Particle Cosmology

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences, Moscow
and
Moscow State University

Lecture 2

Outline of Lecture 2

- Axions
  - Theory
  - Cosmology
  - Search
- Warm dark matter
  - Gravitino
- Dark matter summary

Axions

Motivation: solution of strong CP problem

What's the problem?

Quark Yukawa interactions $\Longrightarrow$ quark mass matrix

$$L_Y = y_{ij}^{(d)} \bar{Q}_L^i H d_R^j + y_{ij}^{(u)} \bar{Q}_L^i \tilde{H} u_R^j + \text{h.c.} \Longrightarrow L_m = m_{ij}^{(d)} \bar{d}_L^i d_R^j + m_{ij}^{(u)} \bar{u}_L^i u_R^j + \text{h.c.}$$

$m_{ij}^{(u,d)} = y_{ij}^{(u,d)} v/\sqrt{2}$ complex; $i, j = 1, 2, 3 =$ generation label.

Standard lore: diagonalize $\Longrightarrow$ CKM matrix, 3 angles, 1 phase.

This is not quite true

One more phase: common phase of all Yukawa couplings/masses,

$$m_{ij} = \mathrm{e}^{i\theta} \cdot m_{ij}^{CKM} \qquad \text{or} \qquad \theta = \text{Arg det } m$$

At first sight: rotate away,

$$q_L^i \to \mathrm{e}^{i\theta/2} q_L^i, \quad q_R^i \to \mathrm{e}^{-i\theta/2} q_R^i, \quad \text{i.e.,} \quad q^i \to \mathrm{e}^{i\gamma^5 \theta/2} q^i$$

But this is does not remove phase θ from QCD!

To see this, let us calculate vacuum energy density for non-zero θ, call it $V(\theta)$ (useful in what follows). Keep only u, d-quarks, take their masses equal for simplicity, $m_u = m_d \equiv m_q \sim 10$ MeV (heavier quarks make smaller contribution),

$$L_m = \mathrm{e}^{i\theta} m_q (\bar{u}_L u_R + \bar{d}_L d_R) + \text{h.c.}$$

Perturbation theory in m_q: $V(\theta) = -\langle L_m \rangle$.

Quark condensate in QCD vacuum in chiral limit, $m_q = 0$:

$$\langle \bar{u}_L u_R \rangle = \langle \bar{d}_L d_R \rangle = \frac{1}{2} \langle \bar{q} q \rangle = \text{real} \sim \Lambda_{QCD}^3$$

NB: No arbitrary phase here, otherwise η' would be pseudo-Goldstone!

Get

$$V(\theta) = -\langle L_m \rangle = -m_q \langle \bar{q} q \rangle \cos\theta$$

θ is a physical parameter! Violates CP.

NB: Minimum of $V(\theta)$ at $\theta = 0$. No help if θ is just a free parameter.

Q: What's wrong with the field redefinition $q^i \to e^{i\gamma^5\theta/2} q^i$?

A: It modifies QCD Lagrangian.

QCD Lagrangian:

$$L_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \tau \frac{\alpha_s}{16\pi} \varepsilon_{\mu\nu\lambda\rho} F^a_{\mu\nu} F^a_{\lambda\rho} + L_{\text{quark}}$$

τ: an extra coupling constant. Violates CP.

NB: No extra term in QED: $\varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$ is total derivative. But total derivatives do sometimes matter in quantum theory!

Field redefinition $q^i \to e^{i\gamma^5\theta/2} q^i \Longrightarrow \tau \to \tau + \theta$.

By field redefinition can reshuffle τ (parameter in gluon Lagrangian) and θ (common phase in quark mass matrix).

But cannot get rid of both! $\bar{\theta} = \tau + \text{Arg det } m$ is invariant under field redefinitions.

NB: We actually calculated $V(\bar{\theta})$ (implicitly assumed $\tau = 0$).

