Particle Cosmology

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences, Moscow
and
Moscow State University

Lecture 3

Outline of Lecture 3

- Baryon asymmetry of the Universe.
  - Electroweak baryogenesis.
    - Electroweak baryon number violation
    - Electroweak transition
    - What can make electroweak mechanism work?
  - Leptogenesis
    - See-saw in nutshell
    - Asymmetry from decays
    - Washout in scattering
    - Conclusions on leptogenesis
- Overall conclusions

Baryon asymmetry of the Universe

- There is matter and no antimatter in the present Universe.
- Baryon-to-photon ratio, almost constant in time:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 6 \cdot 10^{-10}$$

Baryon-to-entropy, constant in time: $n_B/s = 0.9 \cdot 10^{-10}$

What's the problem?

Early Universe ($T > 10^{12}$ K $= 100$ MeV):
creation and annihilation of quark-antiquark pairs $\Rightarrow$

$$n_q, n_{\bar{q}} \approx n_\gamma$$

Hence

$$\frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \sim 10^{-9}$$

How was this excess generated in the course of the cosmological evolution?

Sakharov conditions

To generate baryon asymmetry, three necessary conditions should be met at the same cosmological epoch:

- B-violation
- C- and CP-violation
- Thermal inequilibrium

NB. Reservation: L-violation with B-conservation at $T \gg 100$ GeV would do as well $\Longrightarrow$ Leptogenesis.

Can baryon asymmetry be due to electroweak physics?

Baryon number is violated in electroweak interactions.

Non-perturbative effect

Hint: triangle anomaly in baryonic current B^μ:

$$\partial_\mu B^\mu = \left(\frac{1}{3}\right)_{B_q} \cdot 3_{colors} \cdot 3_{generations} \cdot \frac{g_W^2}{32\pi^2} \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

$F^a_{\mu\nu}$: $SU(2)_W$ field strength; g_W: $SU(2)_W$ coupling

Likewise, each leptonic current ($n = e, \mu, \tau$)

$$\partial_\mu L_n^\mu = \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho}$$

Large field fluctuations, $F^a_{\mu\nu} \propto g_W^{-1}$ may have

$$Q \equiv \int d^3x dt \; \frac{g_W^2}{32\pi^2} \cdot \varepsilon^{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} \neq 0$$

Then

$$B_{fin} - B_{in} = \int d^3x dt \; \partial_\mu B^\mu = 3Q$$

Likewise

$$L_{n,\; fin} - L_{n,\; in} = Q$$

B is violated, $B - L$ is not.

How can baryon number be not conserved without explicit B-violating terms in Lagrangian?

Consider massless fermions in background gauge field $\vec{A}(\mathbf{x},t)$ (gauge $A_0 = 0$). Let $\vec{A}(\mathbf{x},t)$ start from vacuum value and end up in vacuum.

NB: This can be a fluctuation

Dirac equation

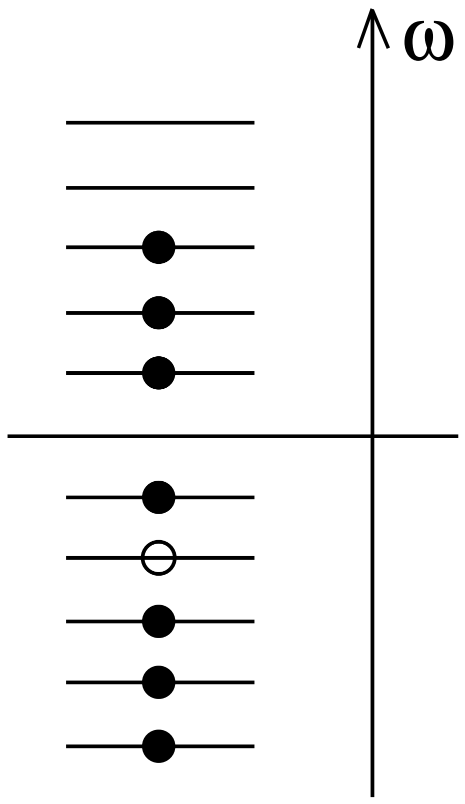
$$i\frac{\partial}{\partial t}\psi = i\gamma^0\vec{\gamma}(\vec{\partial} - ig\vec{A})\psi = H_{Dirac}(t)\psi$$

