

Standard Model

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44. Herbstschule für Hochenergiephysik, Maria Laach
04. - 14. September 2012

- Lecture 1: Formulation of the Standard Model
- Lecture 2: Precision tests of the Standard Model
- Lecture 3: NLO Calculations for the LHC
- Lecture 4: Higgs physics at the LHC

Precision tests of the Standard Model

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- Relevance of radiative corrections
- Evaluation of radiative corrections
- Radiative corrections to muon decay
- Z physics at LEP and SLC
- Precision tests of the Standard Model

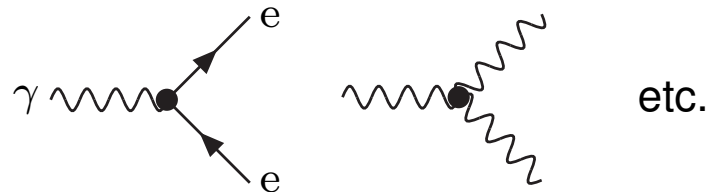
Relevance of radiative corrections

Standard Model is defined via its Lagrangian density
 predictions are obtained within perturbation theory
 Lagrangian is equivalent to **Feynman rules**

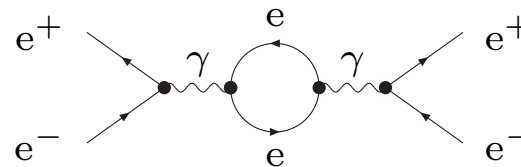
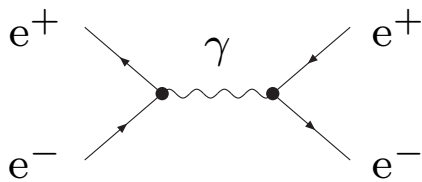
propagators for free fields



vertices for interactions



Feynman diagrams provide an exact formulation of perturbation theory
 leading order (LO): **tree diagrams**
 next-to-leading order (NLO): **loop diagrams**



matrix element = $\mathcal{M}_{if} = \langle f | \mathcal{M} | i \rangle = \sum$ all Feynman diagrams for $|i\rangle \rightarrow |f\rangle$
 perturbative series = series in coupling constant e or g_s or both
 = expansion in # of loops of Feynman diagrams
 = expansion in powers of \hbar (quantum corrections)

e^+e^- collider 1989–2000

events

$$1.5 \times 10^7 \quad e^+e^- \rightarrow Z \rightarrow \text{hadrons}, \quad 1.7 \times 10^6 \quad e^+e^- \rightarrow Z \rightarrow \text{leptons}$$

$$4.8 \times 10^4 \quad e^+e^- \rightarrow W^+W^-$$

typical accuracies $\lesssim 0.1\%$

$$e^+e^- \rightarrow Z \rightarrow \text{hadrons}: \Delta\sigma/\sigma \sim 0.09\%$$

$$\Delta \sin^2 \theta_{\text{eff}}^{\text{lept}} \sim 0.03\%$$

$$e^+e^- \rightarrow W^+W^-: \Delta\sigma/\sigma \sim 1\%$$

$$\Delta M_W/M_W \sim 0.05\%$$

predictions are calculated within perturbation theory

$$\text{expansion parameter: } \alpha/\pi \sim 0.23\%, \quad \alpha/(\pi \sin^2 \theta_w) \sim 1\%$$

$$\text{generic one-loop corrections: } \sim 2\%$$

$$\text{generic two-loop corrections: } \sim 0.05\%$$

implications

complete one-loop electroweak radiative corrections (EWRC) mandatory for $e^+e^- \rightarrow Z$ (at least leading) two-loop corrections needed

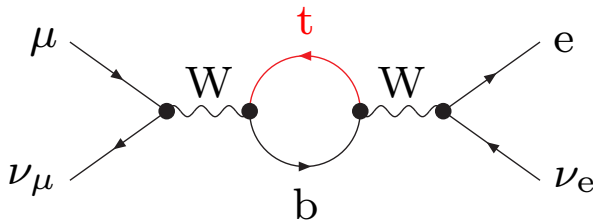
Order of magnitude

$$\sim \left[\frac{\alpha}{\pi} \dots \frac{\alpha}{\pi} \ln \frac{E^2}{m_e^2} \right] \times \mathcal{O}(1) \sim [0.2\% \dots 6\%] \times \mathcal{O}(1) \text{ for } E = 100 \text{ GeV}$$

⇒ **mandatory for precision tests of the electroweak (EW) Standard Model (SM)**

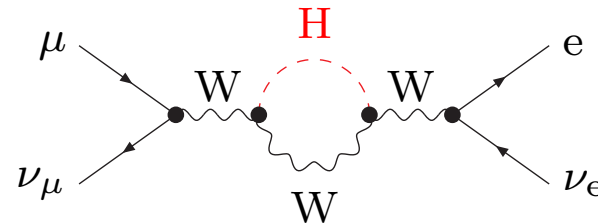
depend on all details of the theory

- **top quark**



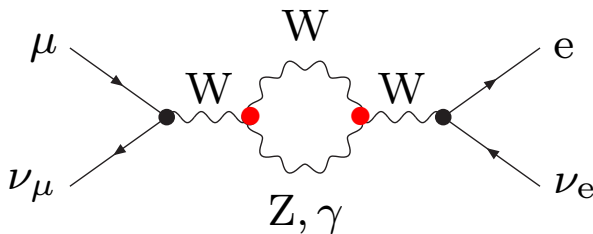
$$\sim \frac{\alpha}{\pi} \frac{m_t^2}{s_w^2 M_W^2} \approx 0.05$$