CP violation within QCD. Neutron edm $d_n < 3 \cdot 10^{-26} e \cdot \mathrm{cm} \Longrightarrow$

$$\bar{\theta} \lesssim 10^{-10}$$

Either fine tune, or find mechanism that ensures $\bar{\theta} = 0$

Peccei–Quinn mechanism: promote $\bar{\theta}$ to a field.

Simple version: two Higgs fields plus singlet S (DFSZ)

$$L = y^{(d)} \bar{Q}_L H_1 d_R + y^{(u)} \bar{Q}_L H_2 u_R + |D_\mu H_1|^2 + |D_\mu H_2|^2 + \frac{1}{2}(\partial_\mu S)^2 - V(H_1, H_2, S)$$

Classical level: require global symmetry (PQ symmetry)

$$q^i \to \mathrm{e}^{i\gamma^5 \theta/2} q^i \,, \quad H_1 \to \mathrm{e}^{i\theta} H_1 \,, \quad H_2 \to \mathrm{e}^{i\theta} H_2 \,, \quad S \to \mathrm{e}^{2i\theta} S$$

Vev's: $\langle H_1 \rangle = v_1/\sqrt{2}$, $\langle H_2 \rangle = v_2/\sqrt{2}$, $\langle S \rangle = v_S$, break PQ symmetry spontaneously.

Parametrize

$$H_1 = \mathrm{e}^{i\theta(x)} v_1/\sqrt{2}\,, \quad H_2 = \mathrm{e}^{i\theta(x)} v_2\sqrt{2}\,, \quad S = \mathrm{e}^{2i\theta(x)} v_S$$

If not for QCD, $\langle\theta(x)\rangle$ would be arbitrary. Quark masses $m = yv_{1,2}\mathrm{e}^{i\langle\theta(x)\rangle}$. CP-violating parameter $\bar{\theta} = \tau + \text{Arg det } yv + \langle\theta(x)\rangle$, vev of a field. $\theta(x)$ would be a massless Goldstone boson. Kinetic term

$$\frac{1}{2}v_1^2(\partial_\mu\theta)^2 + \frac{1}{2}v_2^2(\partial_\mu\theta)^2 + \frac{1}{2}v_S^2(\partial_\mu\theta)^2 = \frac{1}{2}f_{PQ}^2(\partial_\mu\theta)^2$$

$f_{PQ}^2 = v_1^2 + v_2^2 + v_S^2$ can be large, if v_S is large.

Turn on QCD: shift $\theta \to \theta + \text{const}$ is NOT a symmetry.

Consequences

- $\langle\theta(x)\rangle$ is such that $V(\bar{\theta})$ is at minimum $\Longrightarrow \bar{\theta} = 0$ automatically. Strong CP problem solved.
- $\theta(x)$ gets a mass

$$L_\theta = \frac{1}{2} f_{PQ}^2 (\partial_\mu \theta)^2 - V(\theta) \, , \qquad V(\theta) \simeq -m_q \langle \bar{q} q \rangle \cos\theta = \frac{1}{2} m_q \langle \bar{q} q \rangle \theta^2$$

Axion field $\theta(x) = a(x)/f_{PQ}$:

$$m_a^2 \simeq \frac{m_q \langle \bar{q} q \rangle}{f_{PQ}^2} \simeq \frac{m_q \Lambda_{QCD}^3}{f_{PQ}^2} \quad \Longrightarrow \quad m_a = 0.6 \text{ eV} \cdot \left(\frac{10^7 \text{ GeV}}{f_{PQ}} \right)$$

Interactions:

- Axion-gluon-gluon:

$$C_{agg} \frac{\alpha_s}{16\pi} \theta(x) \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu}^a F_{\lambda\rho}^a \, , \qquad C_{agg} \sim 1 \quad \text{roughly}$$

- Likewise: axion-photon-photon

$$C_{a\gamma\gamma} \frac{\alpha}{16\pi} \theta(x) \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho} \, , \qquad C_{a\gamma\gamma} \sim 1 \quad \text{roughly}$$

To summarize

Peccei–Quinn solution to strong CP problem predicts axion with mass

$$m_a = 0.6\ \text{eV} \cdot \left(\frac{10^7\ \text{GeV}}{f_{PQ}} \right)$$

and $a\gamma\gamma$ interaction

$$C_{a\gamma\gamma} \frac{\alpha}{16\pi} \frac{a(x)}{f_{PQ}} \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}$$

where $C_{a\gamma\gamma} \sim 1$ is model-dependent, and f_{PQ} is the only free parameter. Larger $f_{PQ} \Longrightarrow$ smaller m_a, weaker interactions.

