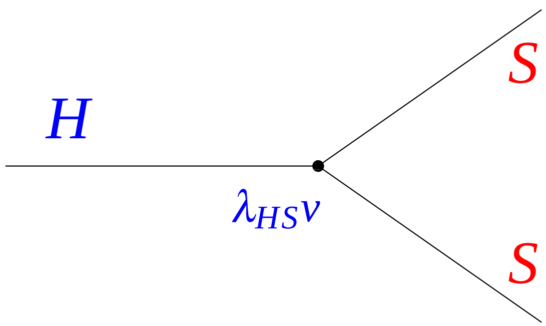
- Light S, $m_S \ll m_H/2$.

  Fairly popular a year ago

  No longer viable: main annihilation channel $SS \to b\bar{b}$. Cross section suppressed by $y_b^2 \propto m_b^2$



$\langle \sigma v \rangle = 1$ pb $\Longrightarrow$ quite large λ_{SH} $\Longrightarrow$
Invisible Higgs decay $H \to SS$ by far dominant.



Nice excersize:
calculate $\langle \sigma v \rangle$
and $\Gamma(H \to SS)$

- Degeneracy: m_s just below $m_H/2$.

  Pole enhanced $\langle \sigma v \rangle \Longrightarrow$ not so large $\lambda_{SH} \Longrightarrow$

  + threshold suppression of invisible Higgs decay $H \to SS \Longrightarrow$

  viable and interesting.

  Signature: invisible Higgs decay.

- Relatively heavy S: $m_s > m_W$.

  Main annihilation channels $SS \to WW, ZZ, HH$.

  Interesting for direct dark matter detection experiments and LHC at $m_S \lesssim 150$ GeV.

  Signature $pp \to H^* +$ jet $\to$ jet$+SS$;

  jet + missing E_T

Sumary of direct detection limits: backup slide 6

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. Another good DM candidate: axion.

Plus a lot of exotica...

Crucial impact of LHC to cosmology,
direct and indirect dark matter searches

Lecture 2

Outline of Lecture 2

- Axions
  - Theory
  - Cosmology
  - Search
- Warm dark matter
  - Gravitino
- Dark matter summary

Axions

Motivation: solution of strong CP problem

What's the problem?

Quark Yukawa interactions $\Longrightarrow$ quark mass matrix

$$L_Y = y_{ij}^{(d)} \bar{Q}_L^i H d_R^j + y_{ij}^{(u)} \bar{Q}_L^i \tilde{H} u_R^j + \text{h.c.} \Longrightarrow L_m = m_{ij}^{(d)} \bar{d}_L^i d_R^j + m_{ij}^{(u)} \bar{u}_L^i u_R^j + \text{h.c.}$$

$m_{ij}^{(u,d)} = y_{ij}^{(u,d)} v/\sqrt{2}$ complex; $i,j = 1,2,3 =$ generation label.

Standard lore: diagonalize $\Longrightarrow$ CKM matrix, 3 angles, 1 phase.

This is not quite true

One more phase: common phase of all Yukawa couplings/masses,

$$m_{ij} = \mathrm{e}^{i\theta} \cdot m_{ij}^{CKM} \qquad \text{or} \qquad \theta = \text{Arg det } m$$

At first sight: rotate away,

$$q_L^i \to \mathrm{e}^{i\theta/2} q_L^i, \quad q_R^i \to \mathrm{e}^{-i\theta/2} q_R^i, \quad \text{i.e.,} \quad q^i \to \mathrm{e}^{i\gamma^5\theta/2} q^i$$

But this is does not remove phase θ from QCD!

To see this, let us calculate vacuum energy density for non-zero θ, call it $V(\theta)$ (useful in what follows). Keep only u, d-quarks, take their masses equal for simplicity, $m_u = m_d \equiv m_q \sim 10$ MeV (heavier quarks make smaller contribution),

$$L_m = \mathrm{e}^{i\theta} m_q (\overline{u_L} u_R + \overline{d_L} d_R) + \text{h.c.}$$

Perturbation theory in m_q: $V(\theta) = -\langle L_m \rangle$.

Quark condensate in QCD vacuum in chiral limit, $m_q = 0$:

$$\langle \overline{u_L} u_R \rangle = \langle \overline{d_L} d_R \rangle = \frac{1}{2} \langle \bar{q} q \rangle = \text{real} \sim \Lambda_{QCD}^3$$

NB: No arbitrary phase here, otherwise η' would be pseudo-Goldstone!

Get

$$V(\theta) = -\langle L_m \rangle = -m_q \langle \bar{q} q \rangle \cos\theta$$

θ is a physical parameter! Violates CP.

NB: Minimum of $V(\theta)$ at $\theta = 0$. No help if θ is just a free parameter.

Q: What's wrong with the field redefinition $q^i \to e^{i\gamma^5\theta/2} q^i$?

A: It modifies QCD Lagrangian.

QCD Lagrangian:

$$L_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \tau \frac{\alpha_s}{16\pi} \varepsilon_{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} + L_{\text{quark}}$$

τ: an extra coupling constant. Violates CP.

NB: No extra term in QED: $\varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$ is total derivative. But total derivatives do sometimes matter in quantum theory!

Field redefinition $q^i \to e^{i\gamma^5\theta/2} q^i \Longrightarrow \tau \to \tau + \theta$.

By field redefinition can reshuffle τ (parameter in gluon Lagrangian) and θ (common phase in quark mass matrix).

But cannot get rid of both! $\bar{\theta} = \tau + \text{Arg det } m$ is invariant under field redefinitions.

NB: We actually calculated $V(\bar{\theta})$ (implicitly assumed $\tau = 0$).

CP violation within QCD. Neutron edm $d_n < 3 \cdot 10^{-26} e \cdot \mathrm{cm} \Longrightarrow$

$$\bar{\theta} \lesssim 10^{-10}$$

Either fine tune, or find mechanism that ensures $\bar{\theta} = 0$

Peccei–Quinn mechanism: promote $\bar{\theta}$ to a field.

Simple version: two Higgs fields plus singlet S (DFSZ)

$$L = y^{(d)} \bar{Q}_L H_1 d_R + y^{(u)} \bar{Q}_L H_2 u_R + |D_\mu H_1|^2 + |D_\mu H_2|^2 + \frac{1}{2}(\partial_\mu S)^2 - V(H_1, H_2, S)$$

Classical level: require global symmetry (PQ symmetry)

$$q^i \to \mathrm{e}^{i\gamma^5 \theta/2} q^i \,, \quad H_1 \to \mathrm{e}^{i\theta} H_1 \,, \quad H_2 \to \mathrm{e}^{i\theta} H_2 \,, \quad S \to \mathrm{e}^{2i\theta} S$$

Vev's: $\langle H_1 \rangle = v_1/\sqrt{2}$, $\langle H_2 \rangle = v_2/\sqrt{2}$, $\langle S \rangle = v_S$, break PQ symmetry spontaneously.

Parametrize

$$H_1 = \mathrm{e}^{i\theta(x)} v_1/\sqrt{2}\,, \quad H_2 = \mathrm{e}^{i\theta(x)} v_2\sqrt{2}\,, \quad S = \mathrm{e}^{2i\theta(x)} v_S$$

If not for QCD, $\langle\theta(x)\rangle$ would be arbitrary. Quark masses $m = yv_{1,2}\mathrm{e}^{i\langle\theta(x)\rangle}$. CP-violating parameter $\bar{\theta} = \tau + \text{Arg det } yv + \langle\theta(x)\rangle$, vev of a field. $\theta(x)$ would be a massless Goldstone boson. Kinetic term

$$\frac{1}{2}v_1^2(\partial_\mu\theta)^2 + \frac{1}{2}v_2^2(\partial_\mu\theta)^2 + \frac{1}{2}v_S^2(\partial_\mu\theta)^2 = \frac{1}{2}f_{PQ}^2(\partial_\mu\theta)^2$$

$f_{PQ}^2 = v_1^2 + v_2^2 + v_S^2$ can be large, if v_S is large.

Turn on QCD: shift $\theta \to \theta + \text{const}$ is NOT a symmetry.

Consequences

- $\langle\theta(x)\rangle$ is such that $V(\bar{\theta})$ is at minimum $\Longrightarrow \bar{\theta} = 0$ automatically. Strong CP problem solved.
- $\theta(x)$ gets a mass

$$L_\theta = \frac{1}{2} f_{PQ}^2 (\partial_\mu \theta)^2 - V(\theta)\ , \qquad V(\theta) \simeq -m_q \langle \bar{q} q \rangle \cos\theta = \frac{1}{2} m_q \langle \bar{q} q \rangle \theta^2$$

Axion field $\theta(x) = a(x)/f_{PQ}$:

$$m_a^2 \simeq \frac{m_q \langle \bar{q} q \rangle}{f_{PQ}^2} \simeq \frac{m_q \Lambda_{QCD}^3}{f_{PQ}^2} \quad \Longrightarrow \quad m_a = 0.6\ \text{eV} \cdot \left(\frac{10^7\ \text{GeV}}{f_{PQ}} \right)$$

Interactions:

- Axion-gluon-gluon:

$$C_{agg} \frac{\alpha_s}{16\pi} \theta(x) \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu}^a F_{\lambda\rho}^a\ , \qquad C_{agg} \sim 1 \quad \text{roughly}$$

- Likewise: axion-photon-photon

$$C_{a\gamma\gamma} \frac{\alpha}{16\pi} \theta(x) \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}\ , \qquad C_{a\gamma\gamma} \sim 1 \quad \text{roughly}$$

To summarize

Peccei–Quinn solution to strong CP problem predicts axion with mass

$$m_a = 0.6\ \text{eV} \cdot \left(\frac{10^7\ \text{GeV}}{f_{PQ}}\right)$$

and $a\gamma\gamma$ interaction

$$C_{a\gamma\gamma}\frac{\alpha}{16\pi}\frac{a(x)}{f_{PQ}}\varepsilon_{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$$

where $C_{a\gamma\gamma} \sim 1$ is model-dependent, and f_{PQ} is the only free parameter. Larger $f_{PQ} \Longrightarrow$ smaller m_a, weaker interactions.

- Why did DFSZ introduce field S? Because $f_{PQ}^2 = v_1^2 + v_2^2 + v_S^2$. No $S \Longrightarrow v_S = 0 \Longrightarrow f_a^2 = 246$ GeV $\Longrightarrow m_a = 15$ keV, fairly strong $a\gamma\gamma$ interaction. Weinberg–Wilczek axion, ruled out.
- There are other ways to make f_{PQ} large, e.g., KSVZ.

Why is this interesting for cosmology?

- Axion is practically stable:

$$\Gamma(a \to \gamma\gamma) = C_{a\gamma\gamma}^2 \left(\frac{\alpha}{8\pi}\right)^2 \frac{m_a^3}{4\pi f_{PQ}^2} \quad \Longrightarrow \quad \tau_a = 10^{17} \left(\frac{\text{eV}}{m_a}\right)^5 \text{ yrs}$$

- Interacts very weakly $\Longrightarrow$ dark matter candidate

- May never be in thermal equilibrium $\Longrightarrow$ cold dark matter if momenta are negligibly small.