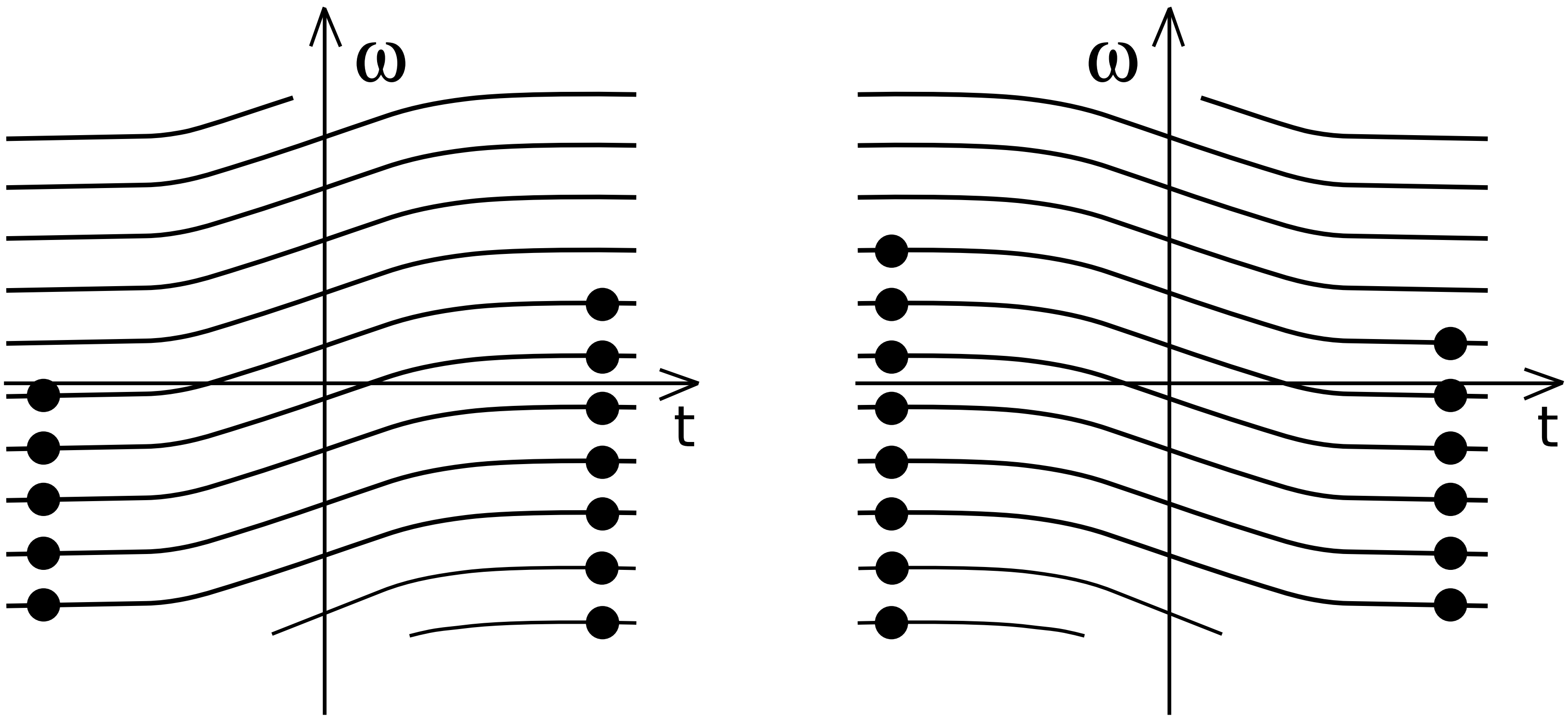
Suppose for the moment that $\vec{A}$ slowly varies in time. Then fermions sit on levels of instantaneous Hamiltonian,

$$H_{Dirac}(t)\psi_n = \omega_n(t)\psi_n$$

How do eigenvalues behave in time?

Fermion energy levels at $\vec{A} = 0$





Left fermions Right fermions

Motion of energy levels in special (topological) gauge field background $\vec{A}(t)$

QCD case, $B = N_L + N_R$ is conserved, $Q^5 = N_L - N_R$ is not

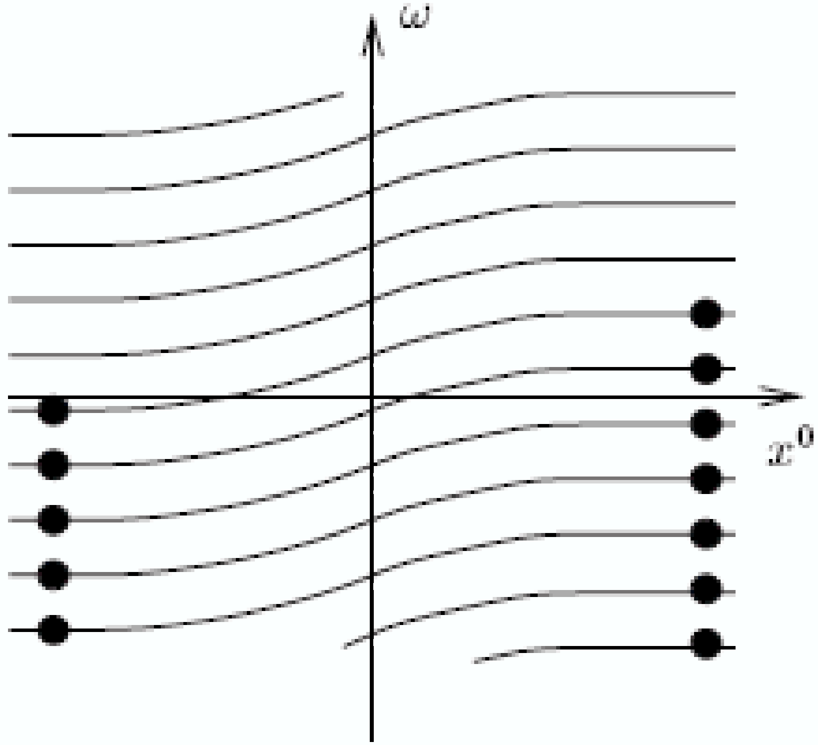
NB: Non-Abelian gauge fields only (in 4 dimensions)

QCD: Violation of Q^5 is a fact.

In chiral limit $m_u, m_d, m_s \to 0$,
global symmetry is $SU(3)_L \times SU(3)_R \times U(1)_B$,
not symmetry of classical Lagrangian
$SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A$

If only left-handed fermions interact with gauge field,

then number of fermions is not conserved



The case for $SU(2)_W$

Fermion number of every doublet changes in the same way

Need large field fluctuations. At zero temprature their rate is suppressed by

$$\mathrm{e}^{-\frac{16\pi^2}{g_W^2}} \sim 10^{-165}$$

High temperatures: large thermal fluctuations (“sphalerons”).

B-violation rapid as compared to cosmological expansion at

$$\langle \phi \rangle_T < T$$

$\langle \phi \rangle_T$: Higgs expectation value at temperature T.

