Particle Cosmology

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Topics

- Basics of Hot Big Bang cosmology
- Dark matter:
  - WIMPs
  - Axions
  - Warm dark matter, gravitinos
- Baryon asymmetry
  - Electroweak mechanism
  - Leptogenesis

Lecture 1

Outline of Lecture 1

- Basics of Hot Big Bang cosmology
- Dark matter: evidence
- WIMPs

Basics of Hot Big Bang cosmology

- The Universe at large is homogeneous, isotropic and expanding. 3d space is Euclidean (observational fact!)
  All this is encoded in space-time metric

$$ds^2 = dt^2 - a^2(t)\mathbf{dx}^2$$

$\mathbf{x}$: comoving coordinates, label distant galaxies.

$a(t)dx$: physical distances.

$a(t)$: scale factor, grows in time; a_0: present value (matter of convention)

$$z(t) = \frac{a_0}{a(t)} - 1 : \quad \text{redshift}$$

Light of wavelength λ emitted at time t has now wavelength $\lambda_0 = \frac{a_0}{a(t)}\lambda = (1+z)\lambda$.

$$H(t) = \frac{\dot{a}}{a} : \quad \text{Hubble parameter, expansion rate}$$

- Present value

$$H_0 = (70.4 \pm 1.4)\ \frac{\text{km/s}}{\text{Mpc}} = (14 \cdot 10^9\ \text{yrs})^{-1}$$

$1\ \text{Mpc} = 3 \cdot 10^6\ \text{light yrs} = 3 \cdot 10^{24}\ \text{cm}$

- Hubble law (valid at $z \ll 1$)

$$z = H_0 r$$

Fig.

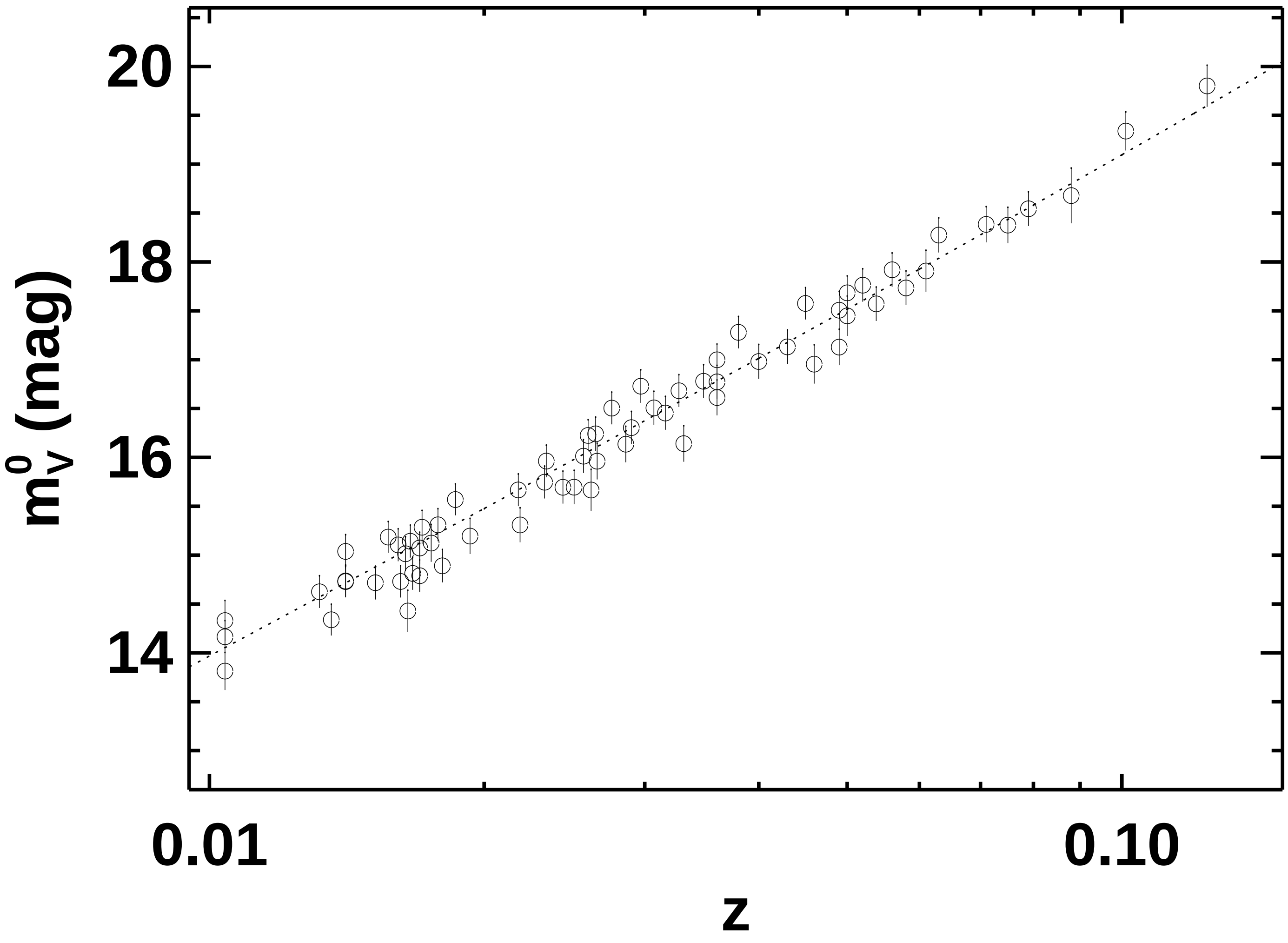
- The Universe is warm: CMB temperature today

$$T_0 = 2.726\ \text{K}$$

Fig.