Q. How can one arrange for negligibly small momenta for particles with sub-eV masses?

A. One way: Condensates
(Not the only option)

Reminder from Lecture 1:

- Expansion rate at radiation domination

$$H(T) = \frac{T^2}{M_{Pl}^*}\,, \qquad M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}}$$

- Mass-to-entropy ratio of dark matter today

$$\frac{\rho_{DM}}{s} = 3 \cdot 10^{-10}\ \text{GeV}$$

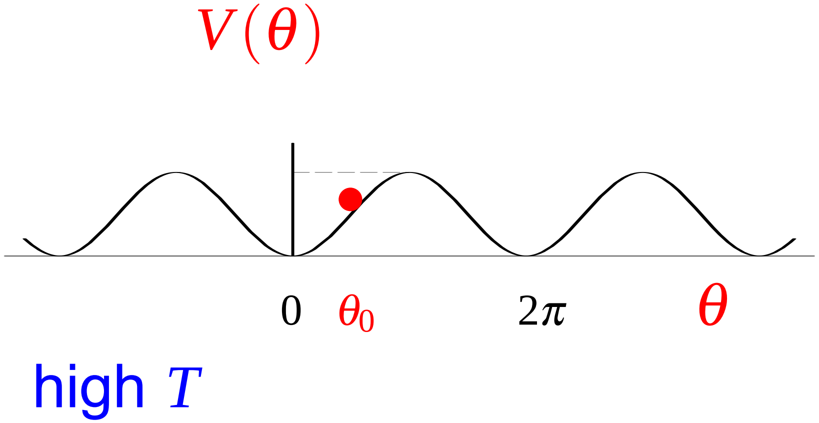
Axion production: misalignment

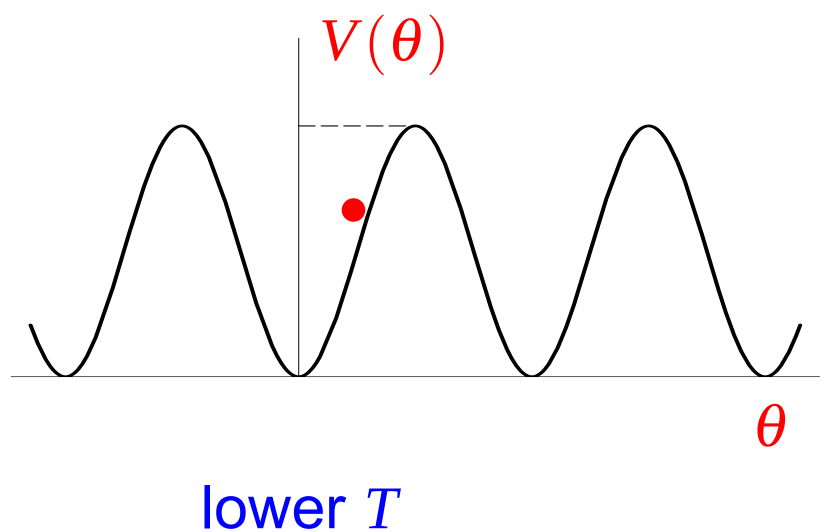
Recall $V(\theta) \simeq -m_q \langle \bar{q}q \rangle \cos\theta$

Early Universe, high T: $\langle \bar{q}q \rangle = 0 \Longrightarrow V(\theta) = 0$.

No preferred value of $\theta \Longrightarrow$ Initial condinion θ_0 anywhere between $-\pi$ and π.

At QCD epoch ($T \sim 200$ MeV) potential $V(\theta)$ builds up. θ starts to roll down.





Rolling down starts when $m_a(T) \sim H(T)$: before that time scale of rolling m_a^{-1} is larger than the cosmological time scale $\sim H^{-1}$.

After initial rolling, θ oscillates about minimum $\theta = 0$.

Homogeneous oscillating field $=$ condensate $=$ collection of quanta with zero spatial momentum. Just what we need for cold dark matter!

Estimate for present mass density:

Energy density at beginning of rolling

$$V(\theta_0, T) = m_a^2(T) a_0^2 = m_a^2(T) f_{PQ}^2 \theta_0^2$$

Number density of quanta at that time

$$n_a(T) = V(\theta_0, T)/m_a(T) = m_a(T) f_{PQ}^2 \theta_0^2$$

Recall $m_a(T) \sim H(T) = T^2/M^*_{Pl} \Longrightarrow$ number-to-entropy

$$\frac{n_a}{s} = \# \frac{H(T) f_{PQ}^2 \theta_0^2}{g_* T^3} = \# \frac{f_{PQ}^2 \theta_0^2}{\sqrt{g_*} M_{Pl} T}$$

with $T = T_{QCD} \sim 200$ MeV and $\# \sim 1$.

Present mass-to-entropy

$$\frac{\rho_a}{s} = m_a^{(T=0)} \cdot \frac{n_a}{s} = \# \frac{m_a^{(T=0)} f_{PQ}^2}{\sqrt{g_*} M_{Pl} T_{QCD}} \theta_0^2$$

Recall $m_a f_{PQ}^2 \propto m_a^{-1}$: the lighter axions, the more dark matter.

$\rho_{DM}/s \sim 3 \cdot 10^{-10}$ GeV is obtained for $m_a = 10^{-5} - 10^{-6}$ eV (for $\theta_0 = \pi/2 - 0.1$).

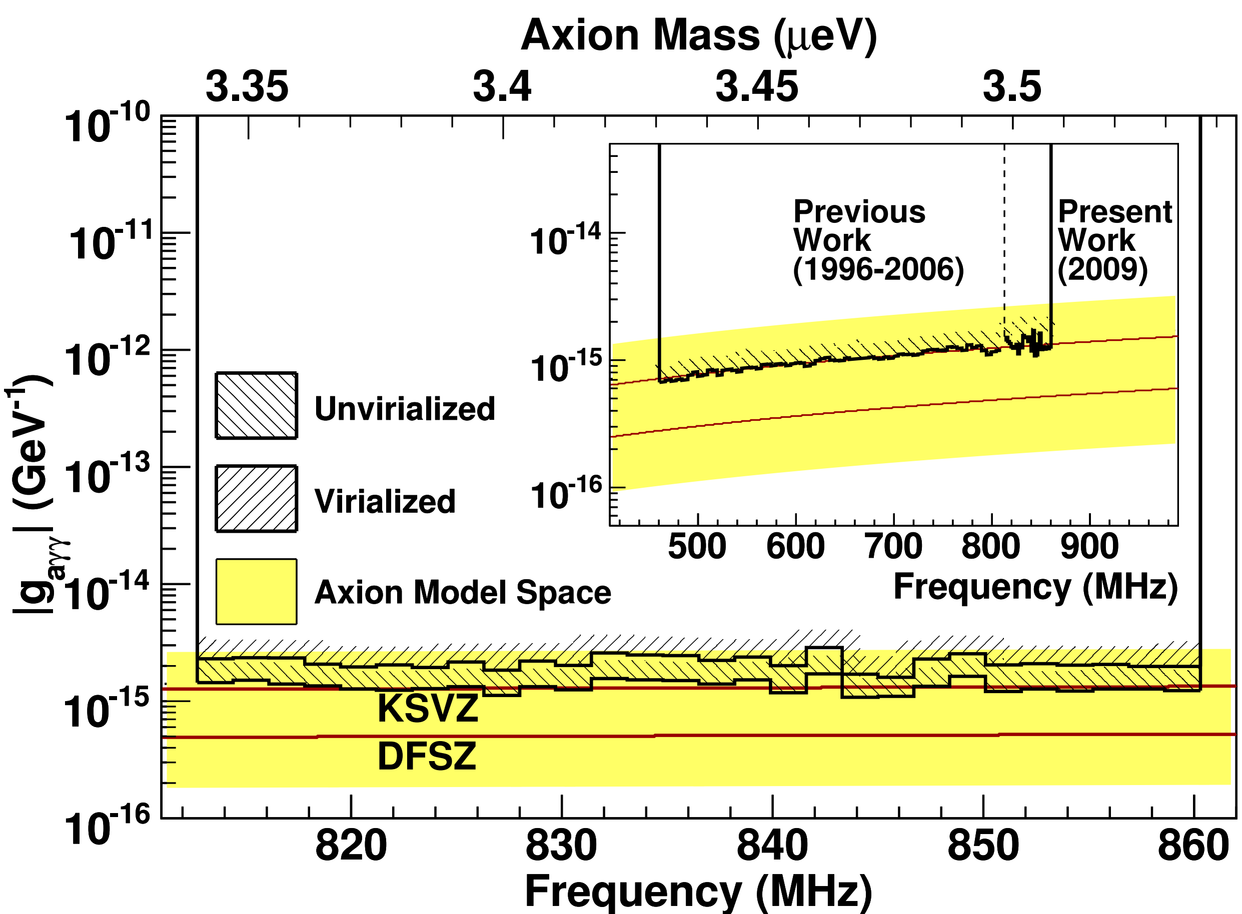
Axions of mass $1 - 10$ μeV are good cold dark matter candidates.

NB: Misalignment is not the only possible production mechanism.

Search

$a\gamma\gamma$ interaction $\qquad C_{a\gamma\gamma}\frac{\alpha}{16\pi}\frac{a(x)}{f_{PQ}}\varepsilon_{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$

Conversion of DM axion into photon in magnetic field in a resonant cavity. 10^{-6} eV/2π = 240 MHz. Need high Q resonator to collect photons, narrow bandwidth, go small steps in m_a. Long story.



ADMX, PRL '2010

Warm dark matter

- Clouds over CDM

  Numerical simulations of structure formation with CDM show

  - Too many dwarf galaxies

    A few hundred satellites of a galaxy like ours —

    But only dozens observed so far

  - Too high density in galactic centers (“cusps”)

- No serious worry yet

  But what if one really needs to suppress small structures?

High initial momenta of DM particles $\Longrightarrow$ Warm dark matter

Warm dark matter

- Decouples when relativistic, $T_f \gg m$.
- Remains relativistic until $T \sim m$ (assuming thermal distribution). Does not feel gravitational potential before that.
- Perturbations of wavelengths shorter than horizon size at that time get smeared out $\Longrightarrow$ small size objects do not form ("free streaming")
- Horizon size at $T \sim m$

$$l(T) = H(T \sim m)^{-1} = \frac{M_{Pl}^*}{T^2} = \frac{M_{Pl}^*}{m^2}$$

Present size of this region

$$l_c = \frac{T}{T_0} l(T) = \frac{M_{Pl}}{mT_0}$$

(modulo g_* factors).