- Why did DFSZ introduce field S? Because $f_{PQ}^2 = v_1^2 + v_2^2 + v_S^2$. No $S \Longrightarrow v_S = 0 \Longrightarrow f_a^2 = 246\ \text{GeV} \Longrightarrow m_a = 15\ \text{keV}$, fairly strong $a\gamma\gamma$ interaction. Weinberg–Wilczek axion, ruled out.
- There are other ways to make f_{PQ} large, e.g., KSVZ.

Why is this interesting for cosmology?

- Axion is practically stable:

$$\Gamma(a \to \gamma\gamma) = C^2_{a\gamma\gamma}\left(\frac{\alpha}{8\pi}\right)^2 \frac{m_a^3}{4\pi f_{PQ}^2} \quad \Longrightarrow \quad \tau_a = 10^{17}\left(\frac{\text{eV}}{m_a}\right)^5 \text{ yrs}$$

- Interacts very weakly $\Longrightarrow$ dark matter candidate
- May never be in thermal equilibrium $\Longrightarrow$ cold dark matter if momenta are negligibly small.

Q. How can one arrange for negligibly small momenta for particles with sub-eV masses?

A. One way: Condensates
(Not the only option)

Reminder from Lecture 1:

- Expansion rate at radiation domination

$$H(T) = \frac{T^2}{M_{Pl}^*}\,, \qquad M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}}$$

- Mass-to-entropy ratio of dark matter today

$$\frac{\rho_{DM}}{s} = 3 \cdot 10^{-10}\ \text{GeV}$$

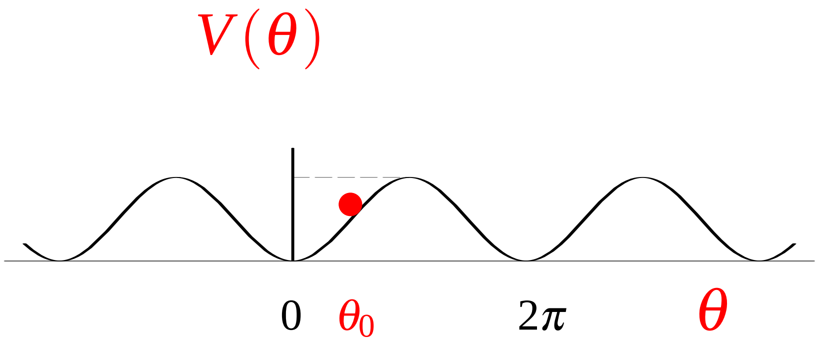
Axion production: misalignment

Recall $V(\theta) \simeq -m_q \langle \bar{q}q \rangle \cos\theta$

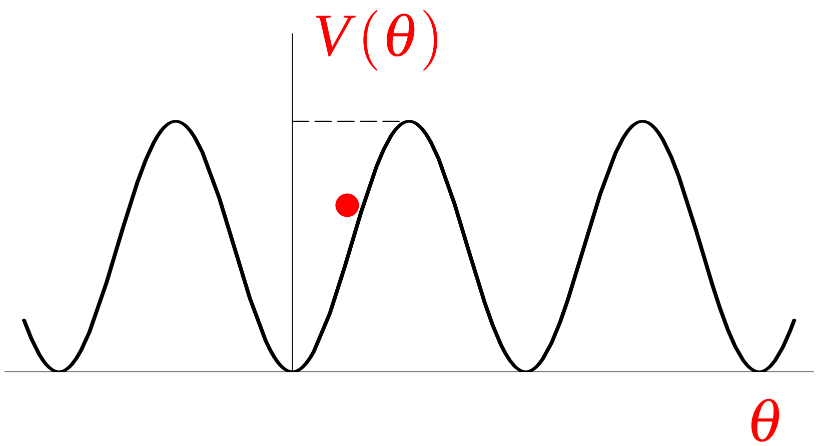
Early Universe, high T: $\langle \bar{q}q \rangle = 0 \implies V(\theta) = 0$.