Possibility to generate baryon asymmetry at electroweak epoch, $T_{EW} \sim 100$ GeV ?

Problem: Universe expands slowly. Expansion time

$$H^{-1} = \frac{M_{Pl}^*}{T_{EW}^2} \sim 10^{14}\ \text{GeV}^{-1} \sim 10^{-10}\ \text{s}$$

Too large to have deviations from thermal equilibrium?

The only chance: 1st order phase transition, highly inequilibrium process

Electroweak symmetry is restored, $\langle \phi \rangle_T = 0$ at high temperatures

Just like superconducting state becomes normal at "high" T

Transition may in principle be 1st order

Fig

1st order phase transition occurs from supercooled state via spontaneous creation of bubbles of new (broken) phase in old (unbroken) phase.

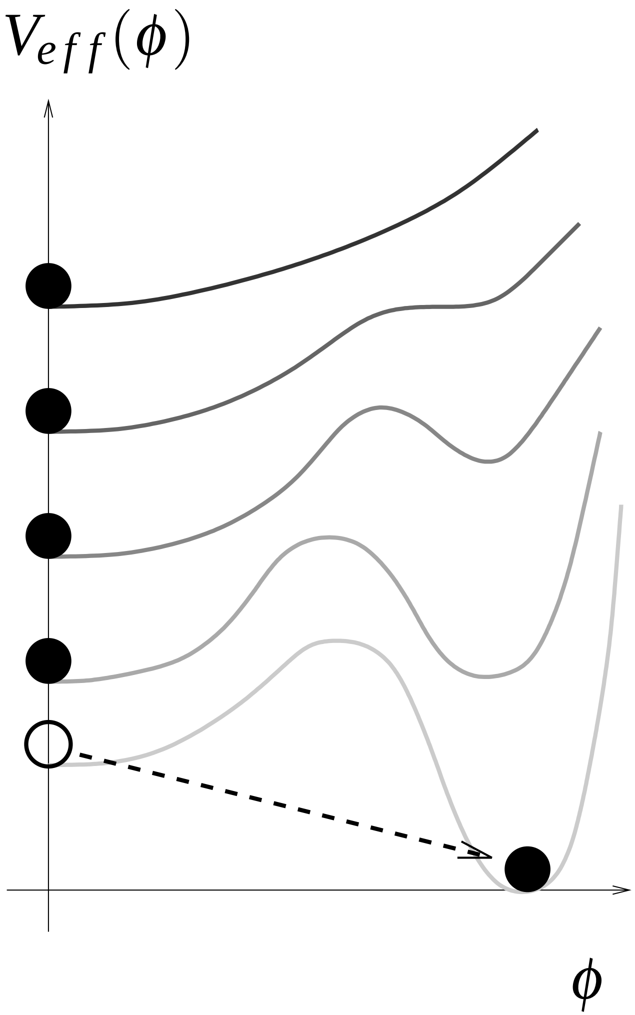
Bubbles then expand at $v \sim 0.1c$

Fig

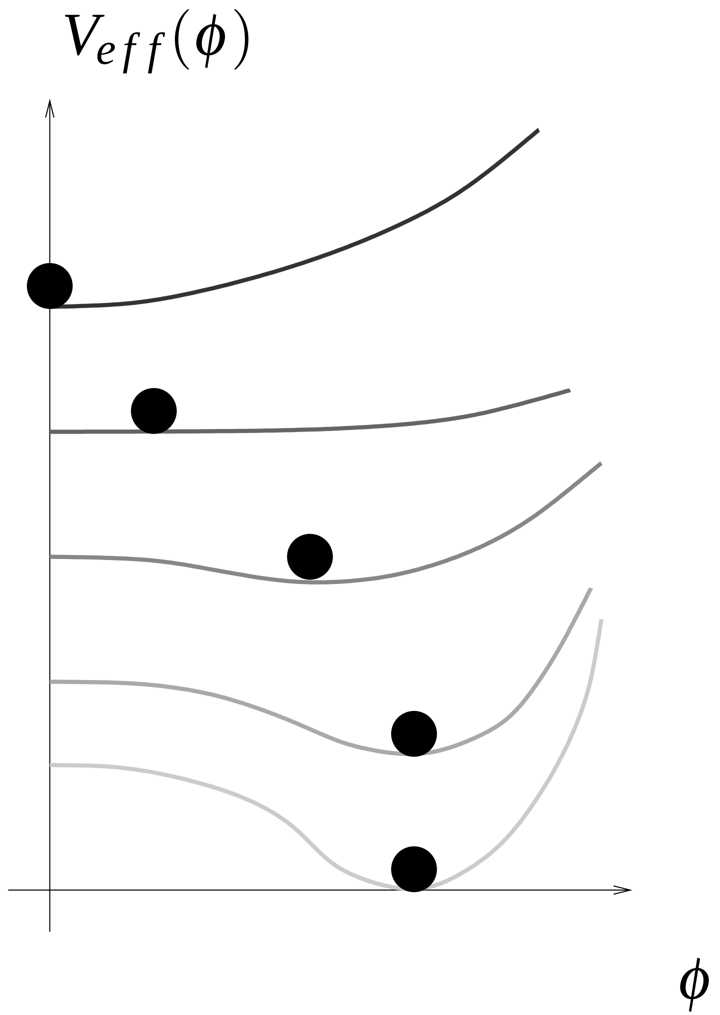
Beginning of transition: about one bubble per horizon

Bubbles born microscopic, $r \sim 10^{-16}$ cm, grow to macroscopic size, $r \sim 0.01 H^{-1} \sim 0.1$ mm, before their walls collide