Objects of initial comoving size smaller than l_c are less abundant

- Initial size of dwarf galaxy $l_{dwarf} \sim 100$ kpc $\sim 3 \cdot 10^{23}$ cm
  Require

$$l_c \simeq \frac{M_{Pl}}{mT_0} \sim l_{dwarf}$$

$\Longrightarrow$ obtain mass of DM particle

$$m \sim \frac{M_{Pl}}{T_0 l_{dwarf}} \sim 3 \text{ keV}$$

($M_{Pl} = 10^{19}$ GeV, $T_0^{-1} = 0.1$ cm).

- Particles of masses in 1 – 10 keV range
  are good warm dark matter candidates
  (assuming their energies at decoupling are of order T)

- Candidates
  - Sterile neutrino
  - Gravitino
  - Exotica

Gravitinos in low energy SUSY

- Rigid supersymmtery: SUSY breaking $\Longrightarrow$ Goldstino, massless Nambu–Goldstone fermion.

- Supergravity = local supersymmetry $\Longrightarrow$ Goldstino eaten up by gravitino, gravitino becomes massive.

- Mass $m_{3/2} \simeq F/M_{Pl}$

  $\sqrt{F} =$ SUSY breaking scale.

  $\Longrightarrow$ Gravitinos light for low SUSY breaking scale.

  E.g. gauge mediation

  From purely theoretical viewpoint, mass may be as low as, say, $\text{TeV}^2/M_{Pl} \sim 10^{-4}$ eV. In concrete models much heavier, but $m_{3/2} \sim$ keV is not unrealistic.

- Light gravitino = LSP $\Longrightarrow$ Stable

- Interactions: mostly due to Goldstino (longitudinal) component. Calculable. Proportional to SUSY breaking parameters, suppressed by $F^{-1} \simeq M_{Pl} m_{3/2}$.

  As an example, interaction with gauge bosons and gaugino

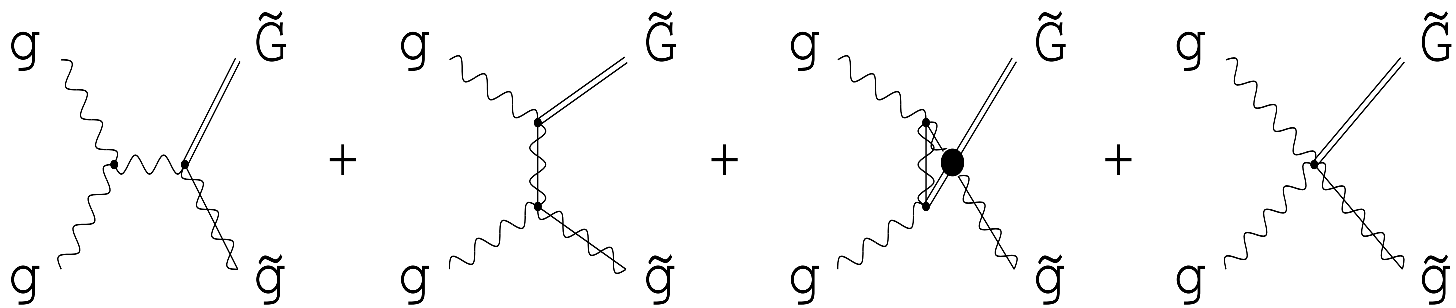
$$L_{int} = \frac{M_{\tilde{g}}}{F} \tilde{G}_{3/2}[\gamma^\mu, \gamma^\nu]\tilde{g}F_{\mu\nu} = \frac{M_{\tilde{g}}}{M_{Pl} m_{3/2}} \tilde{G}_{3/2}[\gamma^\mu, \gamma^\nu]\tilde{g}F_{\mu\nu}$$

  $M_{\tilde{g}} =$ (soft) gaugino mass (due to SUSY breaking).

- Rather weird: the smaller the mass $m_{3/2}$, the stronger interactions.

- Production in the Universe: $E_{3/2} \gtrsim T$.
  Warm dark matter candidate, if $m_{3/2} \sim 10$ keV.
  Heavier gravitino = cold DM candidate.

Production in scattering



Production cross section at $T \gg M_{\tilde{g}}$

$$\sigma_{prod} \simeq C\alpha \frac{M_{\tilde{g}}^2}{M_{Pl}^2 m_{3/2}^2}$$

with $C \sim 10^3$ (matter of debate: infrared logs).

Gravitinos produced in one Hubble time at temperature T

$$\left(\frac{n_{3/2}}{s}\right)_T = \sigma_{prod}\frac{n_g^2}{s}H^{-1}(T) \propto T \ , \quad \text{since } n_g \sim s \propto T^3 \ , \quad H = T^2/M_{Pl}^*$$

Maximum production at maximum possible temperature.

As a result (gluons/gluinos dominate)

$$\frac{m_{3/2}n_{3/2}}{s} \sim \frac{m_{3/2}}{g_*} \cdot C\alpha_s \frac{M_{\tilde{g}}^2}{M_{Pl}^2 m_{3/2}^2} \cdot M_{Pl}^* T_{max}$$

Right abundance, $n_{3/2}m_{3/2}/s = 3 \cdot 10^{-10}$ GeV obtained by tuning T_{max}:

$$T_{max} = 10^4 \text{ GeV} \cdot \frac{m_{3/2}}{10 \text{ keV}} \cdot \left(\frac{1 \text{ TeV}}{M_{\tilde{g}}}\right)^2$$

- "Easy" to obtain right abundance by dialing T_{max}.

  Range of T_{max}:
  from $T_{max} \sim$ a few TeV for $m_{3/2} \sim$ a few keV
  to $T_{max} \sim 10^{12}$ GeV for $m_{3/2} \sim 1$ TeV

  But required abundance appears a coincidence.

  NB: Same for many other dark matter candidadtes.

Competing production mechanism:
Gravitino production in decays of superpartners

$$\frac{d(n_{3/2}/s)}{dt} = \frac{n_{\tilde{S}}}{s}\Gamma_{\tilde{S}}$$

$n_{\tilde{S}}/s =$ const $\sim g_*^{-1}$ for $T \gtrsim M_{\tilde{S}}$, while $n_{\tilde{S}} \propto \mathrm{e}^{-M_{\tilde{S}}/T}$ for $T \ll M_{\tilde{S}}$
$\Longrightarrow$ production most efficient at $T \sim M_{\tilde{S}}$ (slow cosmological expansion with unsuppressed $n_{\tilde{S}}$)

Hence

$$\frac{n_{3/2}}{s} \simeq \frac{\Gamma_{\tilde{S}}}{g_* H(T \sim M_{\tilde{S}})} \simeq \frac{M_{Pl}^*}{g_* M_{\tilde{S}}^2} \cdot \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2}$$

Mass-to-entropy ratio

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \frac{M_{\tilde{S}}^3}{m_{3/2}} \frac{1}{g_*^{3/2} M_{Pl}}$$

For $m_{3/2} =$ a few keV, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV

$M_{\tilde{S}} \simeq 100 \div 300$ GeV

Need: some superpartners light,

others a lot heavier, $m_{\tilde{S}} > T_{max}$.

Rather contrived scenario, but generating warm dark matter is always contrived

Dark matter summary

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. A good non-LHC DM candidate: axion.

Plus a lot of exotica...

Crucial impact of LHC to cosmology, direct and indirect dark matter searches

- WIMP, signal at LHC:
  - Strongest possible motivation for direct and indirect detection
    - Inferred interactions with baryons $\Longrightarrow$ strategy for direct detection
  - A handle on the Universe at

$$T = (\text{a few}) \cdot 10 \text{ GeV} \div (\text{a few}) \cdot 100 \text{ GeV}$$

$$t = 10^{-11} \div 10^{-8} \text{ s}$$

cf. $T = 1$ MeV, $t = 1$ s at nucleosynthesis

- Gravitino-like
  - A lot of work to make sure that it is indeed DM particle
  - Hard time for direct and indirect searches
- No signal at LHC
  - Best guess: axion
  - If not, need more hints from cosmology and astrophysics

Lecture 3

Outline of Lecture 3

- Baryon asymmetry of the Universe.
  - Electroweak baryogenesis.
    - Electroweak baryon number violation
    - Electroweak transition
    - What can make electroweak mechanism work?
  - Leptogenesis
    - See-saw in nutshell
    - Asymmetry from decays
    - Washout in scattering
    - Conclusions on leptogenesis
- Overall conclusions

Baryon asymmetry of the Universe

- There is matter and no antimatter in the present Universe.
- Baryon-to-photon ratio, almost constant in time:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 6 \cdot 10^{-10}$$

Baryon-to-entropy, constant in time: $n_B/s = 0.9 \cdot 10^{-10}$

What's the problem?

Early Universe ($T > 10^{12}$ K $= 100$ MeV):
creation and annihilation of quark-antiquark pairs $\Rightarrow$

$$n_q, n_{\bar{q}} \approx n_\gamma$$

Hence

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-9}$$

How was this excess generated in the course of the cosmological evolution?

Sakharov conditions

To generate baryon asymmetry, three necessary conditions should be met at the same cosmological epoch:

- B-violation
- C- and CP-violation
- Thermal inequilibrium

NB. Reservation: L-violation with B-conservation at $T \gg 100$ GeV would do as well $\Longrightarrow$ Leptogenesis.

Can baryon asymmetry be due to electroweak physics?

Baryon number is violated in electroweak interactions.

Non-perturbative effect

Hint: triangle anomaly in baryonic current B^μ:

$$\partial_\mu B^\mu = \left(\frac{1}{3}\right)_{B_q} \cdot 3_{colors} \cdot 3_{generations} \cdot \frac{g_W^2}{32\pi^2} \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

$F^a_{\mu\nu}$: $SU(2)_W$ field strength; g_W: $SU(2)_W$ coupling

Likewise, each leptonic current ($n = e, \mu, \tau$)

$$\partial_\mu L^\mu_n = \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

Large field fluctuations, $F^{a}_{\mu\nu} \propto g_W^{-1}$ may have

$$Q \equiv \int d^3x dt \ \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^{a}_{\mu\nu} F^{a}_{\lambda\rho} \neq 0$$

Then

$$B_{fin} - B_{in} = \int d^3x dt \ \partial_\mu B^\mu = 3Q$$

Likewise

$$L_{n,\ fin} - L_{n,\ in} = Q$$

B is violated, $B - L$ is not.

How can baryon number be not conserved without explicit B-violating terms in Lagrangian?

Consider massless fermions in background gauge field $\vec{A}(\mathbf{x},t)$ (gauge $A_0 = 0$). Let $\vec{A}(\mathbf{x},t)$ start from vacuum value and end up in vacuum.