No preferred value of $\theta \implies$ Initial condinion θ_0 anywhere between $-\pi$ and π.

At QCD epoch ($T \sim 200$ MeV) potential $V(\theta)$ builds up. θ starts to roll down.



high T



lower T

Rolling down starts when $m_a(T) \sim H(T)$: before that time scale of rolling m_a^{-1} is larger than the cosmological time scale $\sim H^{-1}$.

After initial rolling, θ oscillates about minimum $\theta = 0$.

Homogeneous oscillating field $=$ condensate $=$ collection of quanta with zero spatial momentum. Just what we need for cold dark matter!

Estimate for present mass density:

Energy density at beginning of rolling

$$V(\theta_0, T) = m_a^2(T) a_0^2 = m_a^2(T) f_{PQ}^2 \theta_0^2$$

Number density of quanta at that time

$$n_a(T) = V(\theta_0, T)/m_a(T) = m_a(T) f_{PQ}^2 \theta_0^2$$

Recall $m_a(T) \sim H(T) = T^2/M^*_{Pl} \Longrightarrow$ number-to-entropy

$$\frac{n_a}{s} = \# \frac{H(T) f_{PQ}^2 \theta_0^2}{g_* T^3} = \# \frac{f_{PQ}^2 \theta_0^2}{\sqrt{g_*} M_{Pl} T}$$

with $T = T_{QCD} \sim 200$ MeV and $\# \sim 1$.

Present mass-to-entropy

$$\frac{\rho_a}{s} = m_a^{(T=0)} \cdot \frac{n_a}{s} = \# \frac{m_a^{(T=0)} f_{PQ}^2}{\sqrt{g_*} M_{Pl} T_{QCD}} \theta_0^2$$

Recall $m_a f_{PQ}^2 \propto m_a^{-1}$: the lighter axions, the more dark matter.

$\rho_{DM}/s \sim 3 \cdot 10^{-10}$ GeV is obtained for $m_a = 10^{-5} - 10^{-6}$ eV
(for $\theta_0 = \pi/2 - 0.1$).