- **Higgs boson**

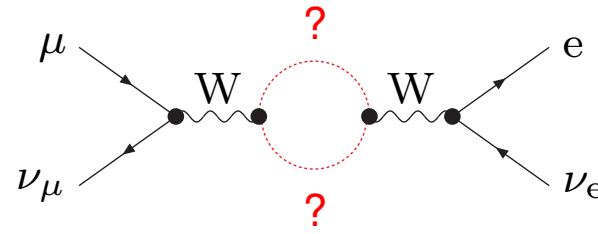


$$\sim \frac{\alpha}{\pi} \ln \frac{M_H}{M_W}$$

- **gauge-boson self couplings**

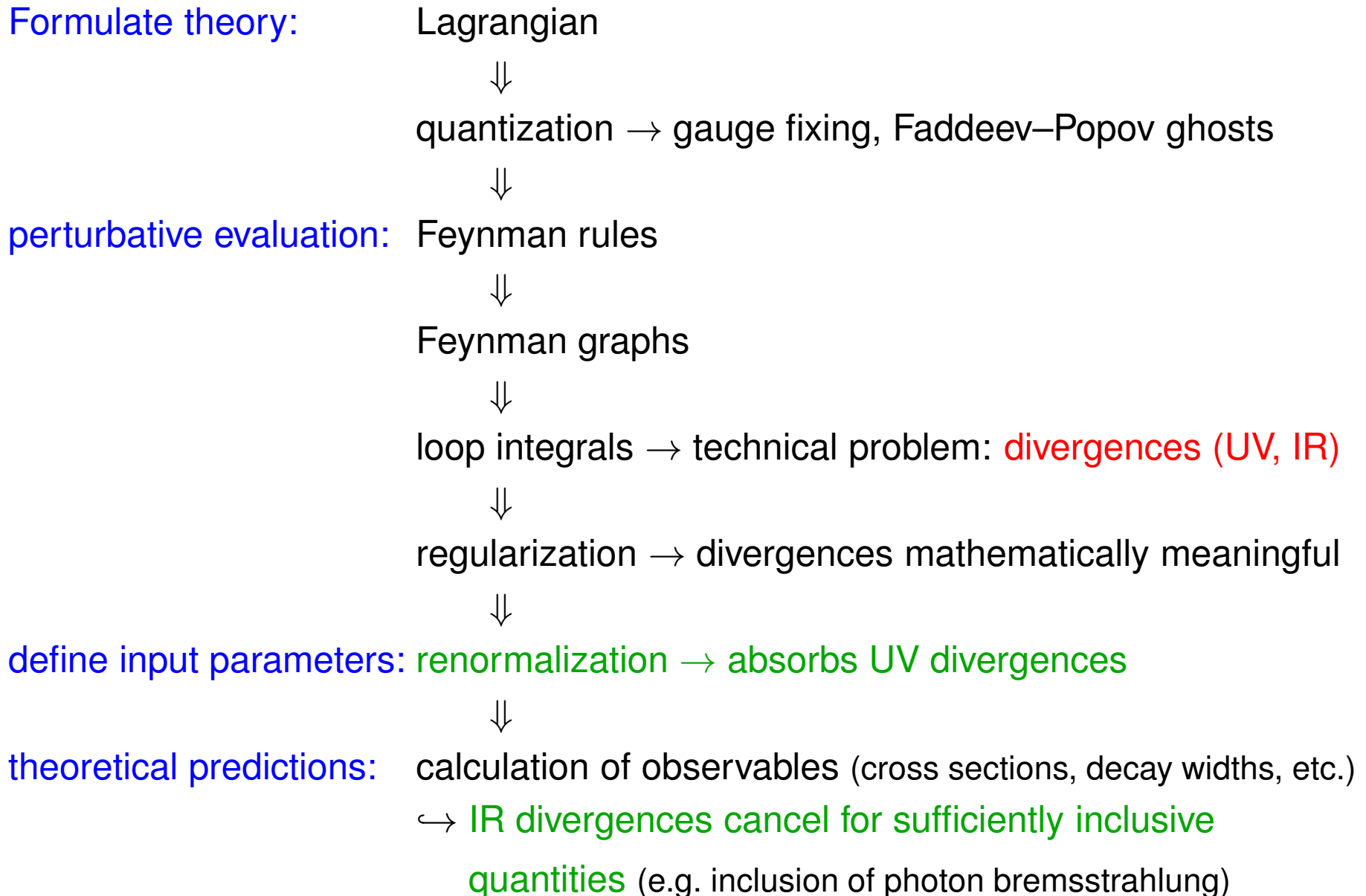


- **New physics (supersymmetry) ?**



⇒ **allow for indirect experimental tests of not directly accessible quantities**

Evaluation of radiative corrections



Observation: **loop integrals involve divergences**

- **ultraviolet (UV) divergences** for $q \rightarrow \infty$ (large momenta), e.g.:

$$\int d^4q \frac{1}{(q^2 - m_0^2)((q+p)^2 - m_1^2)} \sim \int \frac{dq}{q} \text{ for } q \rightarrow \infty \rightarrow \text{logarithmic divergence}$$

- **infrared (IR) divergences** for $q \rightarrow q_0$ (small/collinear momenta), e.g.:

$$\int d^4q \frac{1}{q^2(q^2 + 2qp_1)(q^2 + 2qp_2)} \sim \int \frac{dq}{q} \text{ for } q \rightarrow 0 \rightarrow \text{logarithmic divergence}$$

regularization: extension of theory by free parameter δ (cut-off) such that

- original theory is obtained as limiting case $\delta \rightarrow \delta_0$
- integrals (and thus the theory) become finite, i.e. well defined for $\delta \neq \delta_0$
 - \hookrightarrow fix input parameters x_i of regularized theory ($\delta \neq \delta_0$) by experiment
 - \Rightarrow observables must have finite limit $\delta \rightarrow \delta_0$ as functions of x_i
(independent of regularization scheme)

relations between physical quantities should be finite and indep. of cut-off

if true: **theory is renormalizable**

Cut-off regularization: require $q_0^2 + \mathbf{q}^2 < \Lambda^2$ in momentum space

- UV divergences appear as $\log \Lambda^2, \Lambda^2, \dots$
- **breaks Lorentz invariance and gauge invariance** \Rightarrow not used

dimensional regularization: switch to $D \neq 4$ space-time dimensions

- regularizes UV and IR divergences, respects gauge invariance, easy use
- prescription: (μ = arbitrary reference mass, drops out in observables)

$$\int d^4q \rightarrow (2\pi\mu)^{4-D} \int d^Dq \quad \text{and } D\text{-dim. momenta, metric, Dirac algebra}$$

and analytic continuation to continuous complex D !

- divergences appear as poles $\frac{1}{4-D}$
- \hookrightarrow define $\Delta \equiv \frac{2}{4-D} - \gamma_E + \ln(4\pi) = \frac{2}{4-D} + \text{const.}$

IR regularization by infinitesimal photon mass m_γ

and (if relevant) by small fermion mass m_f

- prescription: photon propagator pole $\frac{1}{q^2} \rightarrow \frac{1}{q^2 - m_\gamma^2}$
- divergences appear as $\ln(m_\gamma)$ and $\ln(m_f)$ terms

Propagators and 2-point functions:

structure of one-loop self-energies (scalar case as example):

$$p \rightarrow \text{---} \bullet \text{---} = \Sigma(p^2) = C_1 p^2 \Delta + C_2 \Delta + \Sigma_{\text{finite}}(p^2) = \text{UV divergent}$$

behaviour of propagator near pole for free propagation:

$$G^{\phi\phi}(p^2) = \frac{i}{p^2 - m^2 + \Sigma(p^2)} \overset{p^2 \rightarrow m^2}{\sim} \frac{1}{1 + \Sigma'(m^2)} \frac{i}{p^2 - m^2 + \Sigma(m^2)}$$

↔ higher-order corrections change location and residue of propagator pole !

interaction vertices:

example: scalar 4-point interaction $\mathcal{L}_{\phi^4} = -\lambda\phi^4/4!$

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = -i\lambda + i\Lambda(p_1, p_2, p_3)$$

momentum-dependent one-loop correction:

$$\Lambda(p_1, p_2, p_3) = C_3 \Delta + \Lambda_{\text{finite}}(p_1, p_2, p_3) = \text{UV divergent}$$

↔ higher-order corrections change coupling strength !

- **Renormalizable field theories:**

UV divergences in vertex functions have analytical form of elementary vertex structures (directly related to \mathcal{L})

↔ idea: absorb divergences in free parameters

⇒ reparametrization of theory = renormalization

different types of renormalizable theories:

- ▶ theories with unrelated couplings of non-negative mass dimensions

↔ renormalizability proven by power counting and “BPHZ procedure”

- ▶ **gauge theories** (couplings unified by gauge invariance)

↔ renormalizability non-trivial consequence of gauge symmetry ‘t Hooft ’71

- **non-renormalizable field theories:**

e.g. theories with couplings of negative mass dimensions

(cf. Fermi model, effective field theories)

operators of higher and higher mass dimensions needed to absorb UV divergences in higher orders

↔ **infinitely many free parameters**, much less predictive power

Consider original (“bare”) parameters and fields as preliminary (denoted with subscripts “0” in the following)

↔ introduce new “renormalized” parameters and fields that obey certain conditions

propagators and 2-point functions:

- **mass renormalization:** $m_0^2 = m^2 + \delta m^2$,
 $m^2 \stackrel{!}{=} \text{location of propagator pole} = \text{“physical mass”} \rightarrow \delta m^2 = \Sigma(m^2)$

- **field renormalization:** rescale fields $\phi_0 = \sqrt{Z_\phi} \phi$, $G^{\phi\phi} = Z_\phi^{-1} G^{\phi_0\phi_0}$
 fix $Z_\phi = 1 + \delta Z_\phi$ such that **residue of $G^{\phi\phi}$ at $p^2 = m^2$ equals 1**
 ↔ $\delta Z_\phi = -\Sigma'(m^2)$

⇒ **renormalized propagator $G^{\phi\phi}$ is UV finite:**

$$G^{\phi\phi}(p^2) = \frac{i}{p^2 - m^2 + \Sigma_{\text{ren}}(p^2)},$$

renormalized self energy

$$\begin{aligned} \Sigma_{\text{ren}}(p^2) &= \Sigma(p^2) - \delta m^2 + (p^2 - m^2)\delta Z_\phi = \Sigma(p^2) - \Sigma(m^2) - (p^2 - m^2)\Sigma'(m^2) \\ &= \Sigma_{\text{finite}}(p^2) - \Sigma_{\text{finite}}(m^2) - (p^2 - m^2)\Sigma'_{\text{finite}}(m^2) = \text{UV finite} \end{aligned}$$

Vertex functions for interactions:

- **coupling-constant renormalization:** $\lambda_0 = \lambda + \delta\lambda$

fix $\delta\lambda$ such that λ assumes a measured value for special kinematics p_i^{exp}

(renormalization point)

note: $\Gamma^{\phi\phi\phi\phi} = Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$

$$\hookrightarrow \delta\lambda = -2\delta Z_\phi \lambda - \Lambda(p_1^{\text{exp}}, p_2^{\text{exp}}, p_3^{\text{exp}})$$

⇒ **renormalized vertex function is UV finite:**

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = -i\lambda + i\Lambda_{\text{ren}}(p_1, p_2, p_3),$$

$$\begin{aligned} \Lambda_{\text{ren}}(p_1, p_2, p_3) &= \Lambda(p_1, p_2, p_3) - \delta\lambda - 2\delta Z_\phi \lambda \\ &= \Lambda_{\text{finite}}(p_1, p_2, p_3) - \Lambda_{\text{finite}}(p_1^{\text{exp}}, p_2^{\text{exp}}, p_3^{\text{exp}}) = \text{UV finite} \end{aligned}$$

↔ **all divergences of ϕ^4 theory can be absorbed by renormalization of parameters (masses and coupling constants) and the fields ($\delta m, \delta\lambda, \delta Z_\phi$)**

Any physical observable calculated in terms of renormalized parameters is finite and well-defined (although affected by a systematic perturbative uncertainty).