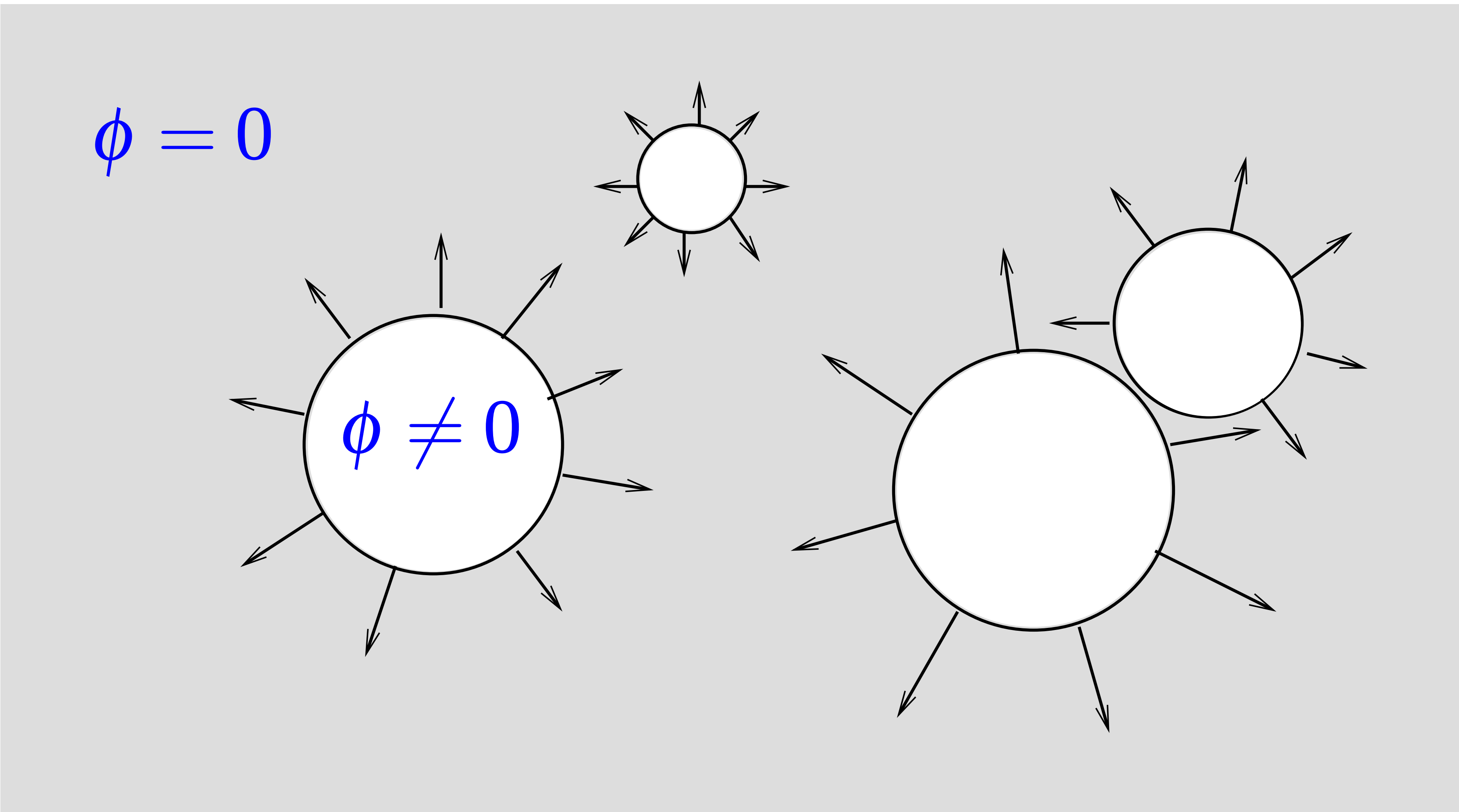
Boiling Universe, strongly out of equilibrium



1st order



2nd order



$\phi = 0$
$\phi \neq 0$

Baryon asymmetry may be generated in the course of phase transition, provided there is enough C- and CP-violation.

Necessary condition:

Baryon asymmetry generated during transition should not be washed out aferwards
$\Longrightarrow$ B-violating processes must be switched off in broken phase
$\Longrightarrow$ Just after transition must have

$$\langle \phi \rangle_T > T$$

Does this really happen?

Not in SM

Temperature-dependent effective potential, one loop

$$V_{eff} = (-m^2 + \alpha T^2)|\phi|^2 - \frac{\beta}{3} T |\phi|^3 + \frac{\lambda}{4}|\phi|^4$$

$\alpha = O(g^2)$, $\beta = O(g^3)$. Cubic term weird,

$$-\frac{\beta}{3} T \left(\phi^\dagger \phi\right)^{3/2}$$

But crucial for 1st order phase transition. Obtains contributions from bosons only

$$f_B = \frac{1}{\mathrm{e}^{E/T} - 1} \simeq \frac{T}{E} \equiv \frac{T}{\sqrt{\mathbf{p}^2 + g^2|\phi|^2}} \simeq \frac{T}{g|\phi|} \quad \text{at} \quad |\mathbf{p}| \ll g|\phi| \ll T$$

Bose enhancement $\Longleftrightarrow$ no analyticity in $g^2|\phi|^2$

At phase transition $(-m^2 + \alpha T^2) = 0$,

$$V_{eff} = -\frac{\beta}{3} T \phi^3 + \frac{\lambda}{4} \phi^4$$

Hence

$$\langle \phi \rangle_T = \frac{\beta}{\lambda} T = \# \frac{g_W^3}{\lambda} T$$

Given the Higgs mass

$$m_H = \sqrt{2\lambda} v = 125 \text{ GeV}$$

one finds $\langle \phi \rangle_T < T$, asymmetry would be washed out even if generated

Furtermore, in SM

- No phase transition at all; smooth crossover
- Way too small CP-violation

What can make EW mechanism work?