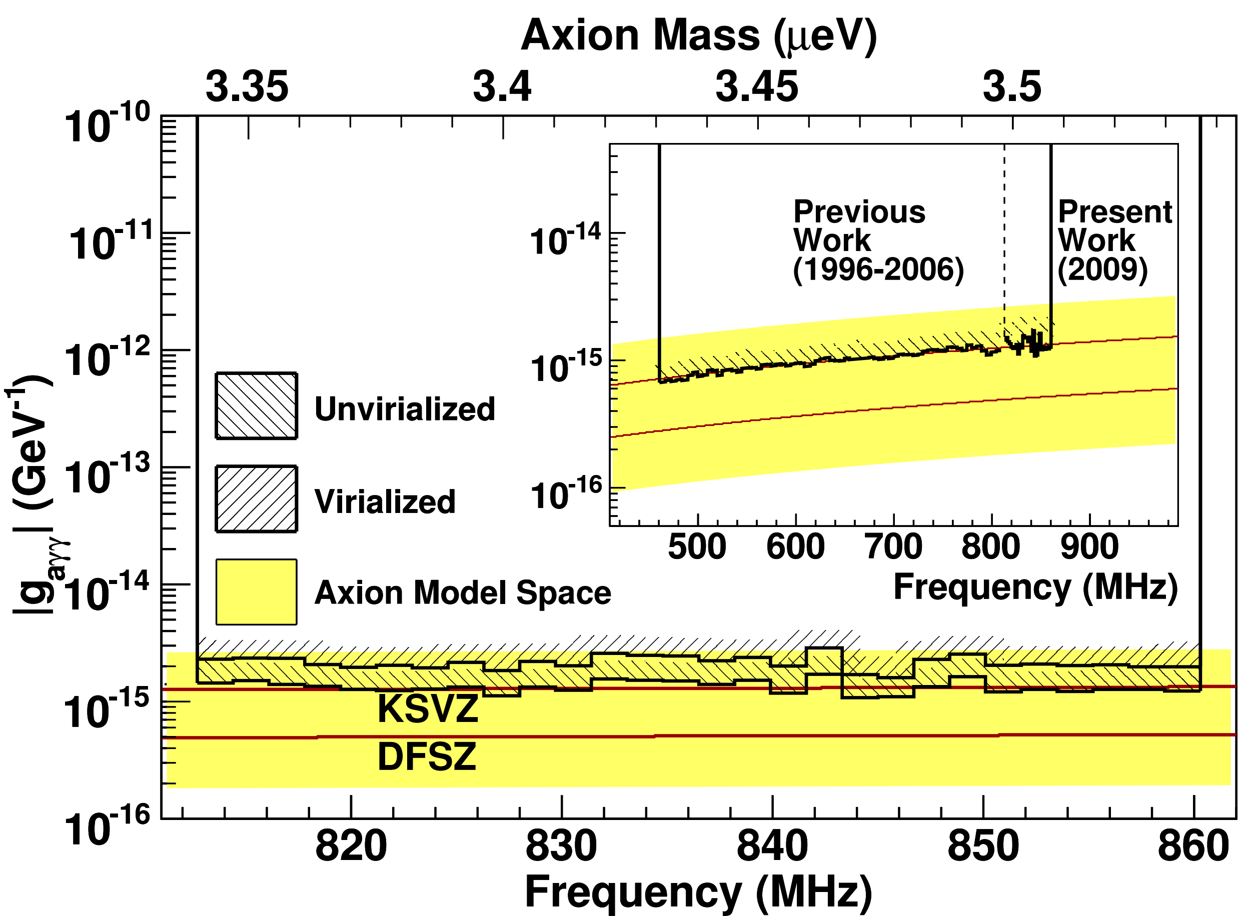
Axions of mass $1 - 10$ μeV are good cold dark matter candidates.

NB: Misalignment is not the only possible production mechanism.

Search

$a\gamma\gamma$ interaction $\quad C_{a\gamma\gamma}\frac{\alpha}{16\pi}\frac{a(x)}{f_{PQ}}\varepsilon_{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}$

Conversion of DM axion into photon in magnetic field in a resonant cavity. 10^{-6} eV/2π = 240 MHz. Need high Q resonator to collect photons, narrow bandwidth, go small steps in m_a. Long story.



ADMX, PRL '2010

Warm dark matter

- Clouds over CDM

  Numerical simulations of structure formation with CDM show

  - Too many dwarf galaxies

    A few hundred satellites of a galaxy like ours —

    But only dozens observed so far

  - Too high density in galactic centers (“cusps”)

- No serious worry yet

  But what if one really needs to suppress small structures?

High initial momenta of DM particles $\Longrightarrow$ Warm dark matter

Warm dark matter

- Decouples when relativistic, $T_f \gg m$.
- Remains relativistic until $T \sim m$ (assuming thermal distribution). Does not feel gravitational potential before that.
- Perturbations of wavelengths shorter than horizon size at that time get smeared out $\Longrightarrow$ small size objects do not form ("free streaming")
- Horizon size at $T \sim m$

$$l(T) = H(T \sim m)^{-1} = \frac{M_{Pl}^*}{T^2} = \frac{M_{Pl}^*}{m^2}$$

Present size of this region

$$l_c = \frac{T}{T_0} l(T) = \frac{M_{Pl}}{m T_0}$$

(modulo g_* factors).

Objects of initial comoving size smaller than l_c are less abundant

- Initial size of dwarf galaxy $l_{dwarf} \sim 100$ kpc $\sim 3 \cdot 10^{23}$ cm
  Require

$$l_c \simeq \frac{M_{Pl}}{mT_0} \sim l_{dwarf}$$

$\Longrightarrow$ obtain mass of DM particle

$$m \sim \frac{M_{Pl}}{T_0 l_{dwarf}} \sim 3 \text{ keV}$$

($M_{Pl} = 10^{19}$ GeV, $T_0^{-1} = 0.1$ cm).