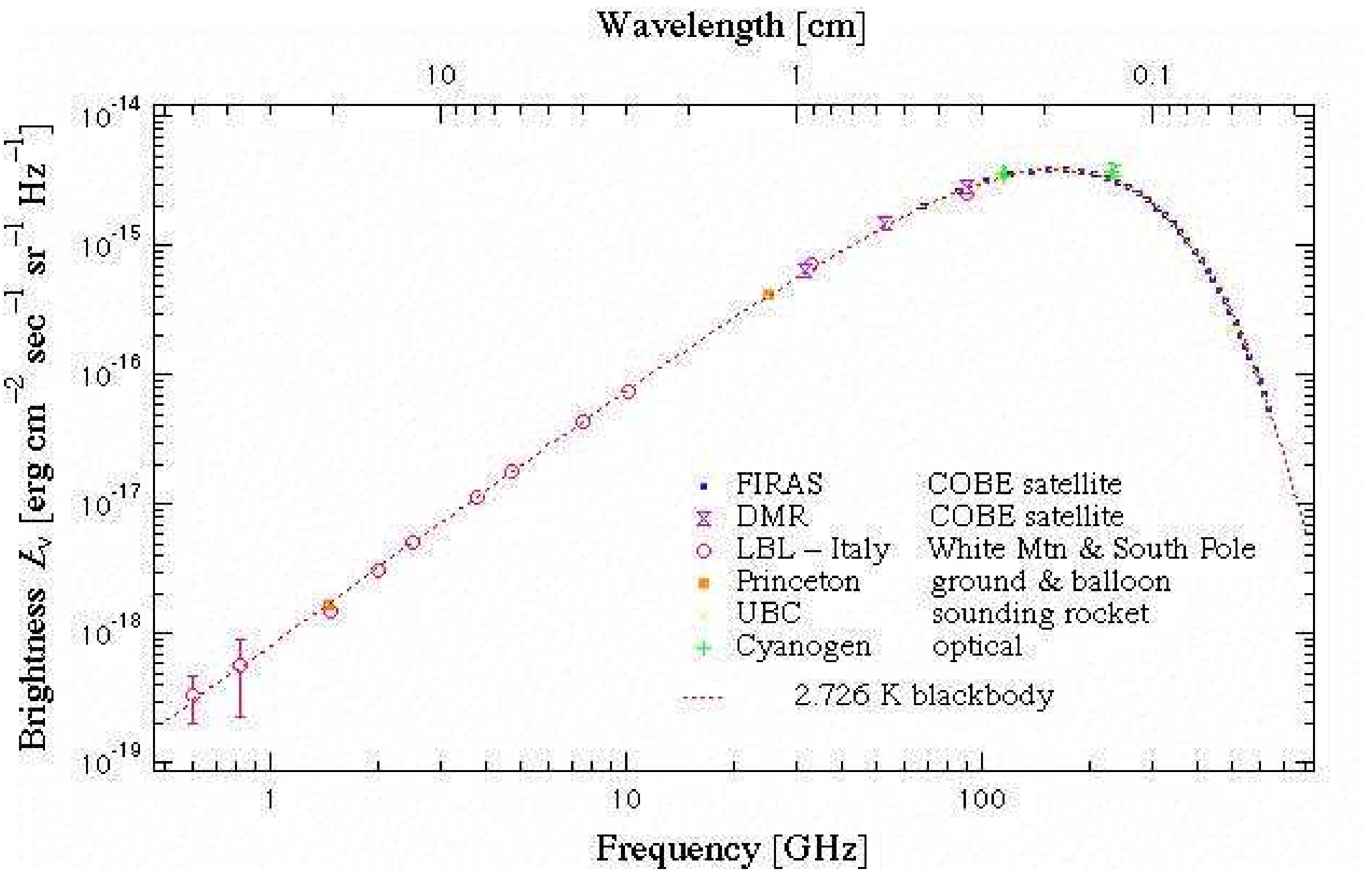
It was denser and warmer at early times.

Hubble diagram for SNe1a



$$\text{mag} = 5\log_{10} r + \text{const}$$

CMB spectrum



$T = 2.726$ K

- Present number density of photons

$$n_\gamma = \# T^3 = 410 \frac{1}{\text{cm}^3}$$

- Present entropy density

$$s = 2 \cdot \frac{2\pi^2}{45} T_0^3 + \text{neutrino contribution} = 3000 \frac{1}{\text{cm}^3}$$

In early Universe

$$s = \frac{2\pi^2}{45} g_* T^3$$

g_*: number of relativistic degrees of freedom with $m \lesssim T$; fermions contribute with factor $7/8$.

Entropy density scales exactly as a^{-3}

Temperature scales approximately as a^{-1}.

- Friedmann equation: expansion rate of the Universe vs total energy density ρ ($M_{Pl} = G^{-1/2} = 10^{19}$ GeV):

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}^2}\rho$$

Einstein equations of General Relativity specified to homogeneous isotropic space-time with zero spatial curvature.

- Present energy density

$$\rho_0 = \rho_c = \frac{3M_{Pl}^2}{8\pi} H_0^2 = 5 \cdot 10^{-6} \; \frac{\text{GeV}}{\text{cm}^3}$$

Present composition of the Universe

$$\Omega_i = \frac{\rho_{i,0}}{\rho_c}$$

present fractional energy density of i-th type of matter.

$$\sum_i \Omega_i = 1$$

- Dark energy: $\Omega_\Lambda = 0.72$
  ρ_Λ stays (almost?) constant in time
- Non-relativistic matter: $\Omega_M = 0.28$
  $\rho_M = mn(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^3$
  - Dark matter: $\Omega_{DM} = 0.23$
  - Usual matter (baryons): $\Omega_B = 0.046$
- Relativistic matter (radiation): $\Omega_{rad} = 8.4 \cdot 10^{-5}$ (for massless neutrinos)
  $\rho_{rad} = \omega(t)n(t)$ scales as $\left(\frac{a_0}{a(t)}\right)^4$

Friedmann equation

$$H^2(t) = \frac{8\pi}{3M_{Pl}^2}\left[\rho_\Lambda + \rho_M(t) + \rho_{rad}(t)\right] = H_0^2\left[\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a(t)}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a(t)}\right)^4\right]$$

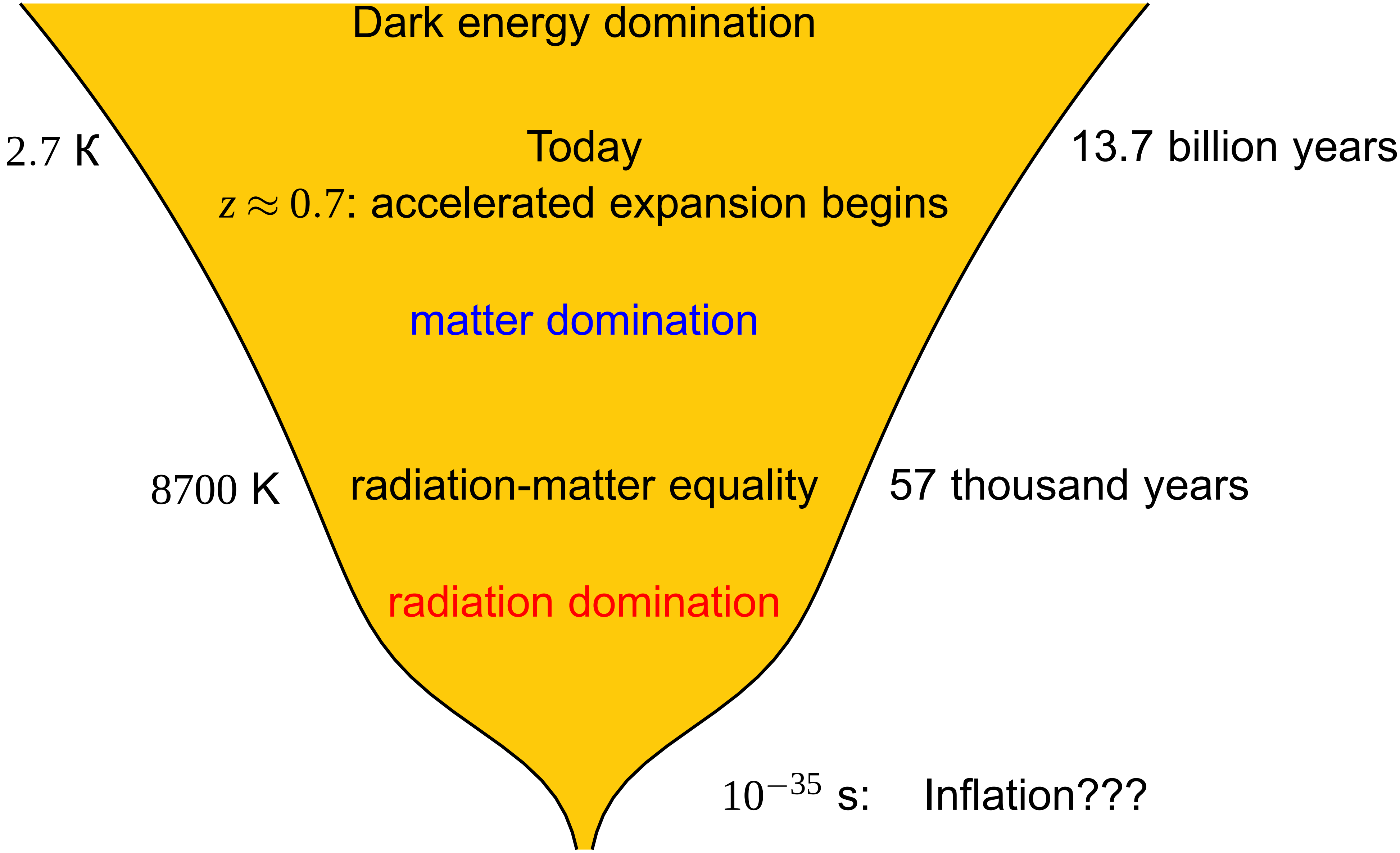
$\ldots \Longrightarrow$ Radiation domination $\Longrightarrow$ Matter domination $\Longrightarrow$ Λ–domination