- Extra bosons
  - Should interact strongly with Higgs(es)
  - Should be present in plasma at $T \sim 100$ GeV

    $\Longrightarrow$ not much heavier than 300 GeV

  E.g. light stop

- Plus extra source of CP-violation.
  Better in Higgs sector $\Longrightarrow$ Several Higgs fields

More generally, EW baryogenesis requires
complex dynamics in EW symmetry breaking sector
at $E \sim (\text{a few}) \cdot 100$ GeV

LHC's FINAL WORD

Is EW the only appealing scenario?

By no means!

— Strong competitor: leptogenesis

— Many other proposals

— Something theorists never thought about

Leptogenesis:

Baryon asymmetry and neutrino masses

B is violated in electroweak interactions,

$B-L$ is conserved

But we know that lepton numbers are violated anyway: neutrino oscillations.

Neutrinos have tiny masses. We know two differences of mass squared:

$$m_2^2 - m_1^2 = (0.01\ \text{eV})^2\ , \qquad |m_3^2 - m_1^2| = (0.05\ \text{eV})^2$$

We also know that all masses are small,

$$m_\nu < 2\ \text{eV} \quad \text{(Experiment)}$$

$$m_\nu < 0.2\ \text{eV} \quad \text{(Cosmology)}$$

Leptogenesis: use the physics responsible for neutrino masses to generate lepton asymmetry in the Universe. Electroweak interactions automatically reprocess part of lepton asymmetry into baryon asymmetry.

Standard Model in thermal equilibrium at $T \gg 100$ GeV with $B-L \neq 1$:

$$B = C \cdot (B-L) , \qquad L = -(1-C) \cdot (B-L)$$

$$C = \frac{8N_{gen} + 4N_{Higgs}}{22N_{gen} + 13N_{Higgs}} = \frac{28}{79} , \quad T \gg 100 \text{ GeV}$$

See-saw in nutshell

Begin with one lepton doublet $L = (\nu, l)$. To generate neutrino mass, add a new left fermion N, singlet under $SU(2)_W \times U(1)_Y$.

Allowed Majorana mass term

$$\frac{M}{2}\bar{N}^c N \equiv \frac{M}{2}\varepsilon_{ij}N_i N_j + \text{h.c.}$$

$i, j = 1, 2$: Lorentzian spinor index of left fermion.

Also allowed Yukawa coupling to Higgs field, so Lagrangian includes

$$\frac{M}{2}\bar{N}^c N + y\bar{N}^c \tilde{H}^\dagger L + \text{h.c}$$

In vacuum $\tilde{H}^\dagger = (v/\sqrt{2}, 0)$, so one gets mass terms

$$\frac{M}{2}\bar{N}^c N + \frac{yv}{\sqrt{2}}\bar{N}^c \nu + \text{h.c}$$

At energies and momenta small compared to M, equation of motion for N is

$$MN + \frac{yv}{\sqrt{2}}\nu = 0 \quad \Longrightarrow \quad N = -\frac{yv}{M\sqrt{2}}\nu$$

Plug N back into Lagrangian, get Majorana mass of ν:

$$L_{m_\nu} = -\frac{y^2v^2}{2M}\bar{\nu}^c\nu + \text{h.c.}$$

Small Majorana neutrino mass for large M and not necessarily small Yukawa coupling y:

$$m_\nu = \frac{y^2v^2}{2M}$$

Three generations:

$$\mathscr{L} = \frac{M_\alpha}{2} \bar{N}_\alpha^c N_\alpha + \left(y_{\alpha\beta} \bar{N}_\alpha^c \tilde{H}^\dagger L_\beta + h.c. \right)$$

N_α: new two-component (left) fermions, $\alpha = 1, 2, 3$.
L_α: SM lepton doublets
H: Higgs doublet
M_α: Majorana masses, large
$y_{\alpha\beta}$: complex Yukawa couplings in basis where M is diagonal and $L_\alpha = (L_e, L_\mu, L_\tau)$.

Once H obtains vev $\tilde{H} = (v/\sqrt{2}, 0)$, SM neutrinos get Majorana masses. Mass matrix

$$m = \frac{v^2}{2} y^T M^{-1} y$$

or

$$m_{\alpha\beta} = \frac{v^2}{2} y_{\gamma\alpha} \frac{1}{M_\gamma} y_{\gamma\beta}$$

Lepton asymmetry from N-decays

Complex $y_{\alpha\beta}$ violate CP $\Longrightarrow$

$$\Gamma(N \to lh) \neq \Gamma(N \to \bar{l}h)$$

(do not distinguish h and $\bar{h}$, no conserved number in the Higgs sector).