Bare input parameters: $e_0, M_{W,0}, M_{Z,0}, M_{H,0}, m_{f,0}, V_{ij,0}$

renormalization transformation:

- parameter renormalization:

$$e_0 = (1 + \delta Z_e)e,$$

$$M_{W,0}^2 = M_W^2 + \delta M_W^2, \quad M_{Z,0}^2 = M_Z^2 + \delta M_Z^2, \quad M_{H,0}^2 = M_H^2 + \delta M_H^2,$$

$$m_{f,0} = m_f + \delta m_f, \quad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad (\text{both } V_{ij,0}, V_{ij} \text{ unitary})$$

note: renormalization of c_w, s_w fixed by on-shell condition $c_w = M_W/M_Z$
(s_w is not a free parameter if M_W, M_Z are used as input parameters)

- field renormalization: (physical fields)

$$W_0^\pm = \sqrt{Z_W} W^\pm, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad H_0 = \sqrt{Z_H} H,$$

$$\psi_{f,0}^L = \sqrt{Z_{ff'}^L} \psi_{f'}^L, \quad \psi_{f,0}^R = \sqrt{Z_{ff'}^R} \psi_{f'}^R,$$

note: matrix renormalization necessary to account for loop-induced mixing

Bare input parameters: $e_0, M_{W,0}, M_{Z,0}, M_{H,0}, m_{f,0}, V_{ij,0}, g_s$

renormalization transformation:

- parameter renormalization:

$$e_0 = (1 + \delta Z_e)e, \quad g_{s,0} = (1 + \delta Z_{g_s})g_s,$$

$$M_{W,0}^2 = M_W^2 + \delta M_W^2, \quad M_{Z,0}^2 = M_Z^2 + \delta M_Z^2, \quad M_{H,0}^2 = M_H^2 + \delta M_H^2,$$

$$m_{f,0} = m_f + \delta m_f, \quad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad (\text{both } V_{ij,0}, V_{ij} \text{ unitary})$$

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- field renormalization: (physical fields)

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$$\psi_{f,0}^L = \sqrt{Z_{ff'}^L} \psi_{f'}^L, \quad \psi_{f,0}^R = \sqrt{Z_{ff'}^R} \psi_{f'}^R, \quad A_0^a = \sqrt{Z_A} A^a$$

note: matrix renormalization necessary to account for loop-induced mixing

- **Mass renormalization:**

on-shell definition: mass² is location of pole in propagator

$$\hookrightarrow \delta M_W^2 = \text{Re}\{\Sigma_T^W(M_W^2)\}, \quad \text{similar expressions for } \delta M_Z^2, \delta M_H^2, \delta m_f$$

note: \diamond location of pole is complex for unstable particles

\hookrightarrow important for processes with unstable-particle production
(gauge-invariant definition: mass² as real part of pole location)

\diamond other definitions of quark masses often more appropriate
(e.g. running masses)

- **field renormalization:** (bosons and leptons)

- ▶ **residues of propagators** (diagonal, transverse parts) **normalized to 1**

$$\hookrightarrow \delta Z_W = -\text{Re}\{\Sigma_T^W'(M_W^2)\},$$

similar expressions for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{L/R}$

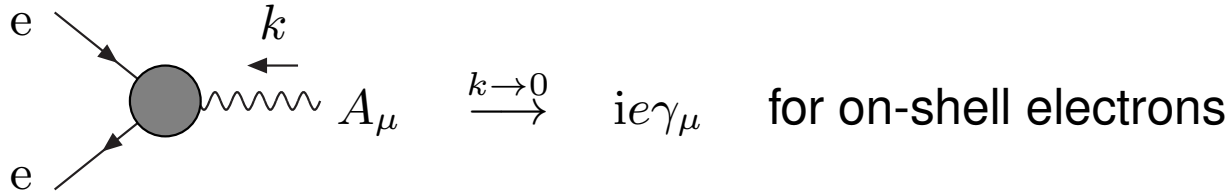
- ▶ **suppression of mixing propagators on particle poles**

physical on-shell particles do not mix

$$\hookrightarrow \text{fixes non-diagonal constants } \delta Z_{AZ}, \delta Z_{ZA}, \delta Z_{ff'}^{L/R} \quad (f \neq f')$$

note: problems for unstable particles beyond one loop
(field-renormalization constants become complex)

- Charge renormalization: define e in Thomson limit (low-energy-elastic scattering)



$\hookrightarrow e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \frac{e^2}{4\pi} = 1/137.03599976$

gauge invariance relates δZ_e to photon field renormalization:

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_w}{2c_w}\delta Z_{ZA} \quad (\text{at one loop})$$

- CKM-matrix renormalization \rightarrow fixes δV_{ij}

rotation to mass eigenstates;

CKM part requires a careful (non-trivial) investigation of mixing energies, mass eigenstates, LSZ reduction, etc.

general result: all renormalization constants can be obtained from self-energies.

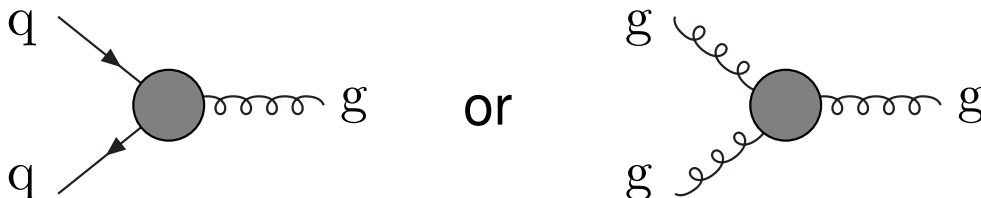
different renormalization schemes are possible

- QCD becomes strongly interacting for small momentum transfers
 - quarks and gluons are confined in hadrons
- ↪ on-shell renormalization makes no sense
- ↪ **no obvious renormalization point for strong coupling**

renormalization schemes for QCD

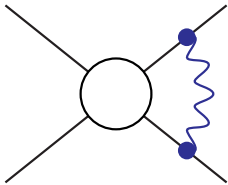
- **momentum subtraction scheme (MOM)**: choose symmetric Euclidean renormalization point, $p_i^2 = -M^2$, for vertex functions
- **minimal subtraction scheme (MS)**: subtract only UV divergences $1/(D - 4)$
- **modified minimal subtraction scheme ($\overline{\text{MS}}$)**: **commonly used**
subtract Δ (UV-divergences plus some universal constants)
renormalized results depend on scale μ of dimensional regularization
 \Rightarrow **renormalization-scale dependence of observables**

renormalization constant δg_s can be determined from one vertex function



Consider processes with charged external particles, e.g., $e^+e^- \rightarrow \mu^+\mu^-$

- **virtual corrections:** loop diagrams

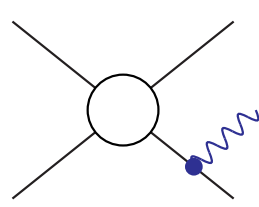


IR divergences from soft virtual photons ($q \rightarrow 0$)

$$\int \frac{d^4q}{(q^2 - m_\gamma^2)(q^2 + 2qp_1)(q^2 + 2qp_2)} \rightarrow C \ln(m_\gamma)$$