$z_{eq} = 3200$

now

$T_{eq} = 8700$ K $= 0.75$ eV

$t_{eq} = 57 \cdot 10^3$ yrs


Dark energy domination
2.7 K
Today
13.7 billion years
$z \approx 0.7$: accelerated expansion begins
matter domination
8700 K
radiation-matter equality
57 thousand years
radiation domination
10^{-35} s: Inflation???

Expansion at radiation domination

- Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{8\pi}{3M_{Pl}}\rho$$

- Radiation energy density: Stefan–Boltzmann

$$\rho = \frac{\pi^2}{30} g_* T^4$$

g_*: number of relativistic degrees of freedom (about 100 in SM at $T \sim 100$ GeV). Hence

$$H(T) = \frac{T^2}{M_{Pl}^*}$$

with $M_{Pl}^* = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Expansion law:

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\rho \quad \Longrightarrow \quad \frac{\dot{a}^2}{a^2} = \frac{\text{const}}{a^4}$$

Solution:

$$a(t) = \text{const} \cdot \sqrt{t}$$

- $t = 0$: Big Bang singularity

$$H = \frac{\dot{a}}{a} = \frac{1}{2t}\,, \qquad \rho \propto \frac{1}{t^2}$$

- Decelerated expansion: $\ddot{a} < 0$.
  - NB: Λ-domination

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi}{3M_{Pl}^2}\rho_\Lambda = \text{const} \Longrightarrow a(t) = \mathrm{e}^{H_\Lambda t}$$

accelerated expansion.

Cornerstones of thermal history

- Big Bang Nucleosynthesis, epoch of thermonuclear reactions

$$\begin{aligned} p+n &\rightarrow {}^2H \\ {}^2H+p &\rightarrow {}^3He \\ {}^3He+n &\rightarrow {}^4He \\ \text{up to} &\quad {}^7Li \end{aligned}$$

  Abundances of light elements: measurements vs theory

  $T = 10^{10} \rightarrow 10^9$ K, $t = 1 \rightarrow 300$ s

  Earliest time in thermal history probed so far Fig.

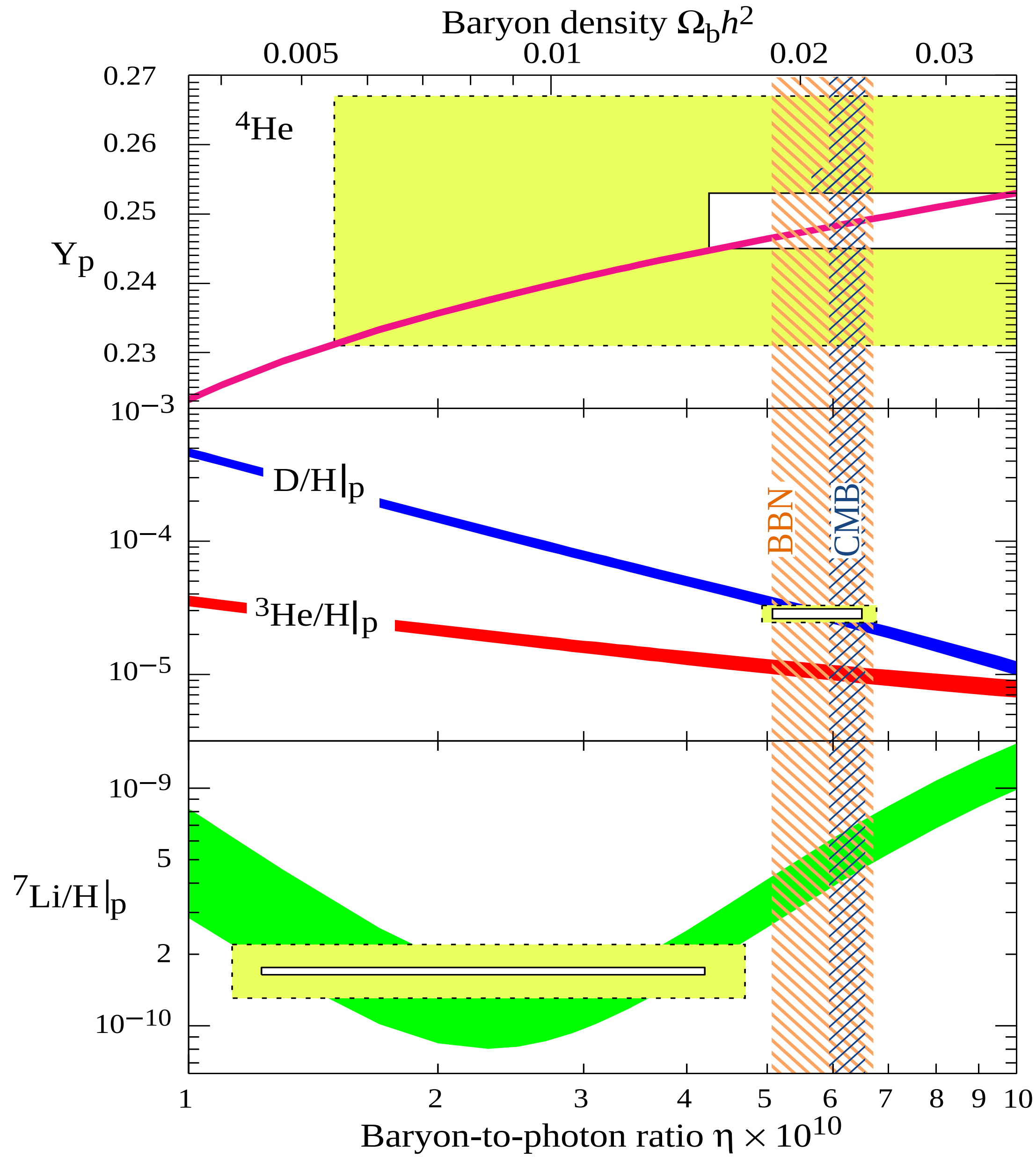
- Recombination, transition from plasma to gas.

  $z = 1090$, $T = 3000$ K, $t = 370\ 000$ years

  Last scattering of CMB photons Fig.

