Standard Model

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44. Herbstschule für Hochenergiephysik, Maria Laach
04. - 14. September 2012

- Lecture 1: Formulation of the Standard Model
- Lecture 2: Precision tests of the Standard Model
- Lecture 3: NLO Calculations for the LHC
- Lecture 4: Higgs physics at the LHC

Formulation of Standard Model

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- Electroweak phenomenology before the Standard Model
- Basic Principles of the Standard Model
  - gauge invariance
  - spontaneous symmetry breaking
- Lagrangian of the Electroweak Standard Model

Electroweak phenomenology before the Standard Model

Some phenomenological facts

- discovery of weak interaction via radioactivity (Becquerel 1896)
  β-decay of heavy nuclei: $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  $\mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e}$ (not possible for free protons)
- terminology "weak": long life time of weakly decaying particles:

strong int.:	$\rho \to 2\pi,$	$\tau \sim 10^{-22}$s
elmg. int.:	$\pi \to 2\gamma,$	$\tau \sim 10^{-16}$s
weak int.:	$\pi^- \to \mu^- + \bar{\nu}_\mu,$	$\tau \sim 10^{-8}$s
	$\mu^- \to \mathrm{e}^- + \bar{\nu}_\mathrm{e} + \nu_\mu,$	$\tau \sim 10^{-6}$s

  $\hookrightarrow$ weak interaction (for $E \lesssim 1\,\mathrm{GeV}$)
  due to very short range at low energies
- lepton-number conservation: $\mu^- \not\to \mathrm{e}^- + \gamma$ (BR $\lesssim 10^{-11}$)
  (For massive νs with different masses, only $L = L_\mathrm{e}+L_\mu+L_\tau$ is conserved.)
- parity violation (predicted by Lee, Yang 1956, detected by Wu et al.1957)
  e.g.: $\pi^+ \to \mu^+ + \nu_\mu$
  μ^+ always left-handed
  $^{60}\mathrm{Co} \to {}^{60}\mathrm{Ni}^* + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$
  electrons are emitted predominantly in direction of spin of $^{60}\mathrm{Co}$
- CP violation (Cronin, Fitch 1964) (C: charge conjugation, P: parity)
  $K_\mathrm{L} \to 2\pi, \qquad \mathrm{CP} = -1 \to \mathrm{CP} = +1$

The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for "current–current interaction" of four fermions:

$$\mathcal{L}_{\text{Fermi}} = -2\sqrt{2} G_\mu J_\rho^\dagger(x) J^\rho(x), \qquad G_\mu = 1.16639 \times 10^{-5}\,\text{GeV}^{-2}$$

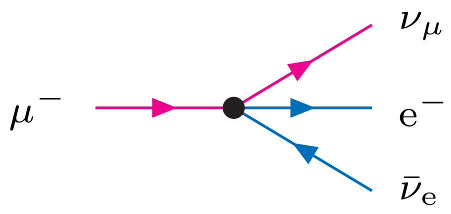
Fermi constant

with $J_\rho(x) = J_\rho^{\text{lep}}(x) + J_\rho^{\text{had}}(x)$ charged weak current

- leptonic current J_ρ^{lep}:

$$J_\rho^{\text{lep}} = \overline{\psi_{\nu_e}} \gamma_\rho \omega_- \psi_e + \overline{\psi_{\nu_\mu}} \gamma_\rho \omega_- \psi_\mu, \qquad \omega_\pm = \tfrac{1}{2}(1 \pm \gamma_5) = \text{chirality projectors}$$

  - only left-handed fermions $(\omega_- \psi)$, right-handed anti-fermions $(\overline{\psi}\omega_+)$ feel (charged-current) weak interactions $\Rightarrow$ maximal P violation
  - doublet structure: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$, later completed by $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$
  - $(J^{\text{lep},\rho})^\dagger J_\rho^{\text{lep}} = \overline{\psi_{\nu_\mu}} \gamma_\rho \omega_- \psi_\mu \, (\overline{\psi_{\nu_e}} \gamma_\rho \omega_- \psi_e)^\dagger$ induces muon decay:



The Fermi model (2)

- Hadronic current J_ρ^{had}:

  formulated in terms of quark fields

  relevant quarks for energies $\lesssim 1\,\mathrm{GeV}$: $\quad \mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}$

  simple doublet structure $\begin{pmatrix}\mathrm{u}\\ \mathrm{d}\end{pmatrix}, \begin{pmatrix}\mathrm{c}\\ \mathrm{s}\end{pmatrix}$ in conflict with experiment

  e.g. observed process $\underbrace{K^+}_{\mathrm{u\bar{s}}\text{ pair in quark model}} \to \mu^+\nu_\mu$ would not be allowed

  solution (Cabibbo 1963):

  u–c-mixing and d–s-mixing in weak interaction

  $\hookrightarrow$ doublets $\begin{pmatrix}\mathrm{u}\\ \mathrm{d}'\end{pmatrix}, \begin{pmatrix}\mathrm{c}\\ \mathrm{s}'\end{pmatrix}$ with $\begin{pmatrix}\mathrm{d}'\\ \mathrm{s}'\end{pmatrix} = U_\mathrm{C}\begin{pmatrix}\mathrm{d}\\ \mathrm{s}\end{pmatrix}$