NB: This can be a fluctuation

Dirac equation

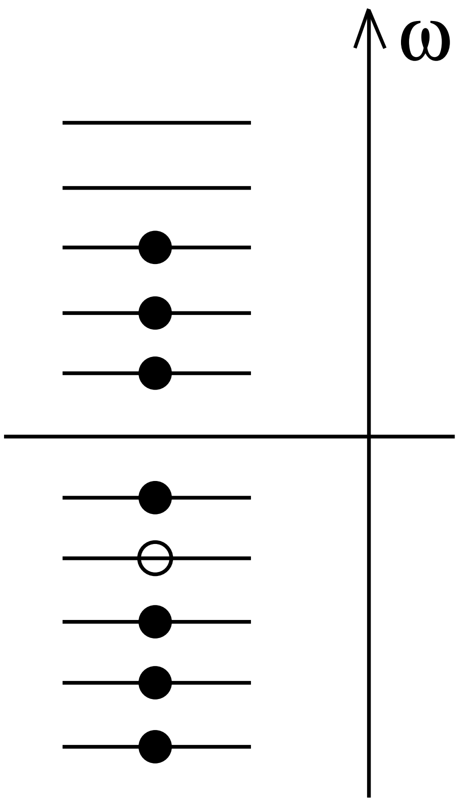
$$i\frac{\partial}{\partial t}\psi = i\gamma^0\vec{\gamma}(\vec{\partial} - ig\vec{A})\psi = H_{Dirac}(t)\psi$$

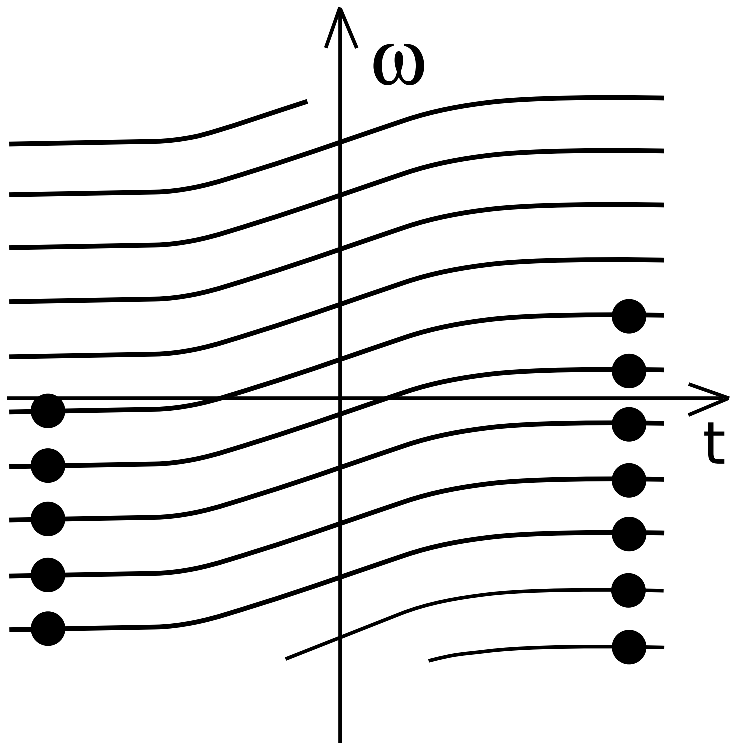
Suppose for the moment that $\vec{A}$ slowly varies in time. Then fermions sit on levels of instantaneous Hamiltonian,

$$H_{Dirac}(t)\psi_n = \omega_n(t)\psi_n$$

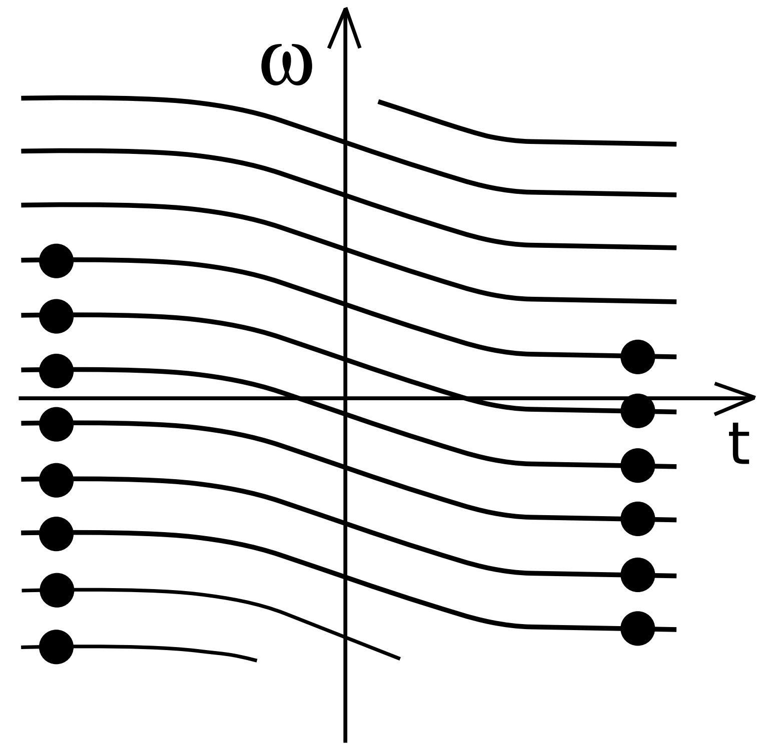
How do eigenvalues behave in time?

Fermion energy levels at $\vec{A} = 0$





Left fermions



Right fermions

Motion of energy levels in special (topological)

gauge field background $\vec{A}(t)$

QCD case, $B = N_L + N_R$ is conserved, $Q^5 = N_L - N_R$ is not

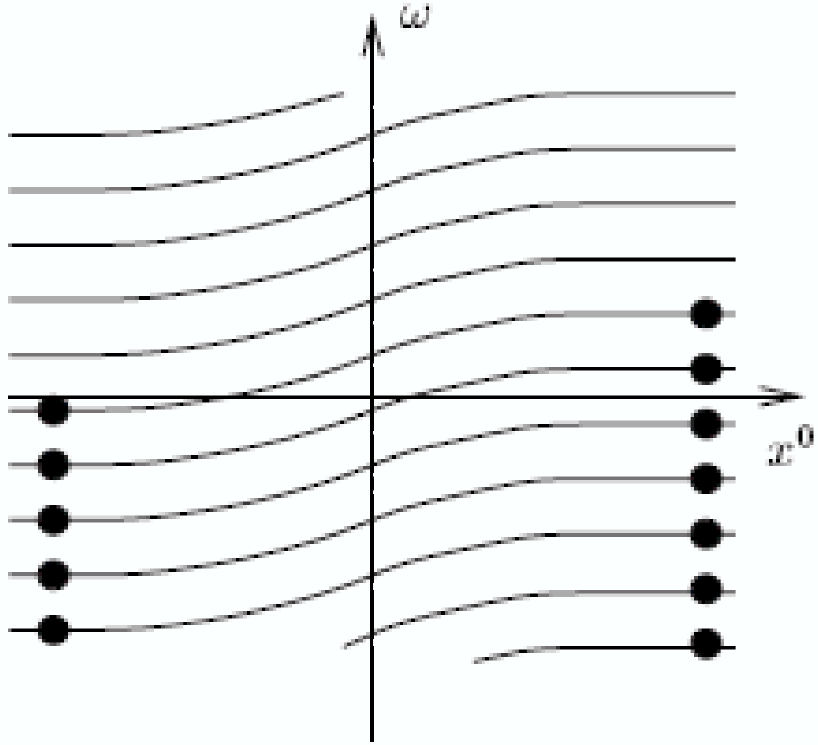
NB: Non-Abelian gauge fields only (in 4 dimensions)

QCD: Violation of Q^5 is a fact.

In chiral limit $m_u, m_d, m_s \to 0$,
global symmetry is $SU(3)_L \times SU(3)_R \times U(1)_B$,
not symmetry of classical Lagrangian
$SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$

If only left-handed fermions interact with gauge field,

then number of fermions is not conserved



The case for $SU(2)_W$

Fermion number of every doublet changes in the same way

Need large field fluctuations. At zero temprature their rate is suppressed by

$$\mathrm{e}^{-\frac{16\pi^2}{g_W^2}} \sim 10^{-165}$$

High temperatures: large thermal fluctuations (“sphalerons”).

B-violation rapid as compared to cosmological expansion at

$$\langle \phi \rangle_T < T$$

$\langle \phi \rangle_T$: Higgs expectation value at temperature T.

Possibility to generate baryon asymmetry at electroweak epoch, $T_{EW} \sim 100$ GeV ?

Problem: Universe expands slowly. Expansion time

$$H^{-1} = \frac{M_{Pl}^*}{T_{EW}^2} \sim 10^{14}\ \text{GeV}^{-1} \sim 10^{-10}\ \text{s}$$

Too large to have deviations from thermal equilibrium?

The only chance: 1st order phase transition, highly inequilibrium process

Electroweak symmetry is restored, $\langle \phi \rangle_T = 0$ at high temperatures

Just like superconducting state becomes normal at "high" T

Transition may in principle be 1st order

Fig

1st order phase transition occurs from supercooled state via spontaneous creation of bubbles of new (broken) phase in old (unbroken) phase.