- **“real” corrections:** photon bremsstrahlung = **“real” radiative corrections**

$$\int \frac{d^3\mathbf{q}}{2q_0} \left| \text{diagram} \right|^2$$



IR divergences from soft real photons ($\mathbf{q} \rightarrow 0$)

$$\int \frac{d^3\mathbf{q}}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \rightarrow -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

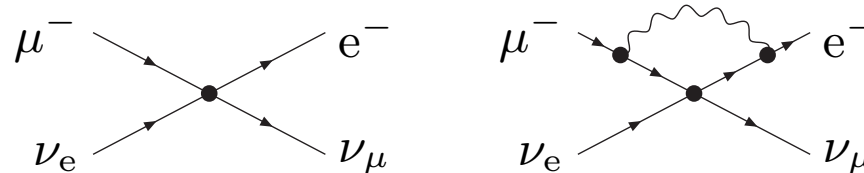
↪ virtual and soft-photon corrections cannot be discussed separately
are related due to limited experimental resolution of soft photons

↪ cross-section predictions necessarily depend on treatment of photon emission (energy and angular cuts)

Radiative corrections to muon decay

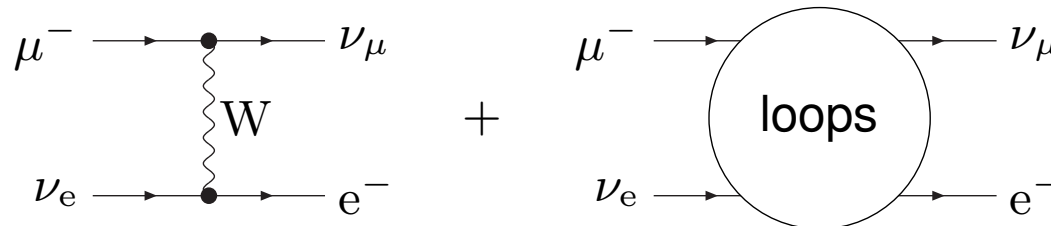
M_W and M_Z correlated via muon lifetime \leftrightarrow Fermi constant G_μ

Fermi model



$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} \left(1 - 8 \frac{m_e^2}{m_\mu^2} \right) (1 + \delta_{\text{QED}})$$

Standard Model

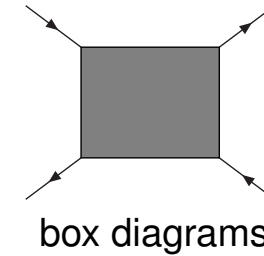
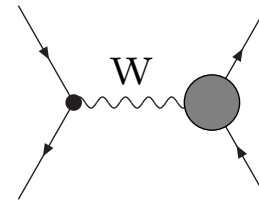
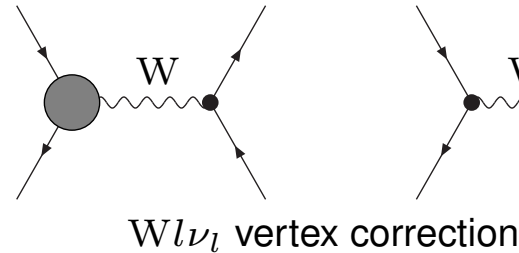
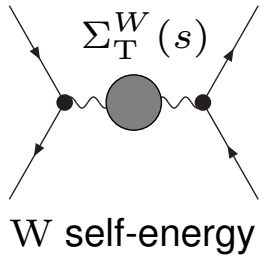


$$G_\mu = \frac{\pi\alpha}{\sqrt{2}} \frac{1}{M_W^2 (1 - M_W^2/M_Z^2)} \frac{1}{1 - \Delta r}$$

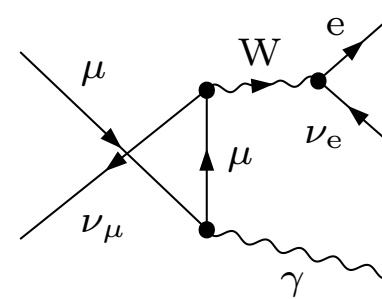
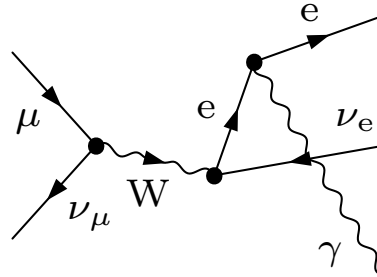
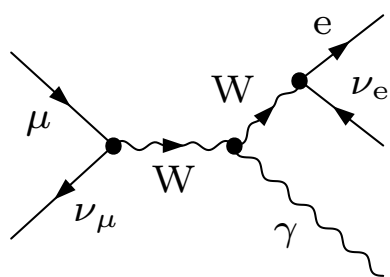
Δr : calculable quantum corrections

(beyond electromagnetic corrections to Fermi model)

Virtual correction – one-loop diagrams:

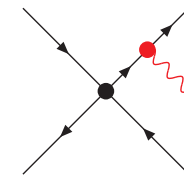
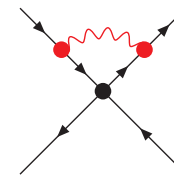


real correction – one-photon bremsstrahlung:



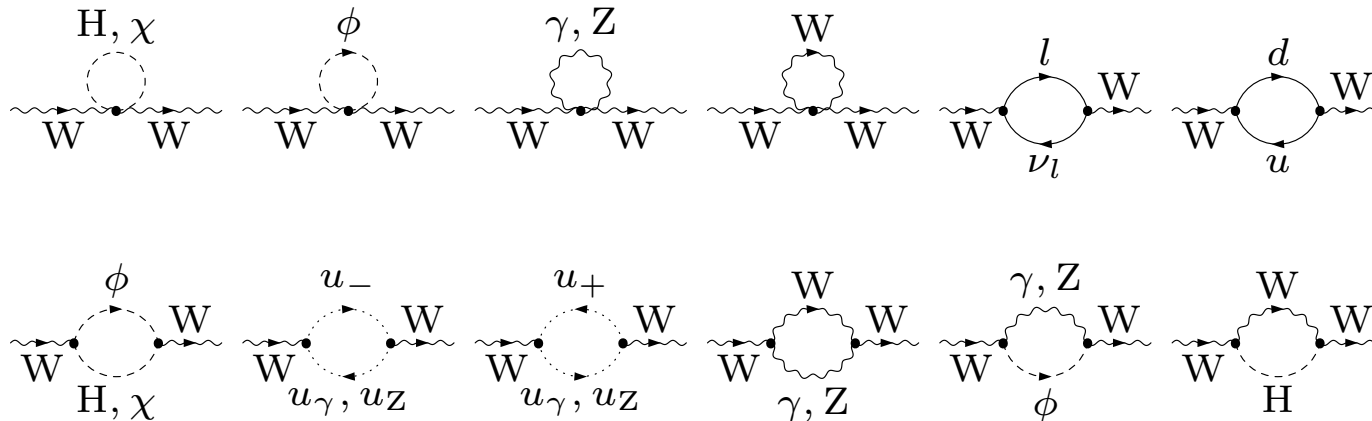
consistent use of G_μ :

photonic QED corrections are treated in the Fermi model and subtracted from Δr

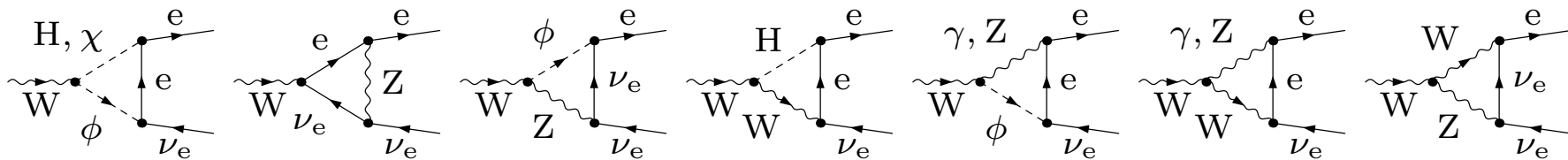


etc.