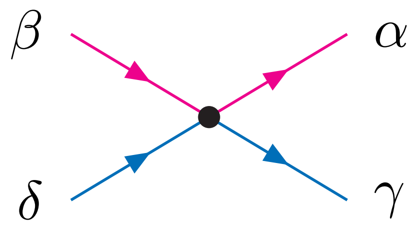
  orthogonal Cabibbo matrix $\quad U_\mathrm{C} = \begin{pmatrix}\cos\theta_\mathrm{C} & \sin\theta_\mathrm{C}\\ -\sin\theta_\mathrm{C} & \cos\theta_\mathrm{C}\end{pmatrix}$

  empirical result: $\quad \theta_\mathrm{C} \approx 13°$

$$
\begin{aligned}
J_\rho^{\mathrm{had}} &= \overline{\psi_\mathrm{u}}\gamma_\rho\omega_-\psi_{\mathrm{d}'} + \overline{\psi_\mathrm{c}}\gamma_\rho\omega_-\psi_{\mathrm{s}'}\\
&= \overline{\psi_\mathrm{u}}\gamma_\rho\omega_-(\cos\theta_\mathrm{C}\psi_\mathrm{d} + \sin\theta_\mathrm{C}\psi_\mathrm{s}) + \overline{\psi_\mathrm{c}}\gamma_\rho\omega_-(\cos\theta_\mathrm{C}\psi_\mathrm{s} - \sin\theta_\mathrm{C}\psi_\mathrm{d})
\end{aligned}
$$

Remarks on the Fermi model

- Elementary interaction:



$$= -\mathrm{i}2\sqrt{2}G_\mu(\gamma_\mu\omega_-)_{\alpha\beta}(\gamma^\mu\omega_-)_{\gamma\delta}$$

- universality of weak interaction:
  universal coupling G_μ for all transitions and $U_\mathrm{C}^\dagger U_\mathrm{C} = \mathbf{1}$
- vector and axial-vector interaction sufficient $(2\gamma_\mu\omega_- = \gamma_\mu - \gamma_\mu\gamma_5)$
  to describe low-energy experiments ($E \lesssim 1\,\mathrm{GeV}$)
  no other couplings like (pseudo-)scalar couplings necessary
  $[(\overline{\psi}\psi)(\overline{\psi}\psi),\ (\overline{\psi}\psi)(\overline{\psi}\gamma_5\psi),\ (\overline{\psi}\sigma^{\mu\nu}\psi)(\overline{\psi}\sigma_{\mu\nu}\psi),\ \ldots]$
- problems:
  - cross sections for $\nu_\mu \mathrm{e} \to \nu_\mathrm{e}\mu$, etc., grow for energy $E \to \infty$ as E^2
    $\hookrightarrow$ unitarity violation !
  - no consistent evaluation of higher perturbative orders possible
    (no cancellation of UV divergences)
    $\hookrightarrow$ non-renormalizability !

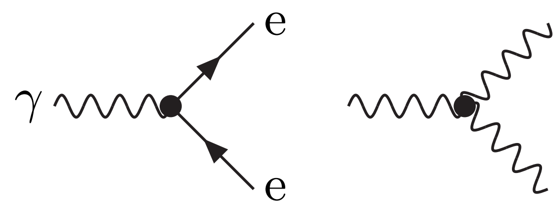
Feynman rules

Lagrangian is equivalent to a set of Feynman rules

propagators for free fields

etc.

vertices = elementary interactions

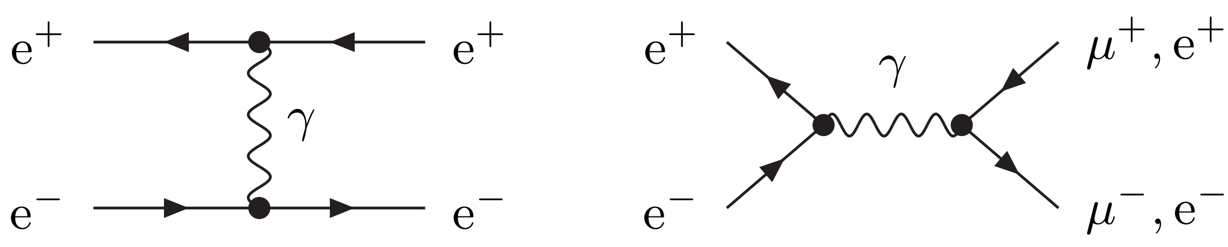


etc.