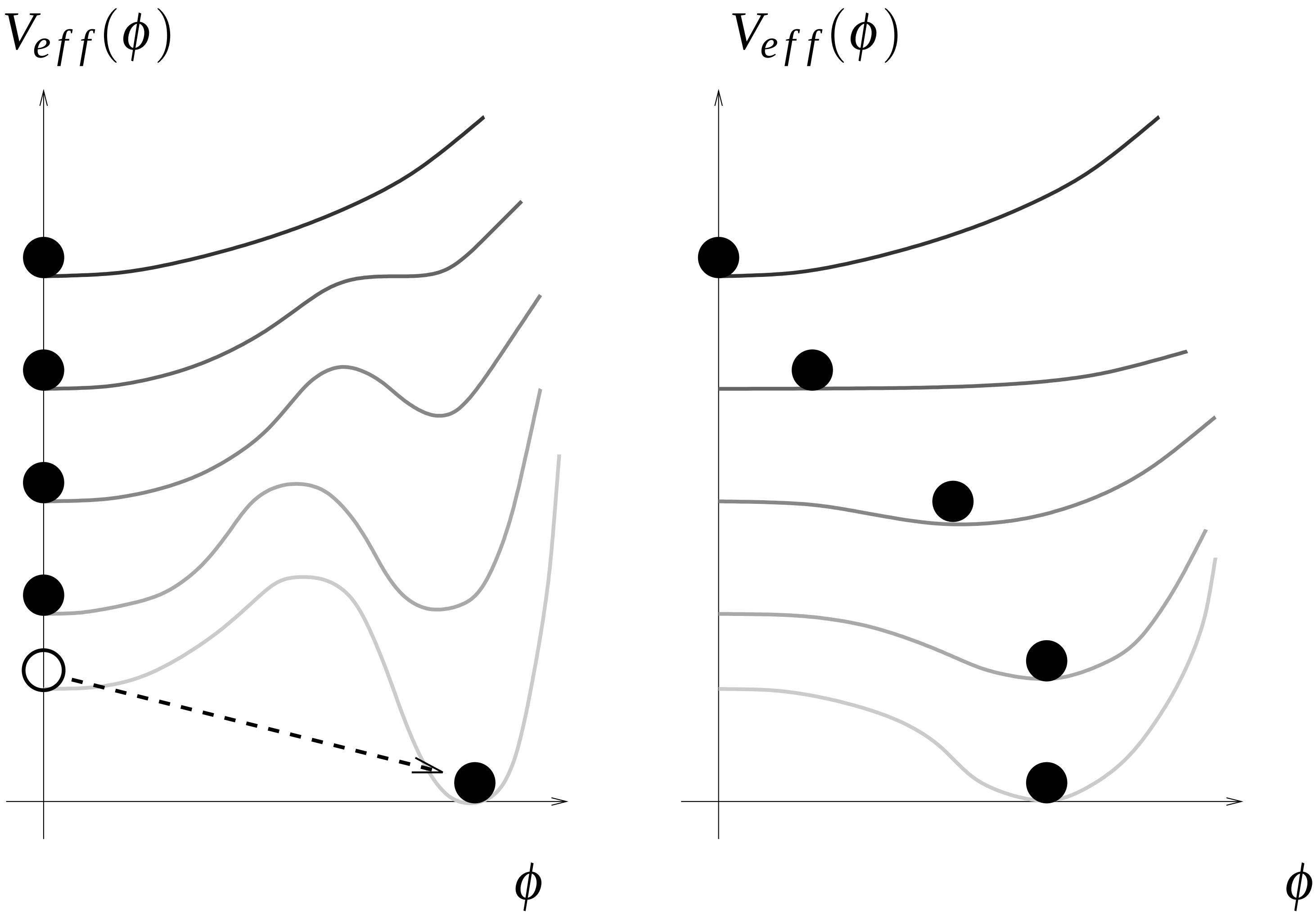
Bubbles then expand at $v \sim 0.1c$

Fig

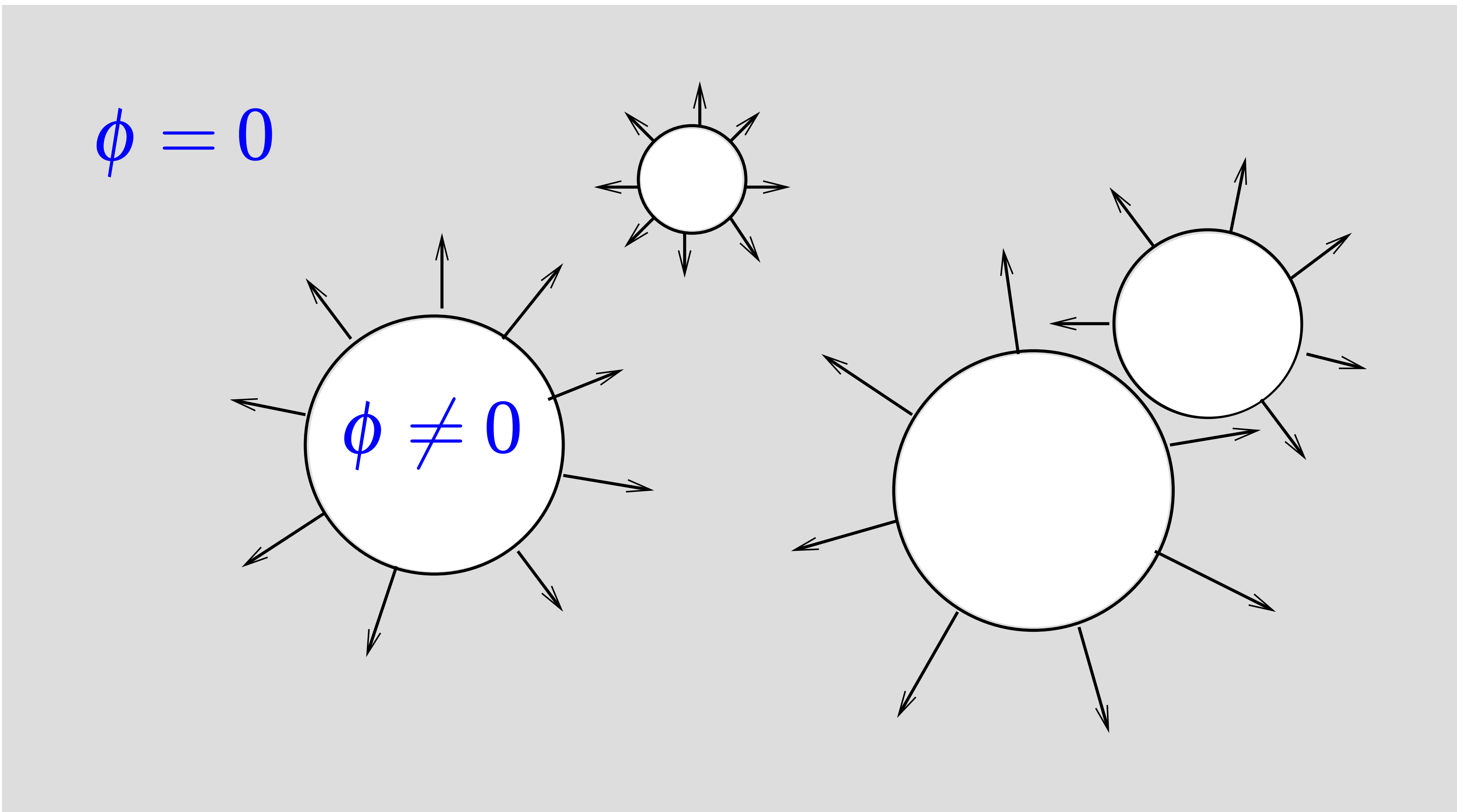
Beginning of transition: about one bubble per horizon

Bubbles born microscopic, $r \sim 10^{-16}$ cm, grow to macroscopic size, $r \sim 0.01 H^{-1} \sim 0.1$ mm, before their walls collide

Boiling Universe, strongly out of equilibrium



1st order   2nd order



$\phi = 0$
$\phi \neq 0$

Baryon asymmetry may be generated in the course of phase transition, provided there is enough C- and CP-violation.

Necessary condition:

Baryon asymmetry generated during transition should not be washed out aferwards
$\Longrightarrow$ B-violating processes must be switched off in broken phase
$\Longrightarrow$ Just after transition must have

$$\langle \phi \rangle_T > T$$

Does this really happen?

Not in SM

Temperature-dependent effective potential, one loop

$$V_{eff} = (-m^2 + \alpha T^2)|\phi|^2 - \frac{\beta}{3} T |\phi|^3 + \frac{\lambda}{4}|\phi|^4$$

$\alpha = O(g^2)$, $\beta = O(g^3)$. Cubic term weird,

$$-\frac{\beta}{3} T \left(\phi^\dagger \phi\right)^{3/2}$$

But crucial for 1st order phase transition. Obtains contributions from bosons only

$$f_B = \frac{1}{\mathrm{e}^{E/T} - 1} \simeq \frac{T}{E} \equiv \frac{T}{\sqrt{\mathbf{p}^2 + g^2|\phi|^2}} \simeq \frac{T}{g|\phi|} \quad \text{at} \quad |\mathbf{p}| \ll g|\phi| \ll T$$

Bose enhancement $\Longleftrightarrow$ no analyticity in $g^2|\phi|^2$

At phase transition $(-m^2 + \alpha T^2) = 0$,

$$V_{eff} = -\frac{\beta}{3} T \phi^3 + \frac{\lambda}{4} \phi^4$$

Hence

$$\langle \phi \rangle_T = \frac{\beta}{\lambda} T = \# \frac{g_W^3}{\lambda} T$$

Given the Higgs mass

$$m_H = \sqrt{2\lambda} v = 125 \text{ GeV}$$

one finds $\langle \phi \rangle_T < T$, asymmetry would be washed out even if generated

Furtermore, in SM

- No phase transition at all; smooth crossover
- Way too small CP-violation

What can make EW mechanism work?

- Extra bosons
  - Should interact strongly with Higgs(es)
  - Should be present in plasma at $T \sim 100$ GeV

    $\Longrightarrow$ not much heavier than 300 GeV

E.g. light stop

- Plus extra source of CP-violation.

Better in Higgs sector $\Longrightarrow$ Several Higgs fields

More generally, EW baryogenesis requires
complex dynamics in EW symmetry breaking sector
at $E \sim (\text{a few}) \cdot 100$ GeV

LHC's FINAL WORD

Is EW the only appealing scenario?

By no means!

— Strong competitor: leptogenesis

— Many other proposals

— Something theorists never thought about

Leptogenesis:

Baryon asymmetry and neutrino masses

B is violated in electroweak interactions,

$B-L$ is conserved

But we know that lepton numbers are violated anyway: neutrino oscillations.

Neutrinos have tiny masses. We know two differences of mass squared:

$$m_2^2 - m_1^2 = (0.01 \text{ eV})^2 \,, \qquad |m_3^2 - m_1^2| = (0.05 \text{ eV})^2$$

We also know that all masses are small,

$$m_\nu < 2 \text{ eV} \quad \text{(Experiment)}$$

$$m_\nu < 0.2 \text{ eV} \quad \text{(Cosmology)}$$

Leptogenesis: use the physics responsible for neutrino masses to generate lepton asymmetry in the Universe. Electroweak interactions automatically reprocess part of lepton asymmetry into baryon asymmetry.

Standard Model in thermal equilibrium at $T \gg 100$ GeV with $B-L \neq 1$:

$$B = C \cdot (B-L) \,, \qquad L = -(1-C) \cdot (B-L)$$

$$C = \frac{8N_{gen} + 4N_{Higgs}}{22N_{gen} + 13N_{Higgs}} = \frac{28}{79} \,, \quad T \gg 100 \text{ GeV}$$

See-saw in nutshell

Begin with one lepton doublet $L = (\nu, l)$. To generate neutrino mass, add a new left fermion N, singlet under $SU(2)_W \times U(1)_Y$.

Allowed Majorana mass term

$$\frac{M}{2}\bar{N}^c N \equiv \frac{M}{2}\varepsilon_{ij}N_i N_j + \text{h.c.}$$

$i, j = 1, 2$: Lorentzian spinor index of left fermion.