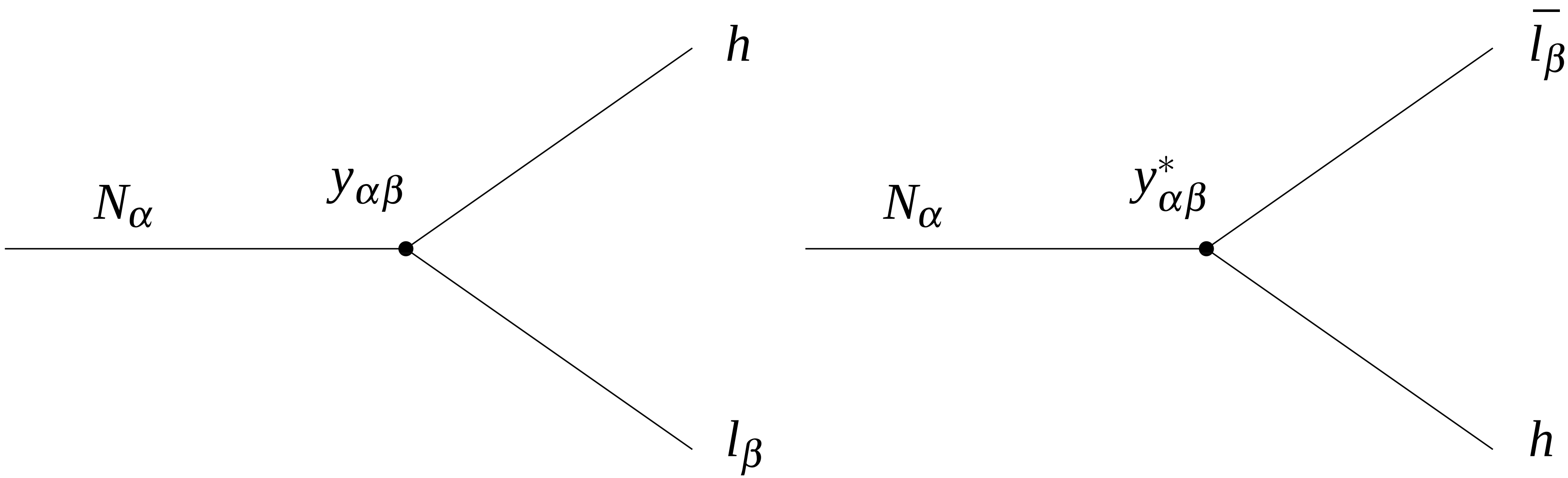
Use to generate lepton asymmetry: as the Universe expands, temperature falls below M_N, heavy N existing in plasma decay and produce lepton asymmetry.

Net asymmetry generated in decays of lightest N, call it N_1.

Equilibrium: decays compensated by inverse decays; decay and inverse decay rates equal to $\Gamma_1 \equiv \Gamma_{N_1}$.

Condition for strong deviation from thermal equilibrium:

$$\Gamma_1 \lesssim H(T = M_1)$$



$$\Gamma_1 = \frac{M_1}{8\pi} \sum_\alpha |y_{1\alpha}|^2 \lesssim H(T = M_1) = \frac{M_1^2}{M^*_{Pl}}$$

$$\Longrightarrow \quad \tilde{m}_1 = \sum_\alpha \frac{|y_{1\alpha}|^2}{2M_1} \cdot v^2 \lesssim \frac{4\pi}{M^*_{Pl}} \cdot v^2 \sim 10^{-3} \text{ eV}$$

$\tilde{m}_1$: contribution of lightest N to neutrino mass matrix $\Longrightarrow$ need small neutrino masses, not many orders of magnitude larger than 10^{-3} eV.

Case $\tilde{m}_1 \lesssim 10^{-3}$ eV: N_1's in thermal equilibrium at $T = M_1$;

$$\frac{n_1}{s} \sim \frac{1}{g_*}$$

N_1 do not decay until very late, inverse decays inoperative at $T \lesssim M_1 \Longrightarrow$

$$\frac{n_L}{s} \equiv \frac{n_l - n_{\bar{l}}}{s} \simeq \delta \cdot \frac{n_1}{s} \simeq \frac{\delta}{g_*}$$

where δ = lepton number generated in a single decay event.

- $\tilde{m}_1 \sim 10^{-3}$ eV needs hierarchy between Yukawas: $y_{1\alpha} \ll y_{2,3\alpha}$ $\Longrightarrow$ fine tuning

NB: $\tilde{m}_1$ is contribution of the lightest N to neutrino masses.
The largest, if no hierarchy between Yukawas.

What if $m_1 \gg 10^{-3}$ eV? (say $\tilde{m}_1 \sim m_{atm} = 0.05$ eV — normal hierarchy without degeneracy), lightest N gives largest contribution to m_ν

In this case $\Gamma_1 \gg H(T = M_1)$.

Mild suppression

$$\frac{n_L}{s} = D \cdot \frac{\delta}{g_*}$$

with

$$D \simeq \frac{1}{K \log K}\,, \qquad K = \frac{\Gamma_1}{H(T = M_1)} = \frac{\tilde{m}_1}{10^{-3}\ \text{eV}}$$

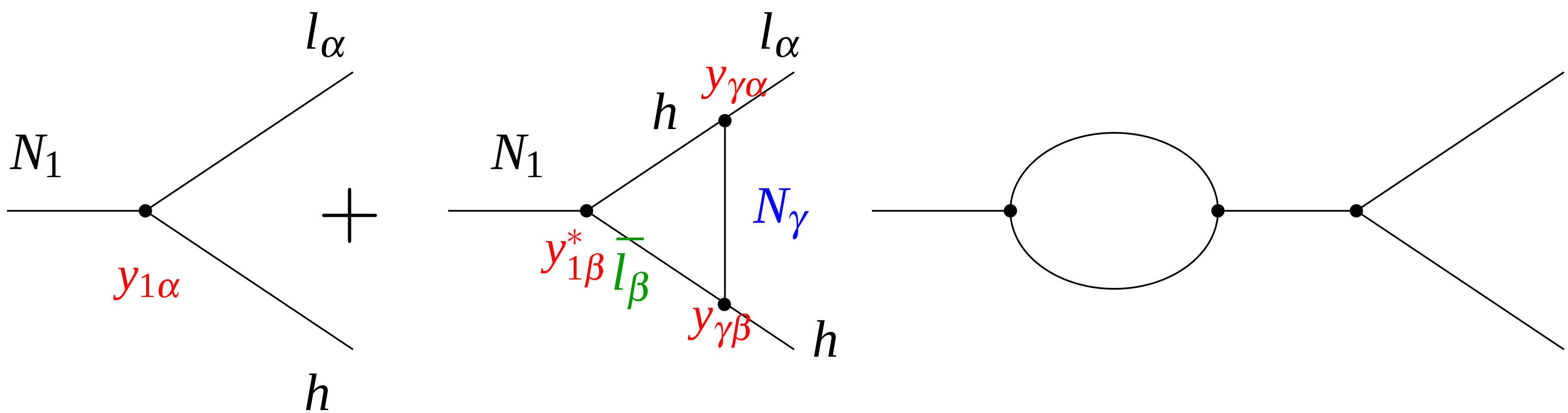
Backup slides for calculating this.