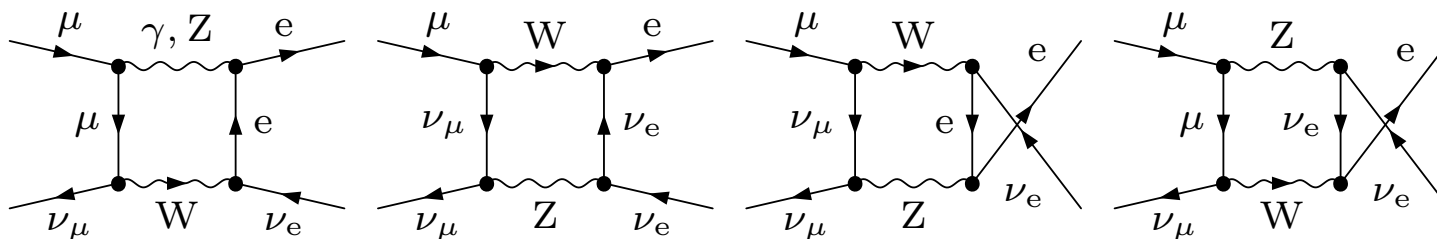
W-boson self-energy:



$W e \nu_e$ vertex correction:



box diagrams:

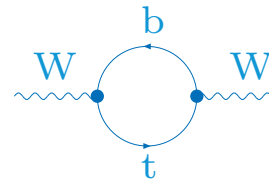
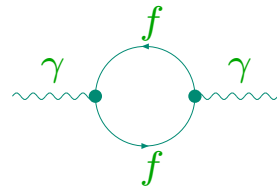


$\mathcal{O}(\alpha)$ corrections:

Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{1\text{-loop}} = \underbrace{\Delta\alpha(M_Z^2)}_{\sim 6\%} - \underbrace{\frac{c_w^2}{s_w^2} \Delta\rho}_{\sim 3\%} + \underbrace{\Delta r_{\text{rem}}(M_H)}_{\sim 1\%}$$

$$\underbrace{\alpha \ln(m_f/M_Z)}_{\sim 6\%} \quad \underbrace{G_\mu m_t^2}_{\sim 3\%} \quad \underbrace{\alpha \ln(M_H/M_Z)}_{\sim 1\%}$$



- $\Delta\alpha(M_Z^2)$: contribution of running electromagnetic coupling

$$\Delta\alpha(M_Z^2) \sim \frac{\alpha}{3\pi} \sum_f Q_f^2 \ln \frac{M_Z^2}{m_f^2}$$

↔ large effects from small fermion masses

- $\Delta\rho$: leading corrections to the ρ -parameter

$$\Delta\rho_{\text{top}} \sim \left(\frac{\Sigma_{\text{T}}^{\text{ZZ}}(0)}{M_Z^2} - \frac{\Sigma_{\text{T}}^{\text{WW}}(0)}{M_W^2} \right)_{\text{top}} \sim \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2}$$

↔ large effects from top–bottom loops in W self-energy

Veltman '77

$$\begin{aligned}\Delta\alpha(s) &= -\operatorname{Re}\{\Sigma_{\text{T,ren}}^{AA}(s)/s\} = -\operatorname{Re}\{\Sigma_{\text{T}}^{AA}(s)/s\} + (\Sigma_{\text{T}}^{AA})'(0) \\ &= \Delta\alpha_{\text{lept}}(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s) \sim \frac{\alpha}{3\pi} \sum_f Q_f^2 \ln \frac{M_Z^2}{m_f^2}\end{aligned}$$

$\Delta\alpha_{\text{had}}^{(5)}$ becomes sensitive to unphysical quark masses m_q

$\hookrightarrow \Delta\alpha_{\text{had}}^{(5)}$ not calculable in perturbation theory

solution: $\Delta\alpha_{\text{had}}^{(5)}$ obtainable from fit to experimental data with subtracted dispersion relation

$$\begin{aligned}\Delta\alpha_{\text{had}}^{(5)} &= -\frac{\alpha}{3\pi} M_Z^2 \operatorname{Re} \left\{ \int_{4m_\pi^2}^{\infty} ds' \frac{R(s')}{s'(s' - M_Z^2 - i\varepsilon)} \right\} \\ R(s) &= \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)}\end{aligned}$$

$R(s)$ is taken from perturbative QCD for high energies ($\sqrt{s} \gtrsim 13 \text{ GeV}$)

$$\Rightarrow \Delta\alpha_{\text{had}}^{(5)} = 0.027498 \pm 0.000135$$

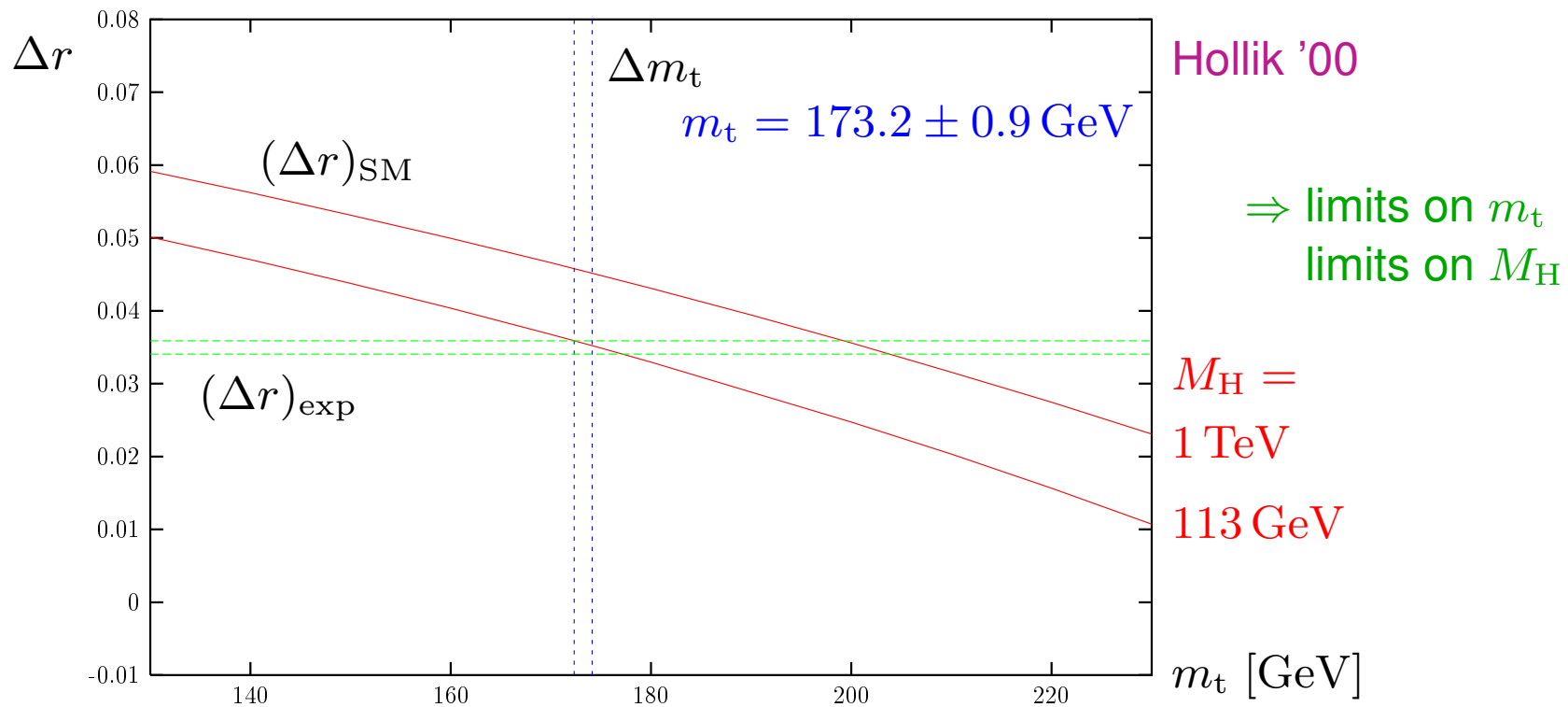
Jegerlehner; Burkhardt, Pietrzyk; Eidelman; Davier, Höcker,...

$$1 - \Delta r = \frac{\pi\alpha}{\sqrt{2}G_\mu} \frac{1}{M_W^2(1 - M_W^2/M_Z^2)}$$