Also allowed Yukawa coupling to Higgs field, so Lagrangian includes

$$\frac{M}{2}\bar{N}^c N + y\bar{N}^c \tilde{H}^\dagger L + \text{h.c}$$

In vacuum $\tilde{H}^\dagger = (v/\sqrt{2}, 0)$, so one gets mass terms

$$\frac{M}{2}\bar{N}^c N + \frac{yv}{\sqrt{2}}\bar{N}^c \nu + \text{h.c}$$

At energies and momenta small compared to M, equation of motion for N is

$$MN + \frac{yv}{\sqrt{2}}\nu = 0 \quad \Longrightarrow \quad N = -\frac{yv}{M\sqrt{2}}\nu$$

Plug N back into Lagrangian, get Majorana mass of ν:

$$L_{m_\nu} = -\frac{y^2 v^2}{2M}\overline{\nu}^c \nu + \text{h.c.}$$

Small Majorana neutrino mass for large M and not necessarily small Yukawa coupling y:

$$m_\nu = \frac{y^2 v^2}{2M}$$

Three generations:

$$\mathscr{L} = \frac{M_\alpha}{2}\bar{N}^c_\alpha N_\alpha + \left(y_{\alpha\beta}\bar{N}^c_\alpha \tilde{H}^\dagger L_\beta + h.c.\right)$$

N_α: new two-component (left) fermions, $\alpha = 1,2,3$.
L_α: SM lepton doublets
H: Higgs doublet
M_α: Majorana masses, large
$y_{\alpha\beta}$: complex Yukawa couplings in basis where M is diagonal and $L_\alpha = (L_e, L_\mu, L_\tau)$.

Once H obtains vev $\tilde{H} = (v/\sqrt{2}, 0)$, SM neutrinos get Majorana masses. Mass matrix

$$m = \frac{v^2}{2} y^T M^{-1} y$$

or

$$m_{\alpha\beta} = \frac{v^2}{2} y_{\gamma\alpha} \frac{1}{M_\gamma} y_{\gamma\beta}$$

Lepton asymmetry from N-decays

Complex $y_{\alpha\beta}$ violate CP $\Longrightarrow$

$$\Gamma(N \to lh) \neq \Gamma(N \to \bar{l}h)$$

(do not distinguish h and $\bar{h}$, no conserved number in the Higgs sector).

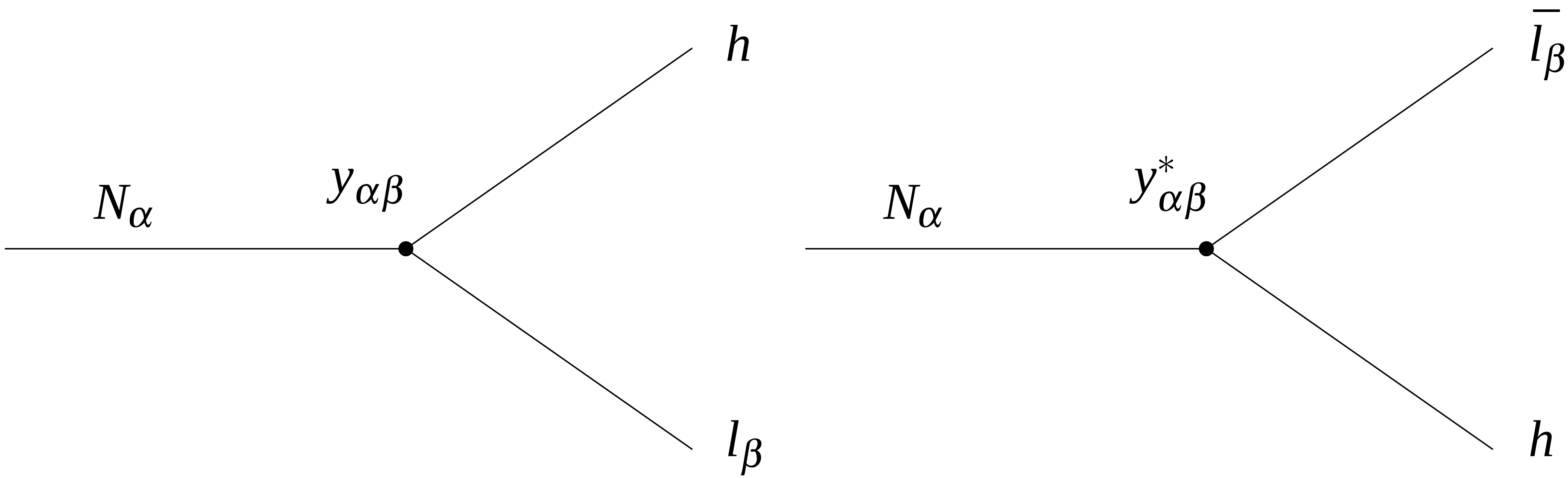
Use to generate lepton asymmetry: as the Universe expands, temperature falls below M_N, heavy N existing in plasma decay and produce lepton asymmetry.

Net asymmetry generated in decays of lightest N, call it N_1.

Equilibrium: decays compensated by inverse decays; decay and inverse decay rates equal to $\Gamma_1 \equiv \Gamma_{N_1}$.

Condition for strong deviation from thermal equilibrium:

$$\Gamma_1 \lesssim H(T = M_1)$$



$$\Gamma_1 = \frac{M_1}{8\pi} \sum_\alpha |y_{1\alpha}|^2 \lesssim H(T = M_1) = \frac{M_1^2}{M_{Pl}^*}$$

$$\Longrightarrow \quad \tilde{m}_1 = \sum_\alpha \frac{|y_{1\alpha}|^2}{2M_1} \cdot v^2 \lesssim \frac{4\pi}{M_{Pl}^*} \cdot v^2 \sim 10^{-3} \text{ eV}$$

$\tilde{m}_1$: contribution of lightest N to neutrino mass matrix $\Longrightarrow$ need small neutrino masses, not many orders of magnitude larger than 10^{-3} eV.

Case $\tilde{m}_1 \lesssim 10^{-3}$ eV: N_1's in thermal equilibrium at $T = M_1$;

$$\frac{n_1}{s} \sim \frac{1}{g_*}$$

N_1 do not decay until very late, inverse decays inoperative at $T \lesssim M_1 \Longrightarrow$

$$\frac{n_L}{s} \equiv \frac{n_l - n_{\bar{l}}}{s} \simeq \delta \cdot \frac{n_1}{s} \simeq \frac{\delta}{g_*}$$

where δ = lepton number generated in a single decay event.

- $\tilde{m}_1 \sim 10^{-3}$ eV needs hierarchy between Yukawas: $y_{1\alpha} \ll y_{2,3\alpha}$ $\Longrightarrow$ fine tuning

NB: $\tilde{m}_1$ is contribution of the lightest N to neutrino masses.
The largest, if no hierarchy between Yukawas.

What if $m_1 \gg 10^{-3}$ eV? (say $\tilde{m}_1 \sim m_{atm} = 0.05$ eV — normal hierarchy without degeneracy), lightest N gives largest contribution to m_ν

In this case $\Gamma_1 \gg H(T = M_1)$.

Mild suppression

$$\frac{n_L}{s} = D \cdot \frac{\delta}{g_*}$$

with

$$D \simeq \frac{1}{K \log K}\,, \qquad K = \frac{\Gamma_1}{H(T = M_1)} = \frac{\tilde{m}_1}{10^{-3}\ \text{eV}}$$

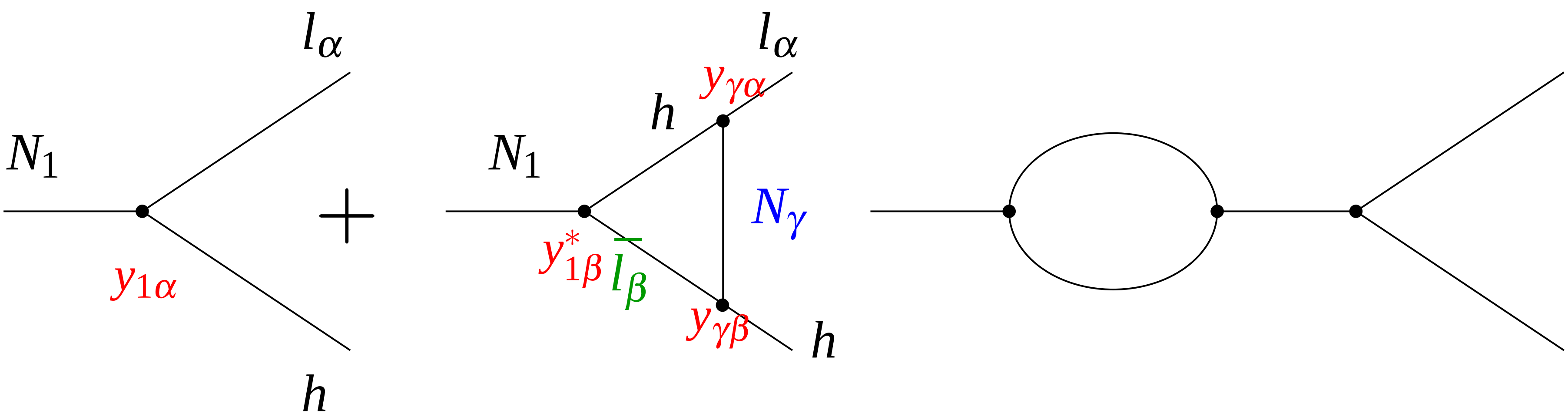
Backup slides for calculating this.

Not so bad even for $m_\nu \sim 0.1$ eV, but not $m_\nu \sim 100$ eV

Microscopic asymmetry

$$\delta = \frac{\Gamma(N_1 \to lh) - \Gamma(N_1 \to \bar{l}h)}{\Gamma(N_1 \to lh) + \Gamma(N_1 \to \bar{l}h)}.$$

Appears due to interferenece of tree and loop



$$\Gamma(N_1 \to lh) = \text{const} \cdot \sum_\alpha \left| y_{1\alpha} + \sum_{\beta,\gamma} D\left(\frac{M_1}{M_\gamma}\right) \cdot y^*_{1\beta} y_{\gamma\alpha} y_{\gamma\beta} \right|^2$$

$D(M_1/M_\gamma)$ = loop factor,

$$\text{Im}D = \frac{1}{24\pi}\frac{M_1}{M_\gamma}\,, \qquad M_\gamma \gg M_1$$

$$\Gamma(N \to \bar{l}h) = \Gamma(N \to lh; y \to y^*)$$

Asymmetry

$$\delta = \frac{\Gamma(N \to lh) - \Gamma(N \to \bar{l}h)}{\Gamma(N \to lh) + \Gamma(N \to \bar{l}h)} = \frac{M_1}{12\pi} \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta\gamma} \mathrm{Im}\left[y_{1\alpha} y_{1\beta} \left(y^*_{\gamma\alpha} \frac{1}{M_\gamma} y^*_{\gamma\beta} \right) \right]$$

For generic Yukawas

$$\delta \sim \frac{M_1 \tilde{m}_{2,3}}{6\pi v^2}$$

where $\tilde{m}_{2,3} =$ contributions of $N_{2,3}$ to neutrino mass matrix.

Comment:

- Relevant phases here are not related to phases of neutrino mass matrix: unitary rotations of $(\nu_e, \nu_\mu, \nu_\tau)$ do not affect δ, but eliminate PMNS mixing.