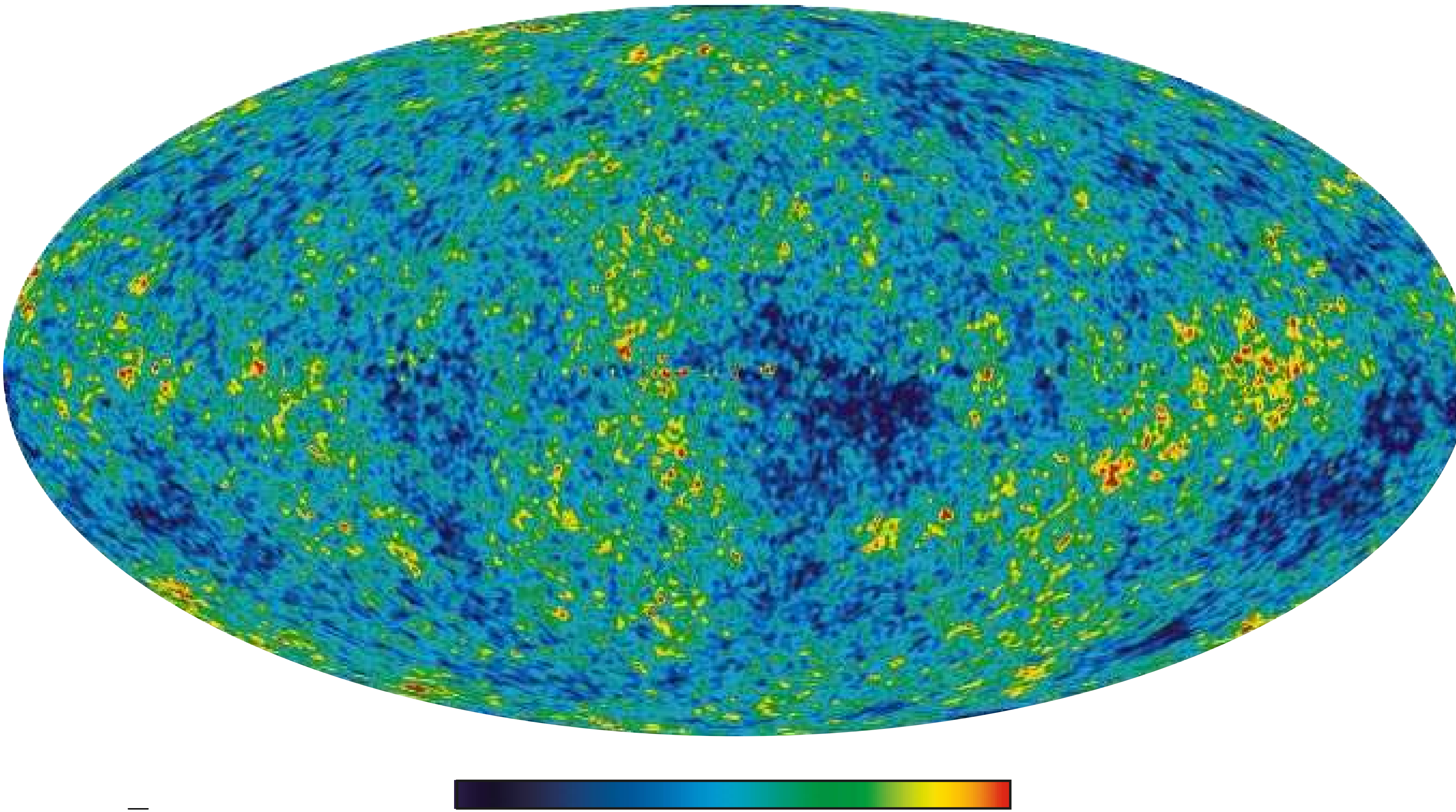
- Neutrino decoupling: $T = 2-3$ MeV $\sim 3 \cdot 10^{10}$K, $t \sim 0.1-1$s
- Generation of dark matter*
- Generation of matter-antimatter asymmetry*

*may have happend before the hot Big Bang epoch

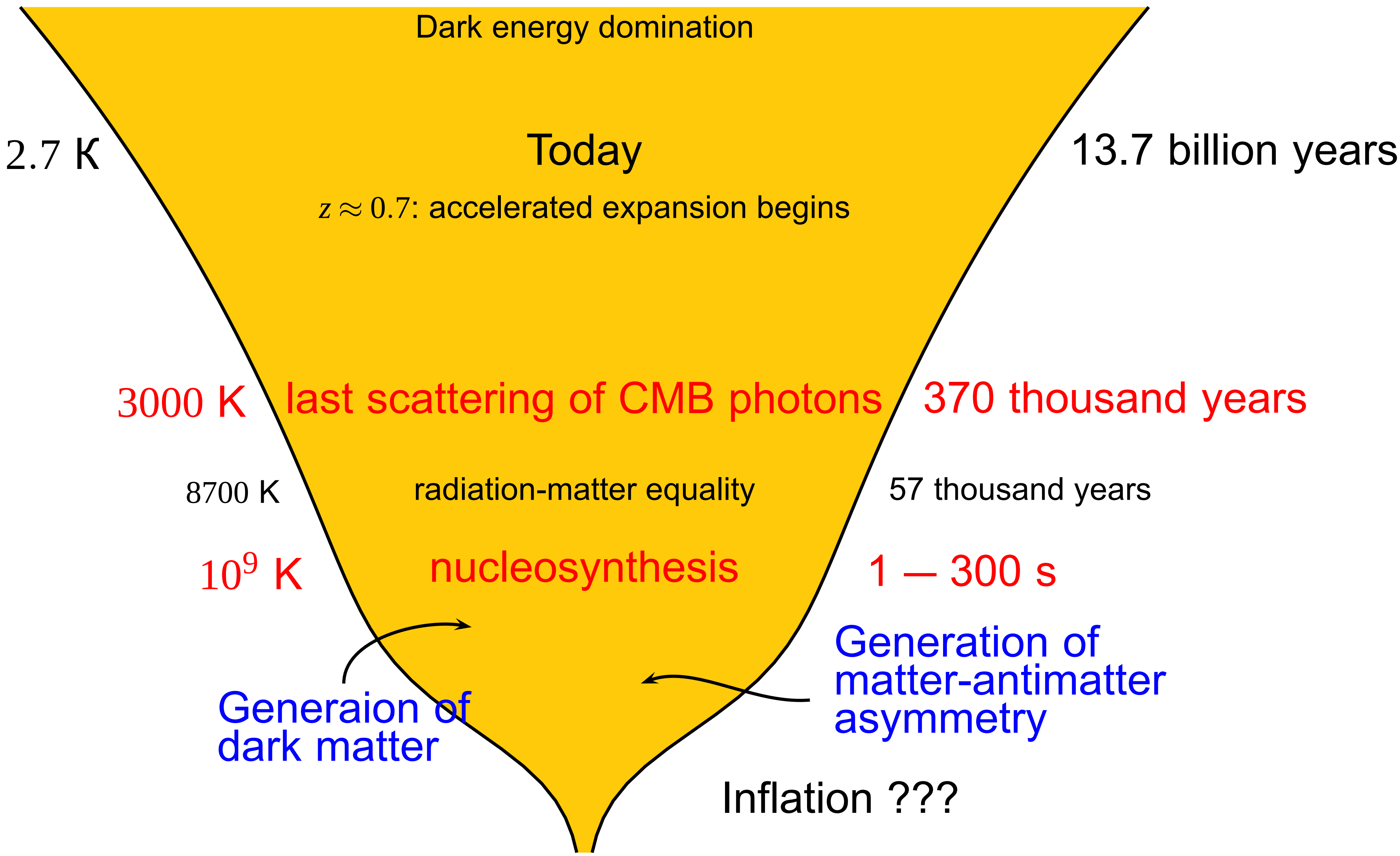


$\eta_{10} = \eta \cdot 10^{-10}$ = baryon-to-photon ratio. Consistent with CMB determination of η