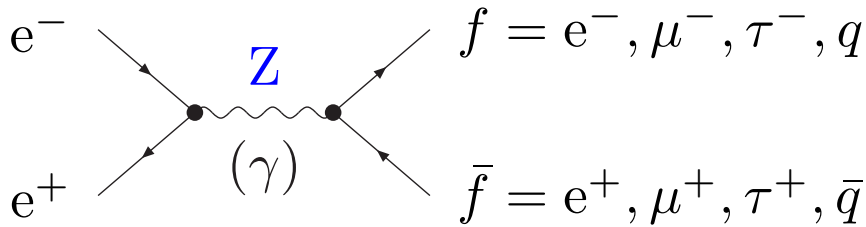
⇒ experimental determination of Δr $\Delta r = 0.0350 \pm 0.0009$

$\Delta r \neq 0$ with 39σ ⇒ confirmation of quantum corrections

$\Delta r \neq \Delta\alpha \sim 0.0594$ with 27σ ⇒ confirmation of weak corrections



Z physics at LEP and SLC

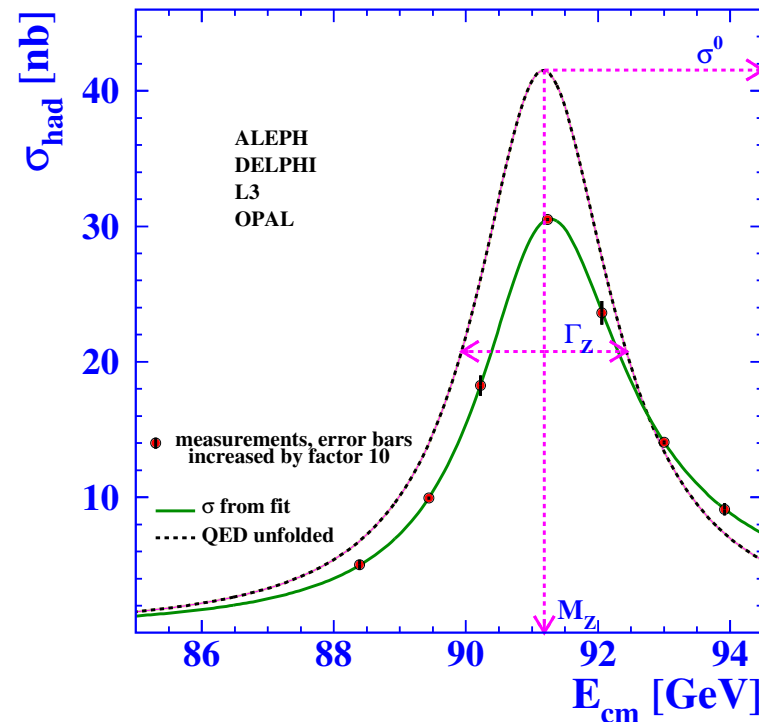


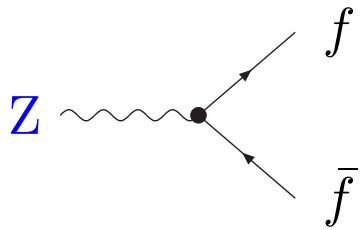
LEP1: $\sim 16 \times 10^7$ events
(1989–1995)

unfolded resonance cross-section $s = E_{\text{CMS}}^2$

$$\sigma_f(s) = 12\pi \frac{s}{M_Z^2} \frac{\Gamma(Z \rightarrow e^+e^-) \Gamma(Z \rightarrow f\bar{f})}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z^2} = \sigma^0 \frac{s \Gamma_Z^2}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z^2}$$

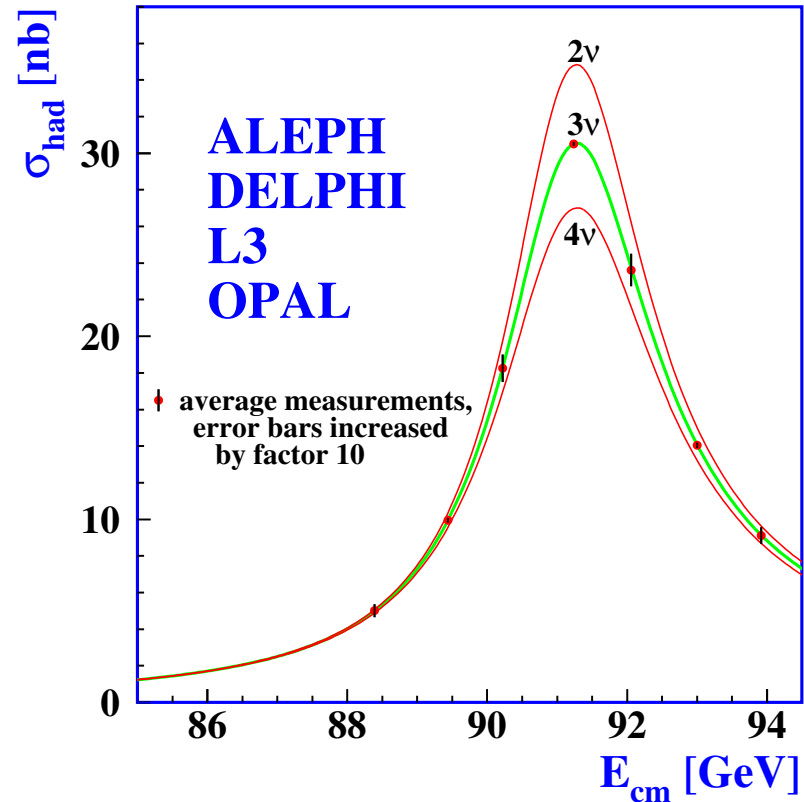
- line shape $\Rightarrow M_Z, \Gamma_Z$
- peak cross section σ^0
 $\Rightarrow \Gamma(Z \rightarrow l^+l^-) / \Gamma_Z,$
 $\Gamma(Z \rightarrow \text{hadrons}) / \Gamma_Z$
- angular distributions,
polarization asymmetries
 \Rightarrow effective $Zf\bar{f}$ vector and
axial-vector couplings g_{Vf}, g_{Af}





$$\Gamma_Z = \underbrace{\Gamma(e^-e^+, \mu^-\mu^+, \tau^-\tau^+)}_{\text{leptonic}} + \underbrace{\sum_q \Gamma(q\bar{q})}_{\text{hadronic}} + \underbrace{N_\nu \Gamma(\nu\bar{\nu})}_{\text{invisible}}$$

$$= \Gamma_{\text{had}} + \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{inv}}$$



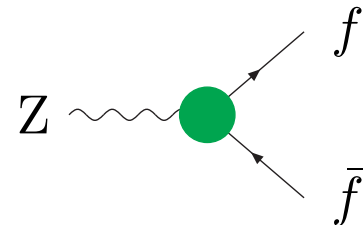
- Γ_Z measured from Z line shape

- Γ_{had} and $\Gamma_{l=e,\mu,\tau}$ from $R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l}$ and $\sigma_{\text{had}}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}$

Fit of Γ_Z , R_l , and σ_{had}^0 yields invisible Z-decay width: $\Gamma_{\text{inv}} = N_\nu \Gamma_{Z \rightarrow \nu\bar{\nu}}^{\text{theory}}$

$\hookrightarrow N_\nu = 2.9840 \pm 0.0082$

Z physics tests predominantly **effective** Z-boson–fermion couplings (**on-shell**)



$$= i \frac{e}{2s_w c_w} \gamma_\mu (g_{v_f} - g_{a_f} \gamma_5)$$

effective couplings contain weak corrections to **on-shell** $Z f \bar{f}$ vertex
Z-boson width

$$\Gamma(Z \rightarrow f \bar{f}) = \frac{G_\mu M_Z^3}{6\pi\sqrt{2}} ((g_{v_f})^2 + (g_{a_f})^2) (1 + \delta_f^{\text{QED}})$$