Feynman diagrams

- provide an exact formulation of perturbation theory
- describe interactions intuitively by scattering processes of free particles

examples for electromagnetic interaction:



transition amplitude (S-matrix element)
$\langle f|S|i\rangle \;=\; \Sigma$ of all Feynman graphs for $|i\rangle \;\rightarrow\; |f\rangle$

Intermediate-vector-boson (IVB) model

Idea: “resolution” of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\mathcal{L}_{\rm IVB} = \mathcal{L}_{0,\rm ferm} + \mathcal{L}_{0,\rm W} + \mathcal{L}_{\rm int}, \qquad \mathcal{L}_{0,\rm ferm} = \sum_f \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f$$

$$\mathcal{L}_{0,\rm W} = -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_{\rm W}^2 W_\mu^+ W^{-,\mu}$$

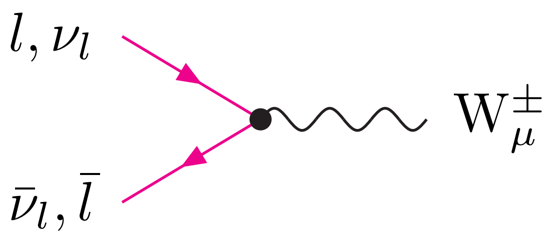
$\mathrm{W}^\pm$ are vector bosons with electric charge $\pm e$ and mass $M_{\rm W}$.

W propagator: $$G_{\mu\nu}^{\rm WW}(k) = \frac{-\mathrm{i}}{k^2 - M_{\rm W}^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_{\rm W}^2}\right),$$ k = momentum

interaction Lagrangian: $$\mathcal{L}_{\rm int} = \frac{g_{\rm W}}{\sqrt{2}}\left(J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-\right)$$

J^ρ = charged weak current as in Fermi model

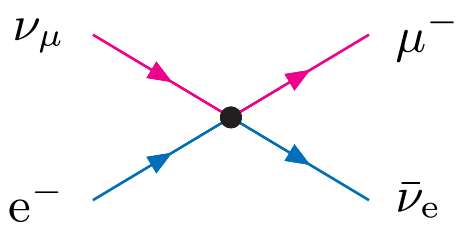
elementary interaction:



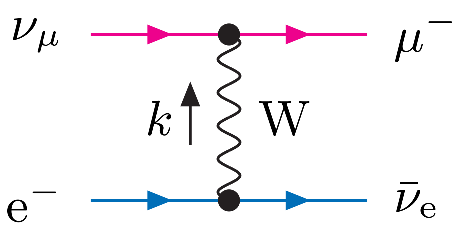
$$= \mathrm{i}\frac{g_{\rm W}}{\sqrt{2}}\gamma_\mu\omega_-$$

Four-fermion interaction in process $\nu_\mu \mathrm{e}^- \to \mu^- \nu_\mathrm{e}$

Fermi model:



IVB model:



$$-\mathrm{i}2\sqrt{2}G_\mu\, g_{\rho\sigma} \qquad \tfrac{\mathrm{i}}{2} g_\mathrm{W}^2 \frac{1}{k^2 - M_\mathrm{W}^2}\left(g_{\rho\sigma} - \frac{k_\rho k_\sigma}{M_\mathrm{W}^2}\right)$$

$$\times \left[\bar{u}_{\mu^-}\gamma^\rho \omega_- u_{\nu_\mu}\right]\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma \omega_- u_{\mathrm{e}^-}\right] \qquad \times \left[\bar{u}_{\mu^-}\gamma^\rho \omega_- u_{\nu_\mu}\right]\left[\bar{u}_{\nu_\mathrm{e}}\gamma^\sigma \omega_- u_{\mathrm{e}^-}\right]$$

identification for $|k| \ll M_\mathrm{W} \quad \Rightarrow \quad 2\sqrt{2}G_\mu = g_\mathrm{W}^2/(2M_\mathrm{W}^2)$

consequences for the high-energy behaviour:

- k^ρ terms: $\quad \bar{u}_{\nu_\mathrm{e}} \not{k} \omega_- u_{\mathrm{e}^-} = \bar{u}_{\nu_\mathrm{e}}(\not{p}_\mathrm{e} - \not{p}_{\nu_\mathrm{e}})\omega_- u_{\mathrm{e}^-} = m_\mathrm{e}\bar{u}_{\nu_\mathrm{e}}\omega_+ u_{\mathrm{e}^-}$
  $\hookrightarrow$ no extra factors of scattering energy E
- propagator $1/(k^2 - M_\mathrm{W}^2) \sim 1/E^2$ for $|k| \sim E \gg M_\mathrm{W}$
  $\hookrightarrow$ damping of amplitude in high-energy limit by factor $1/E^2$

$\Rightarrow$ cross section $\underset{E\to\infty}{\widetilde{}}$ const$/E^2$ $\qquad \Rightarrow$ no unitarity violation !