Collecting things together:

$$\frac{n_L}{s} \sim \frac{1}{g_*} \cdot \delta \cdot \frac{1}{K \log K} \sim 10^{-9} \frac{M_1}{10^{12}\ \text{GeV}} \cdot \frac{\tilde{m}_{2,3}}{10^{-2}\ \text{eV}} \cdot \frac{10^{-1}\ \text{eV}}{\tilde{m}_1}$$

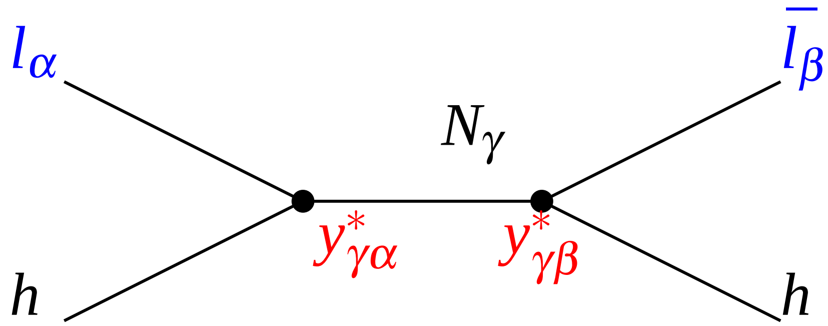
Without hierarchy in Yukawas: $M_1 \gtrsim 10^{12}$ GeV

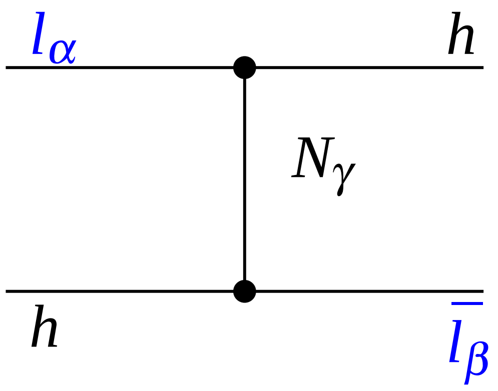
Allowing for hierarchy in Yukawas $y_{1\alpha} \ll y_{2,3\alpha}$ one can have $\tilde{m}_1 \sim 10^{-3}$ eV and get down to $M_1 \sim 10^9$ GeV. But not smaller — we will see in a moment.

Since the generation of asymmetry occurs at $T \sim M_1$ (in fact, slightly below M_1), maximum temperature in the Universe must exceed at least 10^9 GeV. SUSY: Tension with gravitino production

Washout in scattering

Non-resonant scattering with L-violation





Fast processes $\Longrightarrow$ washout, if

$$\langle \sigma v \rangle n_h \gtrsim H(T)$$

At $T \ll M_\alpha$ one has for light h, l

$$\langle \sigma v \rangle \sim \sum_{\alpha\beta\gamma} \left| \frac{y_{\gamma\alpha} y_{\gamma\beta}}{M_\gamma} \right|^2 \sim \frac{\mathrm{Tr}\, m m^\dagger}{v^4} = \frac{\sum m_\nu^2}{v^4}$$

With $n_h \sim T^3$, requirement of absence of washout

$$n_h \langle \sigma v \rangle \sim T^3 \frac{\sum m_\nu^2}{v^4} \ll H(T) = \frac{T^2}{M_{Pl}^*}$$

NB: washout switches off at low T.

Most dangerous at generation of lepton asymmetry, $T \sim M_1$.

$$\sum m_\nu^2 \ll \frac{v^4}{M_{Pl}^* M_1}$$

$M_1 \sim 10^{12}$ GeV $\Longrightarrow$

$$m_\nu = \frac{1}{3} \sum m_\nu^2 < 0.1 \text{ eV}$$

In fact, this bound, when combined with successfull leptogenesis, is valid for virtually all M_1.

Conclusions on leptogenesis

- It is intriguing that the mechanism can work for light neutrinos only. Furthermore, neutrino masses suggested by oscillation data are in right ballpark.
- Needs Majorana neutrino masses
- Highly degenerate neutrino masses $m_\nu \sim 0.3 - 1$ eV would be inconsistent with (simple and appealing versions of) leptogenesis. Watch out Troitsk and Katrin.
- Knowning mass matrix of “our” neutrinos is, generally speaking, insufficient for calculating baryon/lepton asymmetry. Even its sign.
- Reversing the argument, models that relate $y_{\alpha\beta}$ to “our” neutrino mass matrix can be ruled out — or have great success — once the mass matrix is completely known.

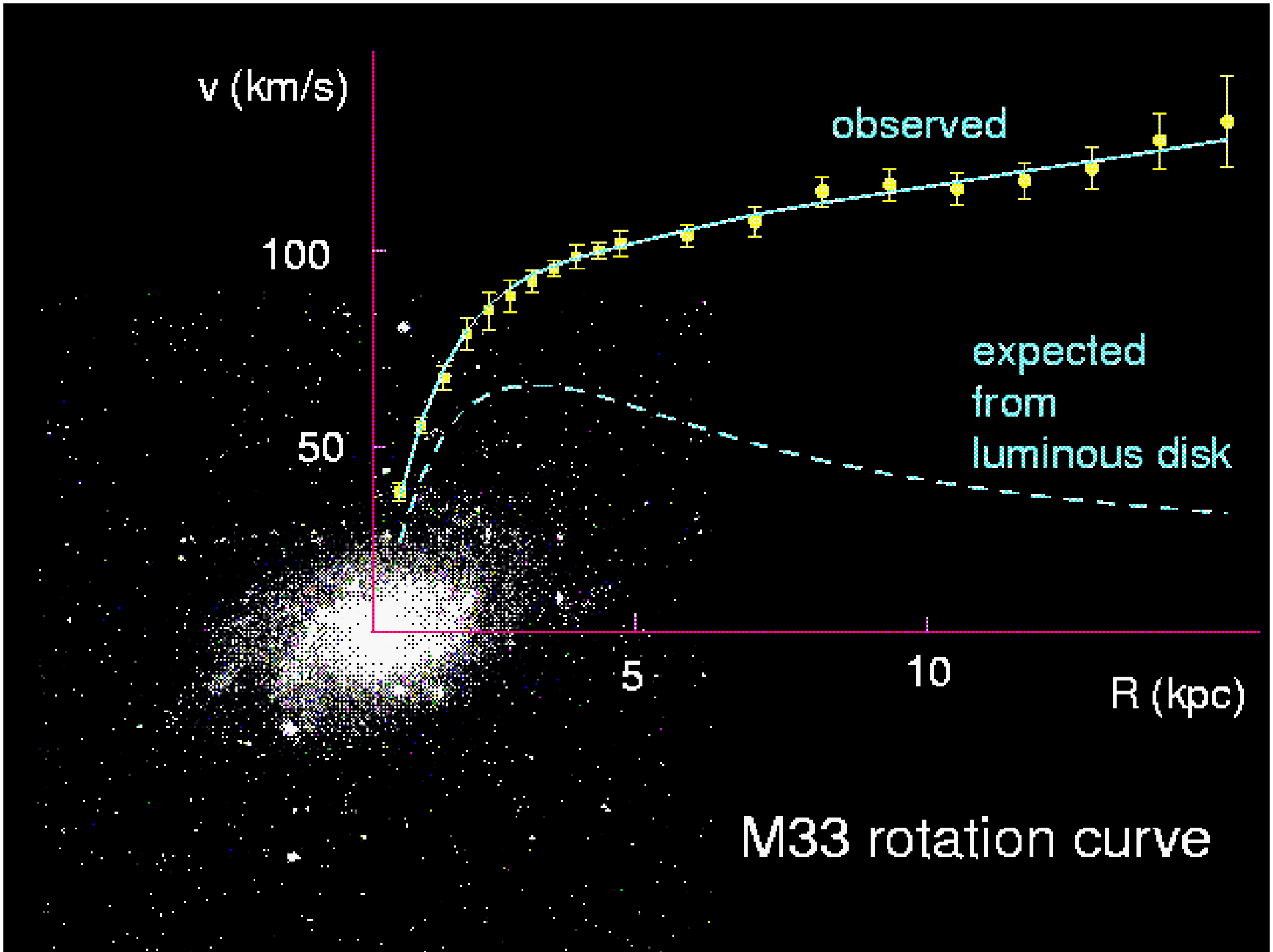
- Mechanism works at high temperatures only, at least $T \sim 10^9$ GeV. This is non-trivial in SUSY models because of the gravitino production. Knowing SUSY extension of SM (if any) will be instrumental.

Overall conclusions

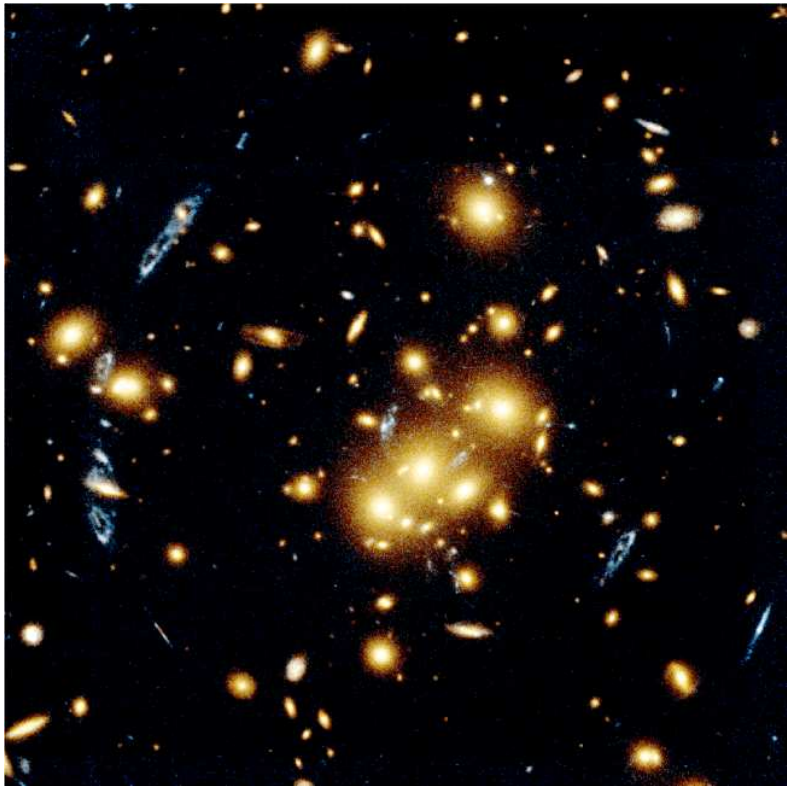
- Cosmology unambiguously tells us that the Sandard Model is incomplete
- There is well motivated expectation that this incompleteness should show up at LHC
- Optimistically, LHC will open up window to the earliest cosmological epoch
- If it does not, many popular scenarios will be ruled out $\Longrightarrow$ great impact anyway.