$$T = 2.726^{\circ}K, \quad \frac{\delta T}{T} \sim 10^{-5}$$



WMAP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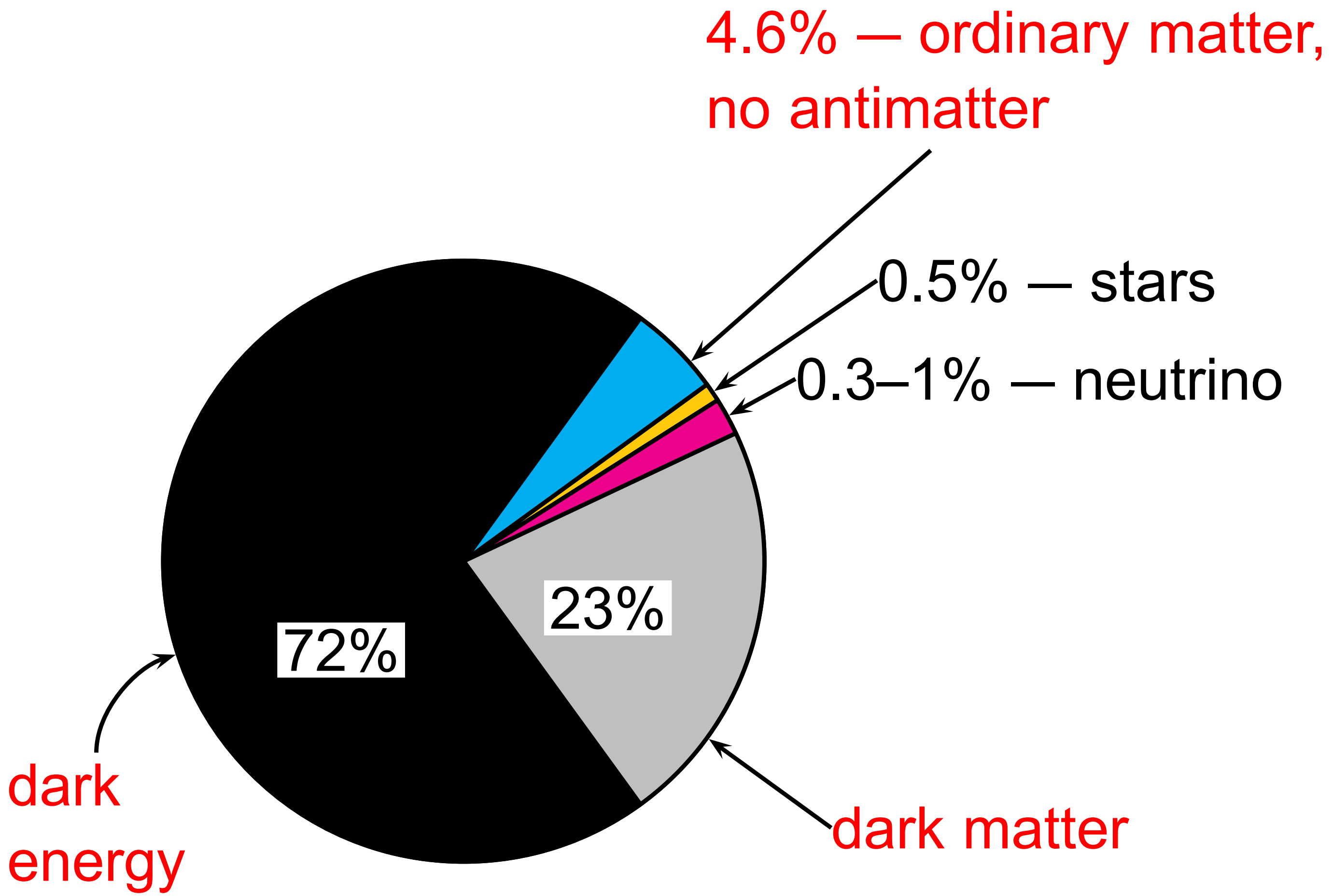
nucleosynthesis
1 — 300 s
Generation of matter-antimatter asymmetry
Generaion of dark matter
Inflation ???

Unknowns


4.6% — ordinary matter,
no antimatter
0.5% — stars
0.3–1% — neutrino
23%
72%
dark
energy
dark matter

Dark matter

- Astrophysical evidence: measurements of gravitational potentials in galaxies and clusters of galaxies
  - Velocity curves of galaxies

    Fig. Backup slide 1
  - Velocities of galaxies in clusters

    Original Zwicky's argument, 1930's

$$v^2 = G\frac{M(r)}{r}$$

  - Temperature of gas in X-ray clusters of galaxies
  - Gravitational lensing of clusters

    Fig. Backup slide 2
  - Etc.

Outcome

$$\Omega_M \equiv \frac{\rho_M}{\rho_c} = 0.2 - 0.3$$

Assuming mass-to-light ratio everywhere the same as in clusters
NB: only 10 % of galaxies sit in clusters

Nucleosynthesis, CMB:

$$\Omega_B = 0.046$$

The rest is non-baryonic, $\Omega_{DM} \approx 0.23$.

Physical parameter: mass-to-entropy ratio. Stays constant in time. Its value

$$\left(\frac{\rho_{DM}}{s}\right)_0 = \frac{\Omega_{DM}\rho_c}{s_0} = \frac{0.2 \cdot 0.5 \cdot 10^{-6}\ \text{GeV cm}^{-3}}{3000\ \text{cm}^{-3}} = 3 \cdot 10^{-10}\ \text{GeV}$$

Cosmological evidence: growth of structure

CMB anisotropies: baryon density perturbations at recombination $\approx$ photon last scattering, $T = 3000$ K, $z = 1100$:

$$\delta_B \equiv \left(\frac{\delta\rho_B}{\rho_B}\right)_{z=1100} \simeq \left(\frac{\delta T}{T}\right)_{CMB} \sim 10^{-4}$$