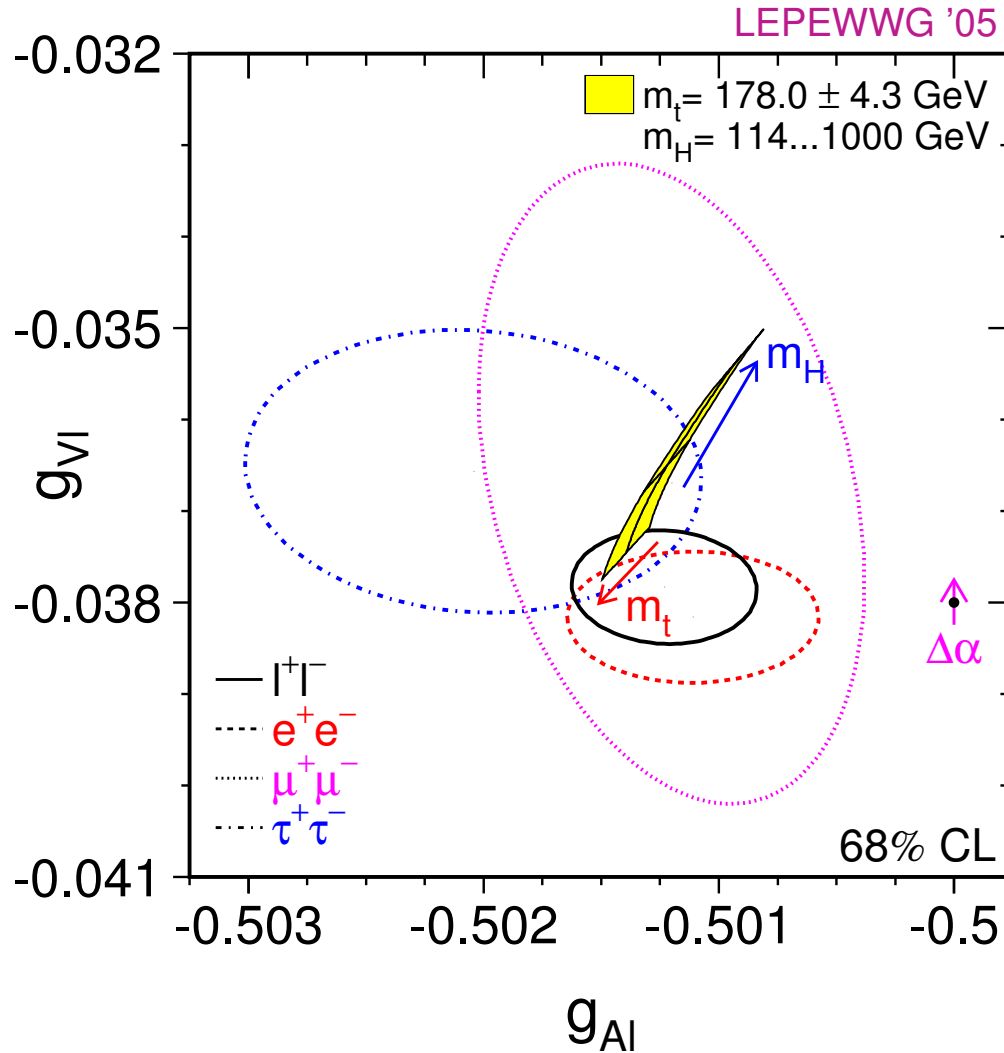
forward–backward asymmetry

$$A_{\text{FB}}^f = \frac{\sigma_f(\theta < 90^\circ) - \sigma_f(\theta > 90^\circ)}{\sigma_f(\theta < 90^\circ) + \sigma_f(\theta > 90^\circ)}$$

contribution of Z-boson exchange (**pseudo observable**)

$$A_{\text{FB}}^f(s = M_Z^2) = \frac{3}{4} \frac{g_{v_e}/g_{a_e}}{(g_{v_e}/g_{a_e})^2 + 1} \frac{g_{v_f}/g_{a_f}}{(g_{v_f}/g_{a_f})^2 + 1}$$

$$\Rightarrow g_{v_f}, g_{a_f}$$



parameters

$$m_t = 178.0 \pm 4.3 \text{ GeV}$$

$$M_H = 114 \dots 1000 \text{ GeV}$$

$$\Delta\alpha_{\text{had}} = 0.02758 \pm 0.00035$$

confirmation of
lepton universality

constraints on m_t and M_H

experimental results
for couplings

$$g_{A_l} = -0.50123 \pm 0.00026$$

$$g_{V_l} = -0.03783 \pm 0.00041$$

Lowest order:

$$g_{a_f,0} = I_{w,f}^3, \quad g_{v_f,0} = (I_{w,f}^3 - 2Q_f \sin^2 \theta_w)$$

definition of **effective fermionic mixing angle** (pseudo observable)

$$g_{a_f} = \sqrt{\rho_f} I_{w,f}^3, \quad g_{v_f} = \sqrt{\rho_f} (I_{w,f}^3 - 2Q_f \sin^2 \theta_{\text{eff}}^f)$$

$$g_{v_f}, g_{a_f} \Rightarrow \sin^2 \theta_{\text{eff}}^f, \rho_f, \quad g_{v_f} / g_{a_f} \Rightarrow \sin^2 \theta_{\text{eff}}^f = \frac{I_{w,f}^3}{2Q_f} \left(1 - \frac{g_{v_f}}{g_{a_f}} \right)$$

lowest-order perturbation theory:

$$\sin^2 \theta_{\text{eff}}^f = \sin^2 \theta_w = 1 - \frac{M_W^2}{M_Z^2}, \quad \rho_f = 1$$

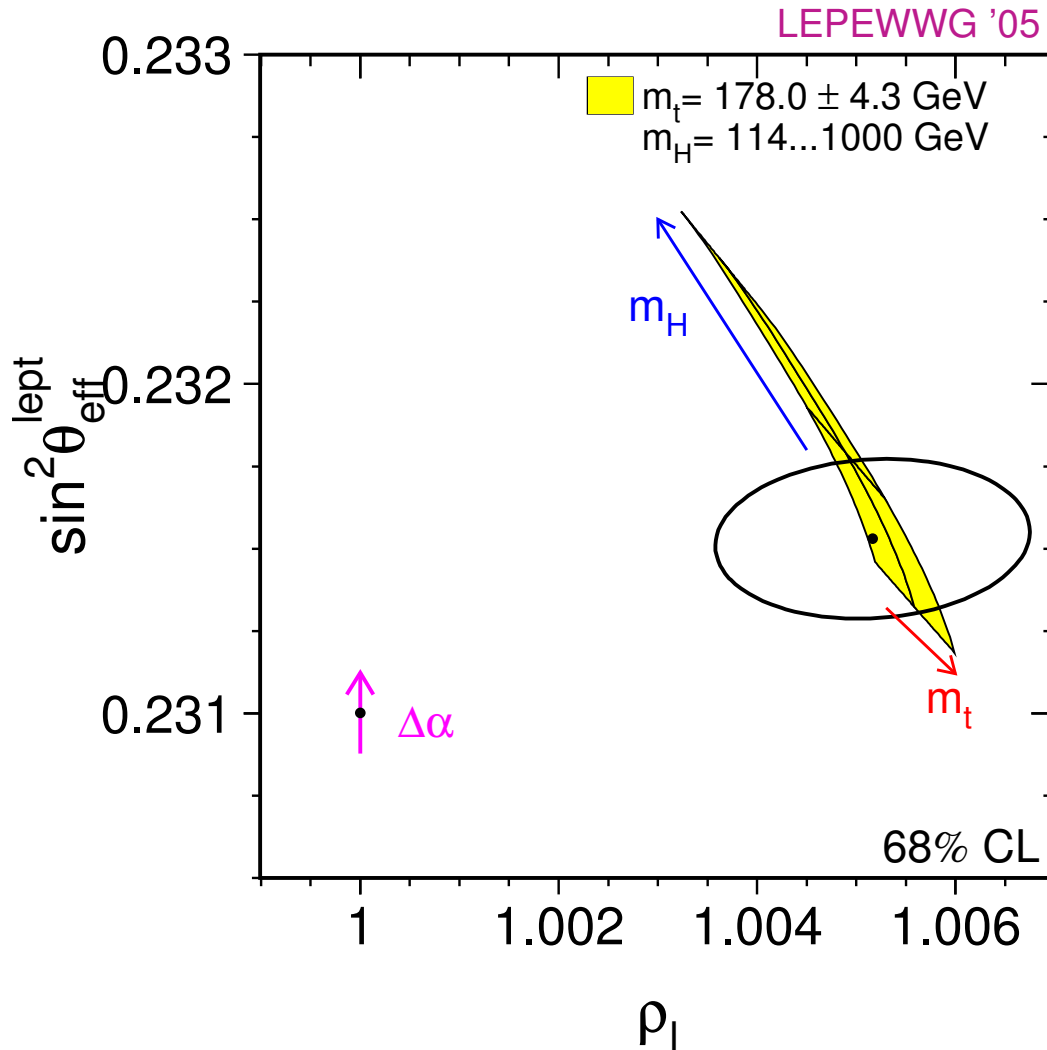
including weak corrections:

$$\sin^2 \theta_{\text{eff}}^f \neq \sin^2 \theta_w, \quad \rho_f \neq 1$$

experiment:

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} \approx 0.2315, \quad \sin^2 \theta_w \approx 0.2229, \quad \sin^2 \theta_{\text{eff}}^{\text{lept}} / \sin^2 \theta_w \approx 1.039$$

$$\rho_f \approx 1.005$$



parameters

$$m_t = 178.0 \pm 4.3 \text{ GeV}$$

$$M_H = 114 \dots 1000 \text{ GeV}$$

$$\Delta\alpha_{\text{had}} = 0.02758 \pm 0.00035$$

preference for light Higgs boson

experimental results

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = 0.23153 \pm 0.00016$$

$$\rho_1 = 1.0050 \pm 0.0010$$

Precision tests of the Standard Model

calculate all precision observables in SM including quantum corrections in terms of $\alpha(M_Z), G_\mu, M_Z, m_{f \neq t}, m_t, M_H, \alpha_s(M_Z)$

determine parameters by fit to all precision data

results

- good agreement between SM and data at the per-mille level (some exceptions)
- indirect determination of parameters

all Z-pole data data, $M_W, \Gamma_W: \Rightarrow m_t = 179^{+12}_{-9}$ GeV

1994: $m_t = 169^{+24}_{-27}$ (top discovery 1995)

present experimental value: $m_t = 173.2 \pm 0.9$ GeV

all Z-pole data data, M_W, Γ_W, m_t

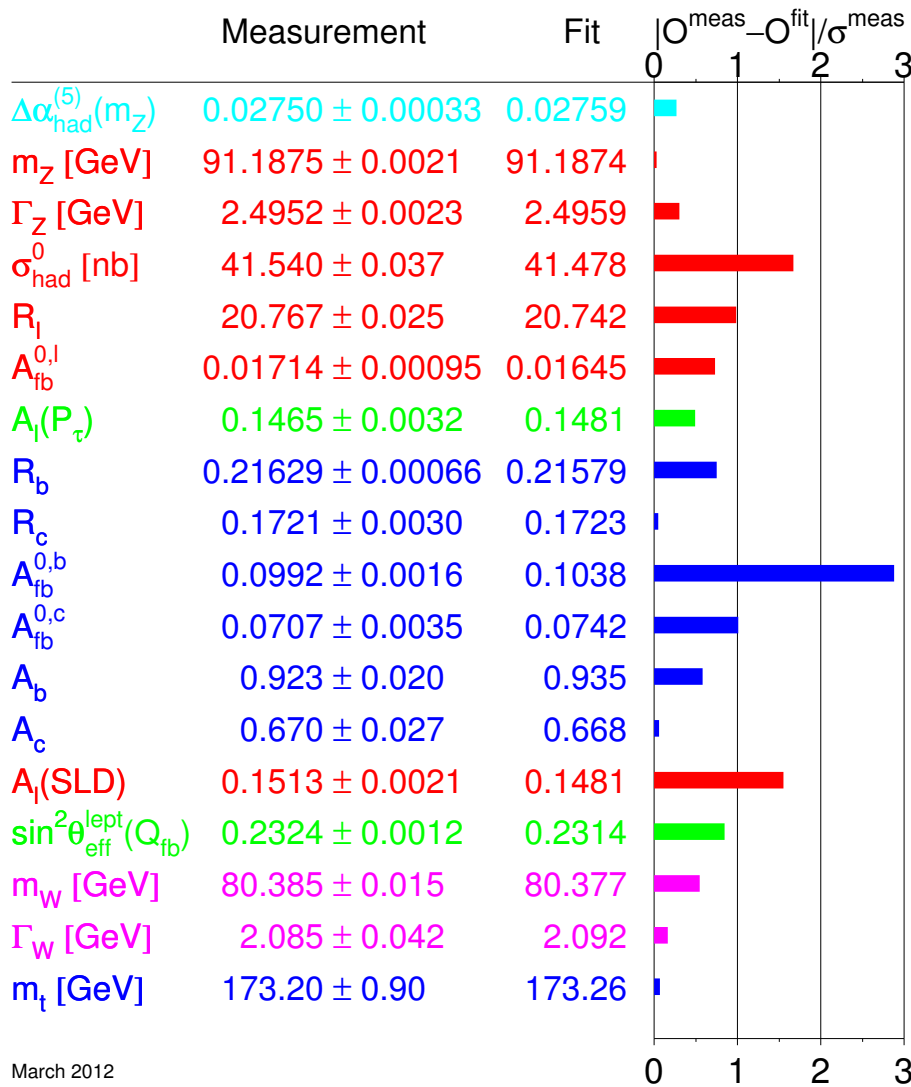
$\Rightarrow M_H = 94^{+29}_{-24}$ GeV

$M_H < 152$ GeV (95% C.L.)

preference for light Higgs boson

$\alpha_s(M_Z) = 0.1185 \pm 0.0026$

LEPEWWG '12



good agreement

best fit for Higgs-boson mass:

$$M_H = 94^{+29}_{-24} \text{ GeV},$$

$$M_H < 152 \text{ GeV @ 95\% CL}$$

direct search at LEP2

($e^+e^- \not\rightarrow ZH$) LEPHIGGS '03

$$M_H > 114.4 \text{ GeV}$$

Higgs discovery at LHC

ATLAS/CMS July '12

$$M_H \approx 125 \text{ GeV}$$

March 2012

- Natural input parameters: $\alpha, M_W, M_Z, m_f, M_H, \alpha_s$
- weak mixing angle: on-shell definition $\sin \theta_w = \sqrt{1 - M_W^2/M_Z^2}$
- alternative input parameter sets: G_μ instead of M_W or α
 G_μ no fundamental parameter, but precisely measured in μ decay
- definition of α
 - ▶ on-shell: $\alpha(0)$
appropriate for external photons
 - ▶ $\alpha(M_Z), \alpha(\sqrt{s})$: $\frac{\alpha(M_Z)}{\alpha(0)} \approx 1.06$
absorbs running of α from $Q = 0$ to EW scale
appropriate for internal photons and weak bosons
 - ▶ G_μ scheme: $\alpha_{G_\mu} = \sqrt{2}G_\mu M_W^2(1 - M_W^2/M_Z^2)/\pi$: $\frac{\alpha_{G_\mu}}{\alpha(0)} \approx 1.03$
absorbs running of α from $Q = 0$ to EW scale and $\Delta\rho$ in $W f \bar{f}'$ coupling
appropriate for processes with W bosons
suitable choice of α reduces missing higher-order corrections

gauge invariance demands unique input-parameter set!

Standard Model established as a quantum field theory

- in agreement with all experiments (accuracy $\gtrsim 0.1\%$)
- quantum corrections = radiative corrections are established
- indirect and direct determinations of m_t agree
- constraints on the Higgs-boson mass \Rightarrow light Higgs boson
- triple-gauge-boson self-interactions established at per-cent level
- candidate for Higgs boson has been found

to be confirmed

- verification of Higgs boson
- measurement of Higgs-boson couplings to gauge bosons and fermions
- Higgs-boson self-interaction \Rightarrow Higgs potential

- Textbooks and Reviews:
 - ▶ Peskin/Schroeder, *An introduction to Quantum Field Theory*
 - ▶ Itzykson/Zuber, *Quantum Field Theory*
 - ▶ Weinberg, *The Quantum Theory of Fields, Vol. 1: Foundations;*
The Quantum Theory of Fields, Vol. 2: Modern Applications
 - ▶ Böhm/Denner/Joos, *Gauge Theories of the Strong and Electroweak Interaction*
 - ▶ Collins, *Renormalization*
- Experimental results:
 - ▶ The ALEPH, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, Phys. Rept. 427 (2006) 257
 - ▶ The ALEPH, CDF, D0, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the Tevatron Electroweak Working Group, and the SLD electroweak and heavy flavour groups, LEPEWWG/2010-01, <http://lepewwg.web.cern.ch/LEPEWWG/>