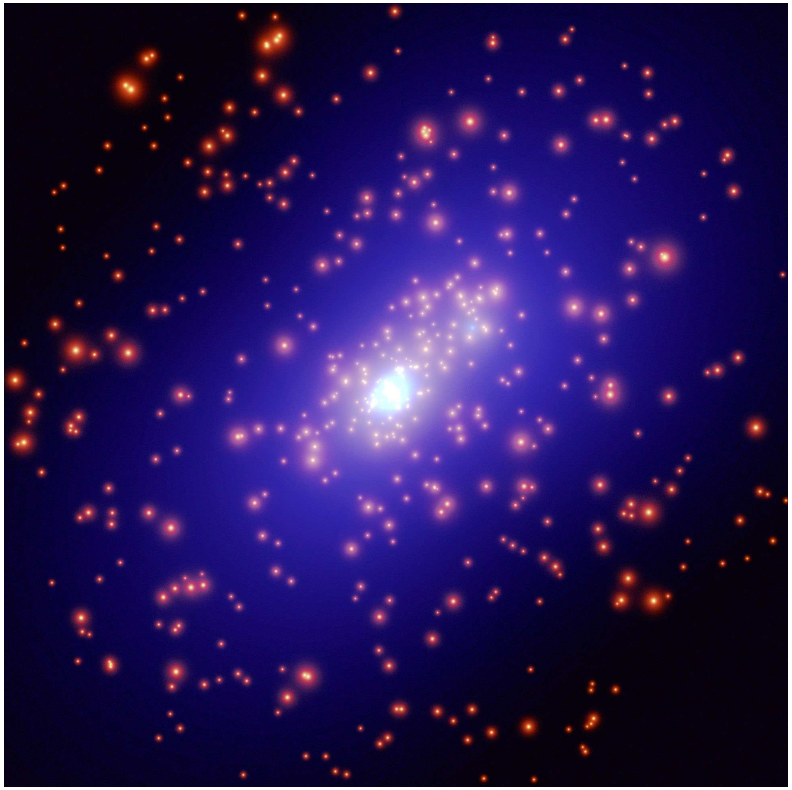
Backup slides, Lecture 1

1. Rotation curves



2. Gravitational lensing





3. Honest calculation of WIMP abundance

Boltzmann equation for $\Delta = n_X/s$ (accounts for expansion):

$$\frac{d\Delta}{dt} = -\langle\sigma v\rangle s(\Delta^2 - \Delta_{eq}^2)$$

First term in r.h.s.: annihilation; second term: creation

Creation terminates when

$$\frac{d\ln n_{X,eq}}{dt} \simeq \langle\sigma v\rangle n_{X,eq}$$

Recall

$$n_{X,eq} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

Then $|\frac{d\ln n_{eq}}{dt}| \simeq (m_X/T)H = m_X T/M_{Pl}^*$, so creation terminates at $T = T_*$ such that

$$n_{X,eq}(T_*) \simeq \frac{m_X T_*}{M_{Pl}^* \langle\sigma v\rangle}$$

4. This again gives (although exact result is slightly different from main lecture, e.g., by argument of log – not shown)

$$T_* \simeq \frac{M_X}{\ln(m_X M_{Pl} \langle \sigma v \rangle)}$$

Since t_*, WIMPs annihilate,

$$\frac{d\Delta}{dt} = -\langle \sigma v \rangle s \Delta^2$$

In terms of T this reads

$$\frac{d\Delta}{dT} = \frac{2\pi^2}{45} g_* M_{Pl}^* \langle \sigma v \rangle \Delta^2$$

This integrates to

$$\Delta^{-1}(T) - \Delta^{-1}(T_*) = \frac{2\pi^2}{45} g_* M_{Pl}^* \langle \sigma v \rangle (T_* - T)$$

5

At late times (low temperatures) we finally have

$$\Delta \quad = \quad \# \frac{1}{T_* \sigma_0 g_* M_{Pl}^*} = \# \frac{\ln(m_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

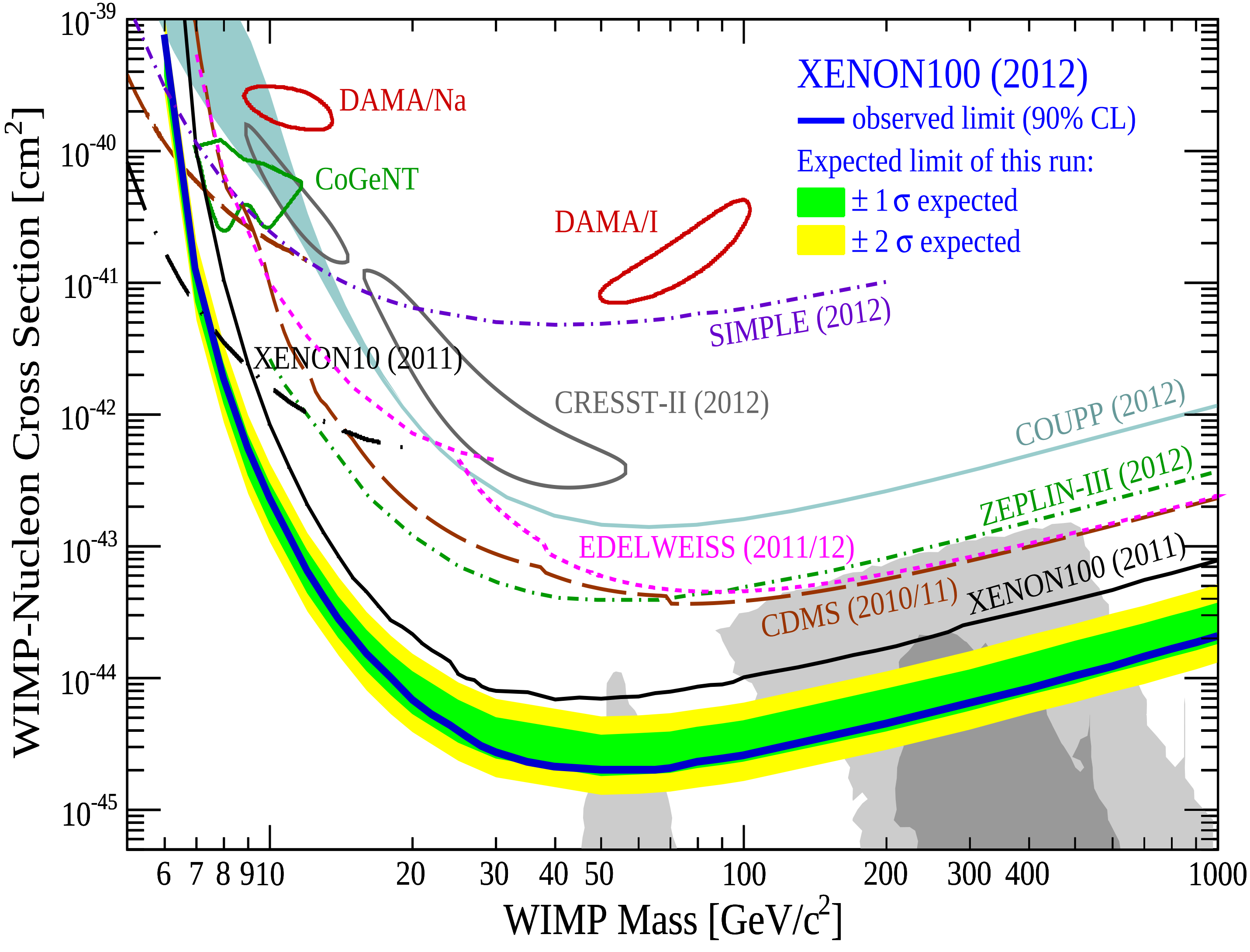
which is precisely the same result as in main lecture, except for precise argument of log (not shown).

Note that this value of Δ is much smaller than

$$\Delta(T_*) \simeq \frac{1}{s(T_*)} \frac{m_X T_*}{M_{Pl}^* \langle \sigma v \rangle} = \# \frac{m_X}{T_*} \frac{1}{T_* \sigma_0 g_* M_{Pl}^*}$$

So, it indeed makes sense to talk about two-stage process: termination of creation and then wash-out by annihilation.

6.Direct detection



Backup slides, Lecture 3

How to calculate D for $\Gamma_1/H \gg 1$

Boltzmann equation for $n_1 \equiv n_{N_1}$:

$$\frac{d\left(n_1 a^3\right)}{dt} = -\Gamma_1 \cdot \left(n_1 \cdot a^3 - n_1^{eq} \cdot a^3\right)$$

$\Gamma_1 n_1 a^3$: decay rate in comoving volume a^3
$\Gamma_1 n_1 a^3$: inverse decay rate in comoving volume; must be equal to decay rate in thermal equilibrium.

Likewise: Boltzmann equation for lepton number density $n_L = n_l - n_{\bar{l}}$

$$\frac{d\left(n_L \cdot a^3\right)}{dt} = \delta \cdot \Gamma_1 \cdot \left(n_1 \cdot a^3 - n_1^{eq} \cdot a^3\right) - c \cdot \Gamma_1 n_1^{eq} a^3 \cdot \frac{n_L}{n_l + n_{\bar{l}}}$$

Last term: wash out of lepton asymmetry in inverse decays: more leptons than antileptons in plasma $\Longrightarrow$ more leptons than antileptons disappear in inverse decays; $c \sim 1$, $n_l, n_{\bar{l}} \sim T^3$.

$$\frac{d\left(n_{\rm L}\cdot a^3\right)}{dt} = -\delta\cdot\frac{d\left(n_1\cdot a^3\right)}{dt} - c\cdot\Gamma_1 n_1^{eq} a^3\cdot\frac{n_L}{n_l+n_{\bar{l}}}$$

Abundance of N_1 nearly in equilibrium $\Longrightarrow$

$$\frac{d\left(n_{\rm L}\cdot a^3\right)}{dt} = -\delta\cdot\frac{d\left(n_1^{eq}\cdot a^3\right)}{dt} - c\cdot\Gamma_1 n_1^{eq} a^3\cdot\frac{n_L}{n_l+n_{\bar{l}}}$$

Thermal inequilibrium peculiar: n_1 is nearly the same as in equilibrium, but still it depends on time.

Recall $\dot{T}/T = H \Longrightarrow$

$$\frac{d\left(n_{\rm L}\cdot a^3\right)}{d\log T} = -\delta\cdot\frac{d\left(n_1^{eq}\cdot a^3\right)}{d\log T} - c\cdot\frac{\Gamma_1}{H} n_1^{eq}\cdot\frac{n_L a^3}{n_l+n_{\bar{l}}}$$

- High T: $\Gamma n_1^{eq}/HT^3$ large, strong washout.
- Low T: $n_1^{eq} \propto \mathrm{e}^{-\frac{M_1}{T}}$ small, generation switched off

Best case (modulo logs): $n_1^{eq} \sim HT^3/\Gamma_1$

$$n_L \sim \delta \cdot n_1^{eq} \sim \delta \cdot \frac{HT^3}{\Gamma_1}$$

$\Longrightarrow$

$$\frac{n_L}{s} = D\frac{\delta}{g_*}\,, \quad D \simeq \frac{H(T = M_1)}{\Gamma_1} = \frac{1}{K}$$