In matter dominated Universe, matter perturbations grow as

$$\frac{\delta\rho}{\rho}(t) \propto a(t)$$

Perturbations in baryonic matter grow after recombination only

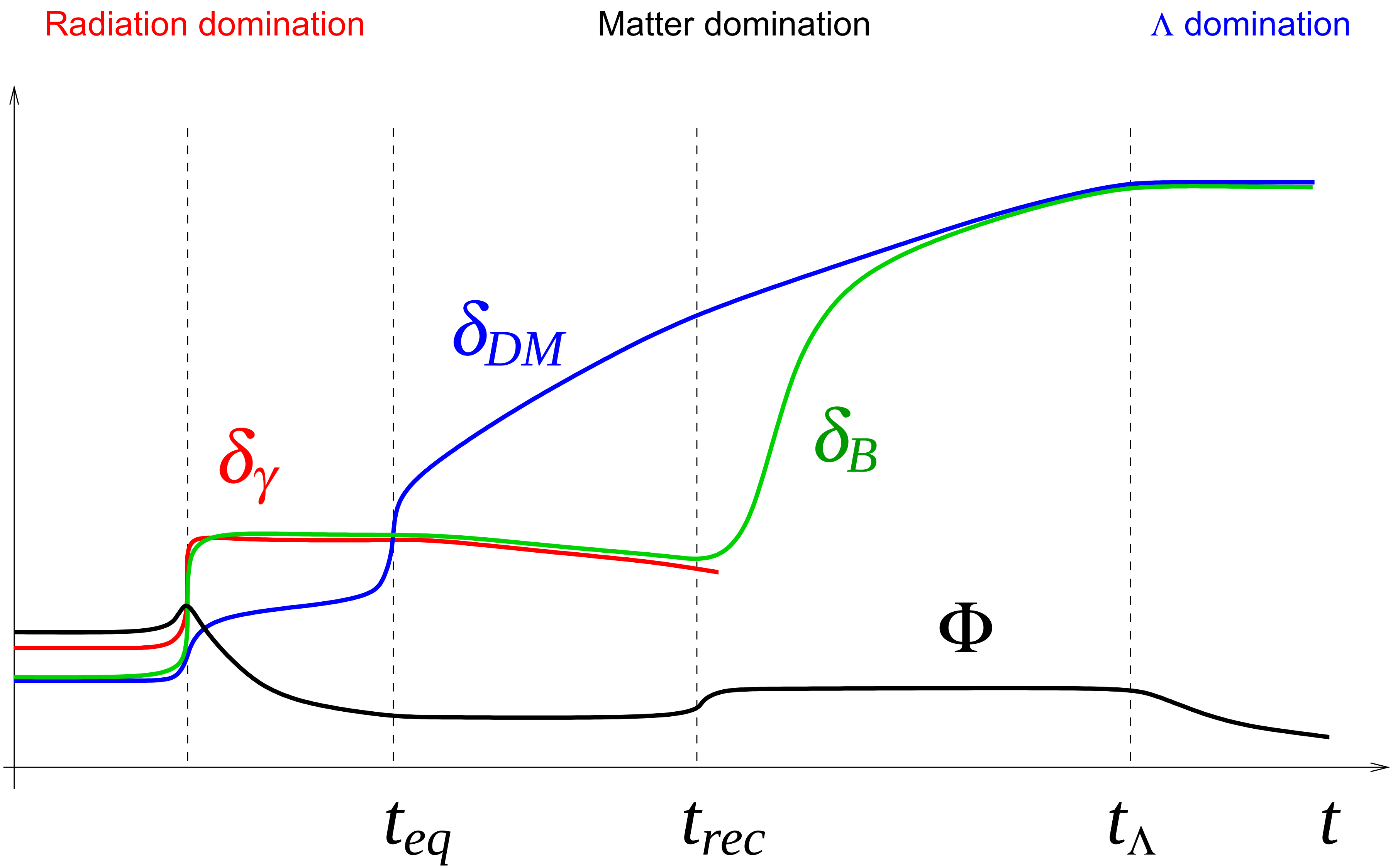
If not for dark matter,

$$\left(\frac{\delta\rho}{\rho}\right)_{today} = 1100 \times 10^{-4} \sim 0.1$$

No galaxies, no stars...

Perturbations in dark matter start to grow much earlier (already at radiation-dominated stage)

Growth of perturbations (linear regime)



NB: Need dark matter particles non-relativistic early on.

Neutrinos are not considerable part of dark matter
(way to set cosmological bound on neutrino mass,
$m_\nu < 0.2$ eV for every type of neutrino)

UNKNOWN DARK MATTER PARTICLES ARE
CRUCIAL FOR OUR EXISTENCE

If thermal relic:

Cold dark matter, CDM

$$m_{DM} \gtrsim 100 \text{ keV}$$

Warm dark matter

$$m_{DM} \simeq 1 - 30 \text{ keV}$$

WIMPs

Simple but very suggestive scenario

- Assume there is a new heavy stable particle X
  - Interacts with SM particles via pair annihilation (and crossing processes)

$$X + X \leftrightarrow q\bar{q}\ \text{, etc}$$

  - Parameters: mass M_X; annihilation cross section at non-relativistic velocity $\sigma(v)$
- Assume that maximum temperature in the Universe was high, $T \gtrsim M_X$
- Calculate present mass density (somewhat oversimplified)
  - Recall

$$H(T) = \frac{T^2}{M^*_{Pl}}$$

with $M^*_{Pl} = M_{Pl}/(1.66\sqrt{g_*}) \sim 10^{18}$ GeV at $T \sim 100$ GeV

- Number density of X-particles in equilibrium at $T < M_X$: Maxwell–Boltzmann

$$n_X = g_X \int \frac{d^3p}{(2\pi)^3} \mathrm{e}^{-\frac{\sqrt{M_X^2+p^2}}{T}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

- Mean free time wrt annihilation: travel distance $\tau_{ann} v$, meet one X particle to annihilate with in volume $\sigma \tau_{ann} v \Longrightarrow$

$$\sigma \tau_{ann} v n_X = 1 \quad \Longrightarrow \quad \tau_{ann} = \frac{1}{n_X \langle \sigma v \rangle}$$

- Freeze-out: $\tau_{ann}^{-1}(T_f) \sim H(T_f) \Longrightarrow n_X(T_f)\langle \sigma v \rangle \sim T_f^2 / M_{Pl}^* \Longrightarrow$

$$T_f \simeq \frac{M_X}{\ln(M_X M_{Pl}^* \langle \sigma v \rangle)}$$

NB: large log $\Longleftrightarrow T_f \sim M_X/20$

Define $\langle \sigma v \rangle \equiv \sigma_0$ (constant for s-wave annihilation)

- Number density at freeze-out

$$n_X(T_f) = \frac{T_f^2}{\sigma_0 M_{Pl}^*}$$

- Number-to-entropy ratio at freeze-out and later on

$$\frac{n_X(T_f)}{s(T_f)} = \# \frac{n_X(T_f)}{g_* T_f^3} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

where $\# = 45/(2\pi^2)$.

- Mass-to-enropy ratio

Confirmed by honest calculation, backup slides 3-5

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Most relevant parameter: annihilation cross section $\sigma_0 \equiv \langle \sigma v \rangle$ at freeze-out

$$\frac{M_X n_X}{s} = \# \frac{\ln(M_X M_{Pl}^* \sigma_0)}{\sigma_0 \sqrt{g_*(T_f)} M_{Pl}}$$

- Correct value, mass-to-entropy$= 3 \cdot 10^{-10}$ GeV, for

$$\sigma_0 \equiv \langle \sigma v \rangle = (1 \div 1.3) \cdot 10^{-36}\ \text{cm}^2 = (1 \div 1.3)\ \text{pb}$$

  - Weak scale cross section.

    Gravitational physics and EW scale physics combine into

$$\text{mass-to-entropy} \simeq \frac{1}{M_{Pl}} \left(\frac{\text{TeV}}{\alpha_W} \right)^2 \simeq 10^{-10}\ \text{GeV}$$

  - Mass M_X should not be much higher than 100 GeV

Weakly interacting massive particles, WIMPs.

Cold dark matter candidates, $T_f \sim M_X/20$, and in kinetic equilibrium long after.