- Particles of masses in 1 – 10 keV range
  are good warm dark matter candidates
  (assuming their energies at decoupling are of order T)

- Candidates
  - Sterile neutrino
  - Gravitino
  - Exotica

Gravitinos in low energy SUSY

- Rigid supersymmtery: SUSY breaking $\Longrightarrow$ Goldstino, massless Nambu–Goldstone fermion.

- Supergravity = local supersymmetry $\Longrightarrow$ Goldstino eaten up by gravitino, gravitino becomes massive.

- Mass $m_{3/2} \simeq F/M_{Pl}$
  $\sqrt{F} =$ SUSY breaking scale.
  $\Longrightarrow$ Gravitinos light for low SUSY breaking scale.
  E.g. gauge mediation

  From purely theoretical viewpoint, mass may be as low as, say, $\text{TeV}^2/M_{Pl} \sim 10^{-4}$ eV. In concrete models much heavier, but $m_{3/2} \sim$ keV is not unrealistic.

- Light gravitino = LSP $\Longrightarrow$ Stable

- Interactions: mostly due to Goldstino (longitudinal) component. Calculable. Proportional to SUSY breaking parameters, suppressed by $F^{-1} \simeq M_{Pl} m_{3/2}$.

  As an example, interaction with gauge bosons and gaugino

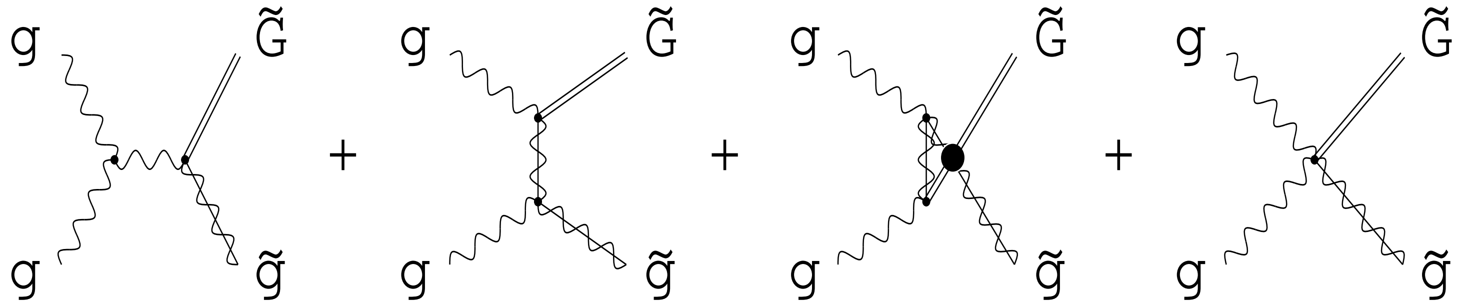
$$L_{int} = \frac{M_{\tilde{g}}}{F} \tilde{G}_{3/2}[\gamma^{\mu}, \gamma^{\nu}] \tilde{g} F_{\mu\nu} = \frac{M_{\tilde{g}}}{M_{Pl} m_{3/2}} \tilde{G}_{3/2}[\gamma^{\mu}, \gamma^{\nu}] \tilde{g} F_{\mu\nu}$$

  $M_{\tilde{g}} =$ (soft) gaugino mass (due to SUSY breaking).

- Rather weird: the smaller the mass $m_{3/2}$, the stronger interactions.

- Production in the Universe: $E_{3/2} \gtrsim T$.
  Warm dark matter candidate, if $m_{3/2} \sim 10$ keV.
  Heavier gravitino = cold DM candidate.

Production in scattering



Production cross section at $T \gg M_{\tilde{g}}$

$$\sigma_{prod} \simeq C\alpha \frac{M_{\tilde{g}}^2}{M_{Pl}^2 m_{3/2}^2}$$

with $C \sim 10^3$ (matter of debate: infrared logs).