Not so bad even for $m_\nu \sim 0.1$ eV, but not $m_\nu \sim 100$ eV

Microscopic asymmetry

$$\delta = \frac{\Gamma(N_1 \to lh) - \Gamma(N_1 \to \bar{l}\bar{h})}{\Gamma(N_1 \to lh) + \Gamma(N_1 \to \bar{l}\bar{h})}.$$

Appears due to interferenece of tree and loop



$$\Gamma(N_1 \to lh) = \text{const} \cdot \sum_\alpha \left| y_{1\alpha} + \sum_{\beta,\gamma} D\left(\frac{M_1}{M_\gamma}\right) \cdot y^*_{1\beta} y_{\gamma\alpha} y_{\gamma\beta} \right|^2$$

$D(M_1/M_\gamma)$ = loop factor,

$$\text{Im}D = \frac{1}{24\pi}\frac{M_1}{M_\gamma}, \qquad M_\gamma \gg M_1$$

$$\Gamma(N \to \bar{l}h) = \Gamma(N \to lh; y \to y^*)$$

Asymmetry

$$\delta = \frac{\Gamma(N \to lh) - \Gamma(N \to \bar{l}h)}{\Gamma(N \to lh) + \Gamma(N \to \bar{l}h)} = \frac{M_1}{12\pi} \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta\gamma} \mathrm{Im}\left[y_{1\alpha} y_{1\beta} \left(y^*_{\gamma\alpha} \frac{1}{M_\gamma} y^*_{\gamma\beta} \right) \right]$$

For generic Yukawas

$$\delta \sim \frac{M_1 \tilde{m}_{2,3}}{6\pi v^2}$$

where $\tilde{m}_{2,3} =$ contributions of $N_{2,3}$ to neutrino mass matrix.

Comment:

- Relevant phases here are not related to phases of neutrino mass matrix: unitary rotations of $(\nu_e, \nu_\mu, \nu_\tau)$ do not affect δ, but eliminate PMNS mixing.

Collecting things together:

$$\frac{n_L}{s} \sim \frac{1}{g_*} \cdot \delta \cdot \frac{1}{K \log K} \sim 10^{-9} \frac{M_1}{10^{12}\ \text{GeV}} \cdot \frac{\tilde{m}_{2,3}}{10^{-2}\ \text{eV}} \cdot \frac{10^{-1}\ \text{eV}}{\tilde{m}_1}$$

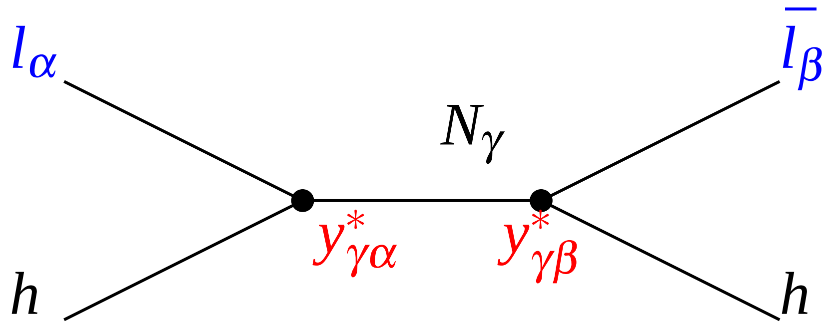
Without hierarchy in Yukawas: $M_1 \gtrsim 10^{12}$ GeV

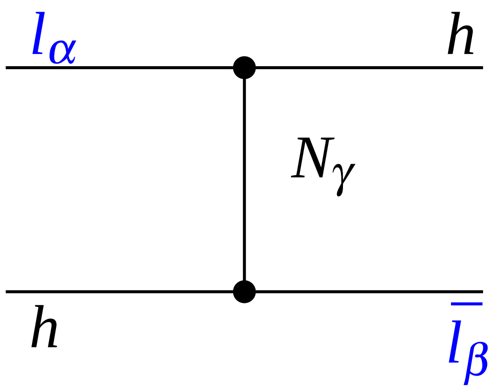
Allowing for hierarchy in Yukawas $y_{1\alpha} \ll y_{2,3\alpha}$ one can have $\tilde{m}_1 \sim 10^{-3}$ eV and get down to $M_1 \sim 10^9$ GeV. But not smaller — we will see in a moment.