WIMP candidates

SUSY: neutralinos, $X = \chi$

But situation is often tense: annihilation cross section is often too low

Important suppersion factor: $\langle \sigma v \rangle \propto v^2 \propto T/M_\chi$ because of p-wave annihilation in case $\chi\chi \to Z^* \to f\bar{f}$:

Relativistic $f\bar{f} \implies$ total angular momentum $J = 1$

$\chi\chi$: identical fermions $\implies L = 0$, parallel spins impossible $\implies$ p-wave

Many other possibilities

Example: Higgs portal

Just to have a WIMP, introduce scalar singlet S.

Renormalizable interaction with Higgs only. Impose symmetry $S \to -S \implies$ stable S .

$$L_S = \frac{1}{2}(\partial_\mu S)^2 - \left(\frac{\mu_s^2}{2} S^2 + \frac{\lambda_{SH}}{4} S^2 H^\dagger H + \frac{\lambda_S}{4} S^4 \right)$$

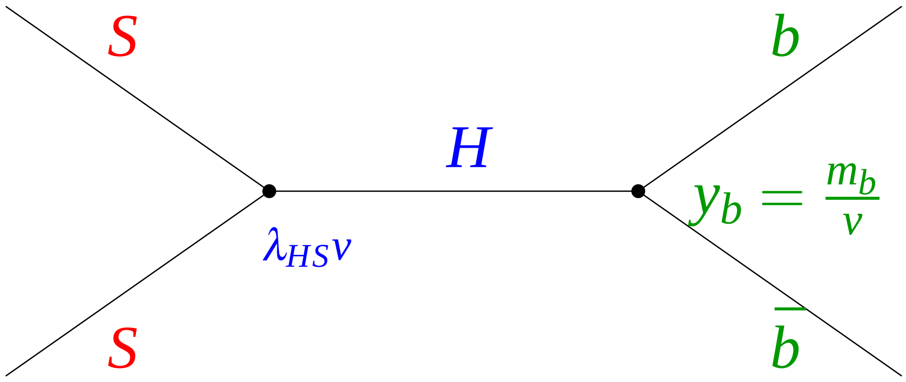
In vacuo $H = v/\sqrt{2} + h/\sqrt{2}$: vertices

$$\frac{\lambda_{SH}}{4} v h S^2 + \frac{\lambda_{SH}}{8} h^2 S^2$$

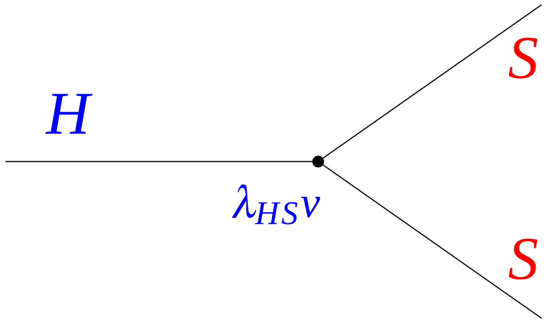
- Light S, $m_S \ll m_H/2$.

  Fairly popular a year ago

  No longer viable: main annihilation channel $SS \to b\bar{b}$. Cross section suppressed by $y_b^2 \propto m_b^2$



  $\langle \sigma v \rangle = 1$ pb $\Longrightarrow$ quite large λ_{SH} $\Longrightarrow$
  Invisible Higgs decay $H \to SS$ by far dominant.



Nice excersize:
calculate $\langle \sigma v \rangle$
and $\Gamma(H \to SS)$

- Degeneracy: m_s just below $m_H/2$.

  Pole enhanced $\langle \sigma v \rangle \Longrightarrow$ not so large $\lambda_{SH} \Longrightarrow$

  + threshold suppression of invisible Higgs decay $H \to SS \Longrightarrow$

  viable and interesting.

  Signature: invisible Higgs decay.

- Relatively heavy S: $m_s > m_W$.

  Main annihilation channels $SS \to WW, ZZ, HH$.

  Interesting for direct dark matter detection experiments and LHC at $m_S \lesssim 150$ GeV.

  Signature $pp \to H^* + \text{jet} \to \text{jet} + SS$;

  jet + missing E_T

Sumary of direct detection limits: backup slide 6

TeV SCALE PHYSICS MAY WELL BE RESPONSIBLE FOR GENERATION OF DARK MATTER

Is this guaranteed?

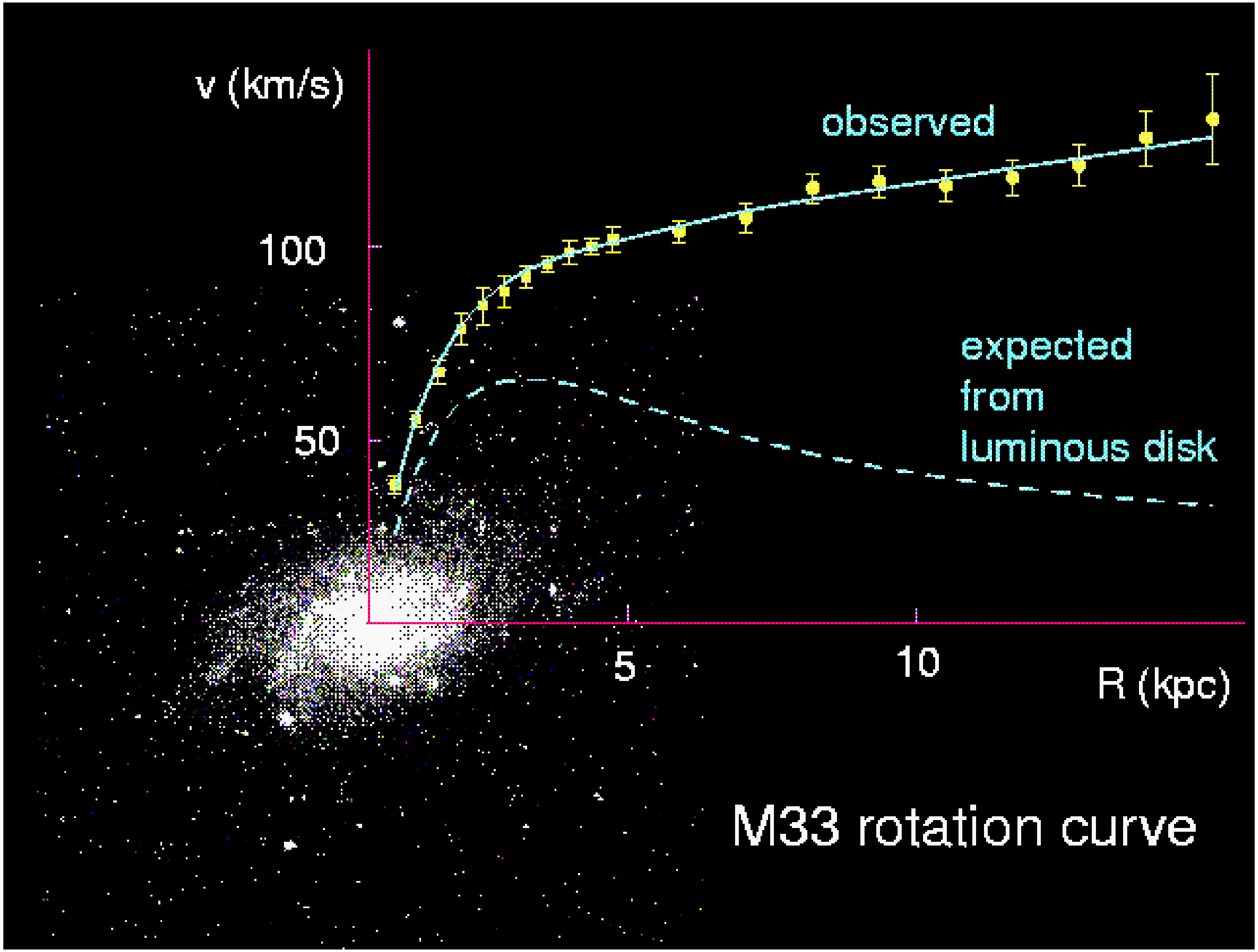
By no means. Another good DM candidate: axion.