Gravitinos produced in one Hubble time at temperature T

$$\left(\frac{n_{3/2}}{s}\right)_T = \sigma_{prod}\frac{n_g^2}{s}H^{-1}(T) \propto T \ , \quad \text{since } n_g \sim s \propto T^3 \ , \quad H = T^2/M_{Pl}^*$$

Maximum production at maximum possible temperature.

As a result (gluons/gluinos dominate)

$$\frac{m_{3/2}n_{3/2}}{s} \sim \frac{m_{3/2}}{g_*} \cdot C\alpha_s \frac{M_{\tilde{g}}^2}{M_{Pl}^2 m_{3/2}^2} \cdot M_{Pl}^* T_{max}$$

Right abundance, $n_{3/2}m_{3/2}/s = 3 \cdot 10^{-10}$ GeV obtained by tuning T_{max}:

$$T_{max} = 10^4 \text{ GeV} \cdot \frac{m_{3/2}}{10 \text{ keV}} \cdot \left(\frac{1 \text{ TeV}}{M_{\tilde{g}}}\right)^2$$

- "Easy" to obtain right abundance by dialing T_{max}.

  Range of T_{max}:
  from $T_{max} \sim$ a few TeV for $m_{3/2} \sim$ a few keV
  to $T_{max} \sim 10^{12}$ GeV for $m_{3/2} \sim 1$ TeV

  But required abundance appears a coincidence.

  NB: Same for many other dark matter candidadtes.

Competing production mechanism:
Gravitino production in decays of superpartners

$$\frac{d(n_{3/2}/s)}{dt} = \frac{n_{\tilde{S}}}{s}\Gamma_{\tilde{S}}$$

$n_{\tilde{S}}/s = \text{const} \sim g_*^{-1}$ for $T \gtrsim M_{\tilde{S}}$, while $n_{\tilde{S}} \propto \text{e}^{-M_{\tilde{S}}/T}$ for $T \ll M_{\tilde{S}}$
$\Longrightarrow$ production most efficient at $T \sim M_{\tilde{S}}$ (slow cosmological expansion with unsuppressed $n_{\tilde{S}}$)

Hence

$$\frac{n_{3/2}}{s} \simeq \frac{\Gamma_{\tilde{S}}}{g_* H(T \sim M_{\tilde{S}})} \simeq \frac{M_{Pl}^*}{g_* M_{\tilde{S}}^2} \cdot \frac{M_{\tilde{S}}^5}{m_{3/2}^2 M_{Pl}^2}$$

Mass-to-entropy ratio

$$\frac{m_{3/2} n_{3/2}}{s} \simeq \frac{M_{\tilde{S}}^3}{m_{3/2}} \frac{1}{g_*^{3/2} M_{Pl}}$$

For $m_{3/2} =$ a few keV, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV

$$M_{\tilde{S}} \simeq 100 \div 300 \text{ GeV}$$

Need: some superpartners light,

others a lot heavier, $m_{\tilde{S}} > T_{max}$.

Rather contrived scenario, but generating warm dark matter is always contrived

Dark matter summary

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

By no means. A good non-LHC DM candidate: axion.

Plus a lot of exotica...

Crucial impact of LHC to cosmology,
direct and indirect dark matter searches

- WIMP, signal at LHC:
  - Strongest possible motivation for direct and indirect detection
    - Inferred interactions with baryons $\Longrightarrow$ strategy for direct detection
  - A handle on the Universe at

$$T = (\text{a few}) \cdot 10 \text{ GeV} \div (\text{a few}) \cdot 100 \text{ GeV}$$

$$t = 10^{-11} \div 10^{-8} \text{ s}$$

cf. $T = 1$ MeV, $t = 1$ s at nucleosynthesis

- Gravitino-like
  - A lot of work to make sure that it is indeed DM particle
  - Hard time for direct and indirect searches
- No signal at LHC
  - Best guess: axion
  - If not, need more hints from cosmology and astrophysics