Since the generation of asymmetry occurs at $T \sim M_1$ (in fact, slightly below M_1), maximum temperature in the Universe must exceed at least 10^9 GeV. SUSY: Tension with gravitino production

Washout in scattering

Non-resonant scattering with L-violation





Fast processes $\Longrightarrow$ washout, if

$$\langle \sigma v \rangle n_h \gtrsim H(T)$$

At $T \ll M_\alpha$ one has for light h, l

$$\langle \sigma v \rangle \sim \sum_{\alpha\beta\gamma} \left| \frac{y_{\gamma\alpha} y_{\gamma\beta}}{M_\gamma} \right|^2 \sim \frac{\mathrm{Tr}\, m m^\dagger}{v^4} = \frac{\sum m_\nu^2}{v^4}$$

With $n_h \sim T^3$, requirement of absence of washout

$$n_h \langle \sigma v \rangle \sim T^3 \frac{\sum m_\nu^2}{v^4} \ll H(T) = \frac{T^2}{M_{Pl}^*}$$

NB: washout switches off at low T.

Most dangerous at generation of lepton asymmetry, $T \sim M_1$.

$$\sum m_\nu^2 \ll \frac{v^4}{M_{Pl}^* M_1}$$

$M_1 \sim 10^{12}$ GeV $\Longrightarrow$

$$m_\nu = \frac{1}{3} \sum m_\nu^2 < 0.1 \text{ eV}$$

In fact, this bound, when combined with successfull leptogenesis, is valid for virtually all M_1.

Conclusions on leptogenesis

- It is intriguing that the mechanism can work for light neutrinos only. Furthermore, neutrino masses suggested by oscillation data are in right ballpark.

- Needs Majorana neutrino masses

- Highly degenerate neutrino masses $m_\nu \sim 0.3 - 1$ eV would be inconsistent with (simple and appealing versions of) leptogenesis. Watch out Troitsk and Katrin.

- Knowning mass matrix of “our” neutrinos is, generally speaking, insufficient for calculating baryon/lepton asymmetry. Even its sign.

- Reversing the argument, models that relate $y_{\alpha\beta}$ to “our” neutrino mass matrix can be ruled out — or have great success — once the mass matrix is completely known.

- Mechanism works at high temperatures only, at least $T \sim 10^9$ GeV. This is non-trivial in SUSY models because of the gravitino production. Knowing SUSY extension of SM (if any) will be instrumental.

Overall conclusions

- Cosmology unambiguously tells us that the Sandard Model is incomplete
- There is well motivated expectation that this incompleteness should show up at LHC
- Optimistically, LHC will open up window to the earliest cosmological epoch
- If it does not, many popular scenarios will be ruled out $\Longrightarrow$ great impact anyway.

Backup slides

How to calculate D for $\Gamma_1/H \gg 1$

Boltzmann equation for $n_1 \equiv n_{N_1}$:

$$\frac{d\left(n_1 a^3\right)}{dt} = -\Gamma_1 \cdot \left(n_1 \cdot a^3 - n_1^{eq} \cdot a^3\right)$$

$\Gamma_1 n_1 a^3$: decay rate in comoving volume a^3
$\Gamma_1 n_1 a^3$: inverse decay rate in comoving volume; must be equal to decay rate in thermal equilibrium.

Likewise: Boltzmann equation for lepton number density $n_L = n_l - n_{\bar{l}}$

$$\frac{d\left(n_\mathrm{L} \cdot a^3\right)}{dt} = \delta \cdot \Gamma_1 \cdot \left(n_1 \cdot a^3 - n_1^{eq} \cdot a^3\right) - c \cdot \Gamma_1 n_1^{eq} a^3 \cdot \frac{n_L}{n_l + n_{\bar{l}}}$$

Last term: wash out of lepton asymmetry in inverse decays: more leptons than antileptons in plasma $\Longrightarrow$ more leptons than antileptons disappear in inverse decays; $c \sim 1$, $n_l, n_{\bar{l}} \sim T^3$.

$$\frac{d\left(n_{\mathrm{L}}\cdot a^{3}\right)}{dt}=-\delta\cdot\frac{d\left(n_{1}\cdot a^{3}\right)}{dt}-c\cdot\Gamma_{1}n_{1}^{eq}a^{3}\cdot\frac{n_{L}}{n_{l}+n_{\bar{l}}}$$

Abundance of N_1 nearly in equilibrium $\Longrightarrow$

$$\frac{d\left(n_{\mathrm{L}}\cdot a^{3}\right)}{dt}=-\delta\cdot\frac{d\left(n_{1}^{eq}\cdot a^{3}\right)}{dt}-c\cdot\Gamma_{1}n_{1}^{eq}a^{3}\cdot\frac{n_{L}}{n_{l}+n_{\bar{l}}}$$

Thermal inequilibrium peculiar: n_1 is nearly the same as in equilibrium, but still it depends on time.

Recall $\dot{T}/T=H$ $\Longrightarrow$

$$\frac{d\left(n_{\mathrm{L}}\cdot a^{3}\right)}{d\log T}=-\delta\cdot\frac{d\left(n_{1}^{eq}\cdot a^{3}\right)}{d\log T}-c\cdot\frac{\Gamma_{1}}{H}n_{1}^{eq}\cdot\frac{n_{L}a^{3}}{n_{l}+n_{\bar{l}}}$$

- High T: $\Gamma n_1^{eq}/HT^3$ large, strong washout.
- Low T: $n_1^{eq}\propto \mathrm{e}^{-\frac{M_1}{T}}$ small, generation switched off

Best case (modulo logs): $n_1^{eq} \sim HT^3/\Gamma_1$

$$n_L \sim \delta \cdot n_1^{eq} \sim \delta \cdot \frac{HT^3}{\Gamma_1}$$

$\Longrightarrow$

$$\frac{n_L}{s} = D \frac{\delta}{g_*}\,, \quad D \simeq \frac{H(T = M_1)}{\Gamma_1} = \frac{1}{K}$$