Plus a lot of exotica...

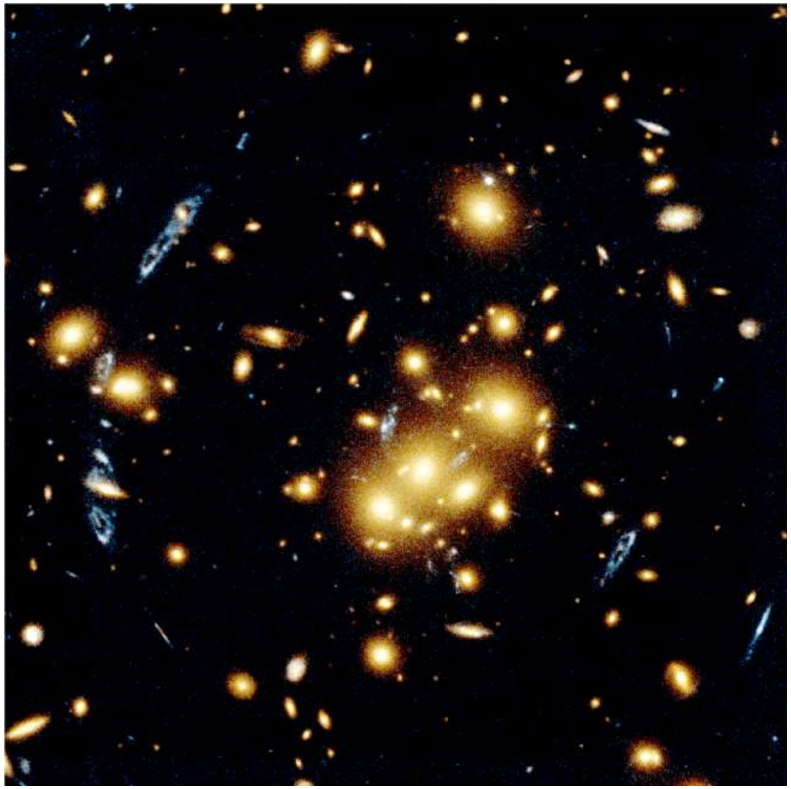
Crucial impact of LHC to cosmology,
direct and indirect dark matter searches

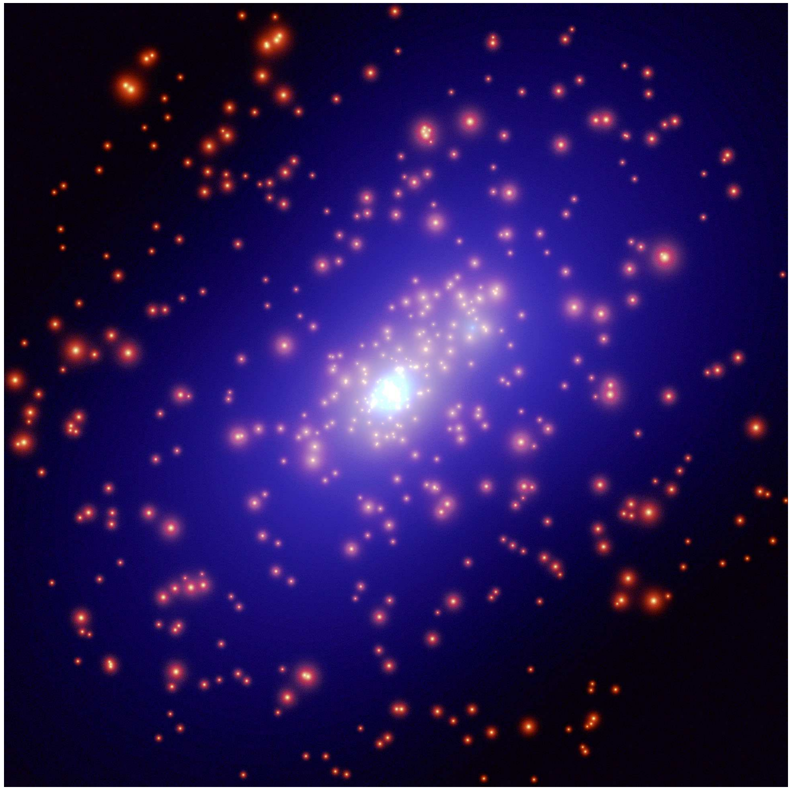
Backup slides

1. Rotation curves



2. Gravitational lensing





3. Honest calculation of WIMP abundance

Boltzmann equation for $\Delta = n_X/s$ (accounts for expansion):

$$\frac{d\Delta}{dt} = -\langle\sigma v\rangle s(\Delta^2 - \Delta_{eq}^2)$$

First term in r.h.s.: annihilation; second term: creation

Creation terminates when

$$\frac{d\ln n_{X,eq}}{dt} \simeq \langle\sigma v\rangle n_{X,eq}$$

Recall

$$n_{X,eq} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-\frac{M_X}{T}}$$

Then $|\frac{d\ln n_{eq}}{dt}| \simeq (m_X/T)H = m_X T/M_{Pl}^*$, so creation terminates at $T = T_*$ such that

$$n_{X,eq}(T_*) \simeq \frac{m_X T_*}{M_{Pl}^* \langle\sigma v\rangle}$$

4. This again gives (although exact result is slightly different from main lecture, e.g., by argument of log – not shown)

$$T_* \simeq \frac{M_X}{\ln(m_X M_{Pl} \langle \sigma v \rangle)}$$

Since t_*, WIMPs annihilate,

$$\frac{d\Delta}{dt} = -\langle \sigma v \rangle s \Delta^2$$

In terms of T this reads

$$\frac{d\Delta}{dT} = \frac{2\pi^2}{45} g_* M_{Pl}^* \langle \sigma v \rangle \Delta^2$$

This integrates to

$$\Delta^{-1}(T) - \Delta^{-1}(T_*) = \frac{2\pi^2}{45} g_* M_{Pl}^* \langle \sigma v \rangle (T_* - T)$$

At late times (low temperatures) we finally have

$$\Delta = \# \frac{1}{T_* \sigma_0 g_* M_{Pl}^*} = \# \frac{\ln(m_X M_{Pl}^* \sigma_0)}{M_X \sigma_0 g_* M_{Pl}^*}$$

which is precisely the same result as in main lecture, except for precise argument of log (not shown).

Note that this value of Δ is much smaller than

$$\Delta(T_*) \simeq \frac{1}{s(T_*)} \frac{m_X T_*}{M_{Pl}^* \langle \sigma v \rangle} = \# \frac{m_X}{T_*} \frac{1}{T_* \sigma_0 g_* M_{Pl}^*}$$

So, it indeed makes sense to talk about two-stage process: termination of creation and then wash-out by annihilation.

6.Direct detection