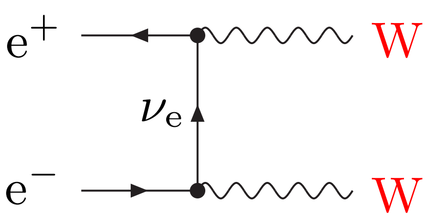
Comments on the IVB model

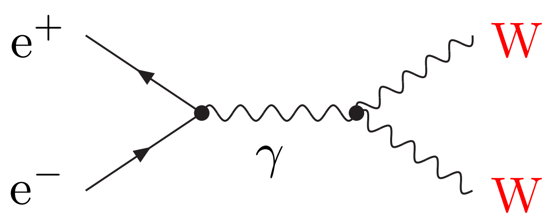
- Formal similarity with QED interaction: $J^\rho W_\rho^+$ + h.c. $\longleftrightarrow$ $j^\rho_{\mathrm{elmg.}} A_\rho$
- intermediate vector bosons can be produced, e.g.

$$\underbrace{\mathrm{u\bar{d}}}_{\text{in } \mathrm{p\bar{p}} \text{ collision}} \longrightarrow \underbrace{\mathrm{W}^+ \to f\bar{f}'}_{\mathrm{W}^\pm \text{ unstable}}$$

(discovery 1983 at CERN)

- problems:
  - ▶ unitarity violations in cross sections with longitudinal W bosons, e.g.





  - ▶ non-renormalizability
    (no consistent treatment of higher perturbative orders)

$\hookrightarrow$ solution by spontaneously broken gauge theories !
$\hookrightarrow$ electroweak Standard Model

Basic principles
of the Standard Model

Basic principles of the SM

Electroweak Standard Model (SM) constructed in analogy to QED

	QED	SM
matter fields	$e^{\pm}$	leptons, quarks
global symmetry	$U(1)_{em}$	$SU(2)_I \times U(1)_Y$

↓ "gauging of the symmetry"
global $\to$ local symmetry

introduction of gauge bosons and interactions	γ	$\gamma, Z, W^{\pm}$

differences to QED

- non-Abelian gauge symmetry $\Rightarrow$ gauge-boson self-interactions
- spontaneous symmetry breaking $SU(2)_I \times U(1)_Y \to U(1)_{em}$
  $\Rightarrow$ massive gauge bosons $Z, W^{\pm}$ and Higgs boson H (spin 0)

$\Rightarrow$ unified description of electromagnetic and weak interaction
Glashow, Salam, Weinberg 1967–1970:
Standard Model of electroweak interaction (GSW model)

The Principle of local gauge invariance

QED as U(1) gauge theory:

free Lagrangian $\mathcal{L}_{0,\mathrm{ferm}} = \overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - m_f)\psi_f$ invariant under global U(1) symmetry:

$$\psi_f \to \psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}\psi_f, \quad \overline{\psi_f} \to \overline{\psi'_f} = \overline{\psi_f}\exp\{+\mathrm{i}Q_f e\theta\}$$

with space-time-independent group parameter θ

“gauging the symmetry”: demand local symmetry, $\theta \to \theta(x)$

to achieve local symmetry, extend theory by “minimal substitution”:

$\partial^\mu \to D^\mu = \partial^\mu + \mathrm{i}Q_f e A^\mu(x)$ = “covariant derivative”

$A^\mu(x)$ = spin-1 gauge field (photon)

Transformation property of photon $A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \to (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-\mathrm{i}Q_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\mathrm{QED}} = \underbrace{\overline{\psi_f}(\mathrm{i}\partial\!\!\!/ - Q_f e A\!\!\!/ - m_f)\psi_f}_{\text{fermion part}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{gauge part}}$$

Non-Abelian gauge theory (Yang–Mills theory)

Starting point: Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting (matter) fields with "internal (non-Abelian) symmetry":

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

  $\Phi \to \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-\mathrm{i}gT^a\theta^a\}$ = unitary

  T^a = (hermitian) group generators, $\quad a = 1, \ldots, N, \quad N$ = dimension of group

  properties of T^a: $\quad [T^a, T^b] = \mathrm{i}f^{abc}T^c, \quad \mathrm{Tr}\, T^aT^b = \frac{1}{2}\delta^{ab}$

  f^{abc} structure constants of G

  infinitesimal transformation: $\delta\Phi = \Phi' - \Phi = -\mathrm{i}gT^a\theta^a\Phi$

- $\mathcal{L}_\Phi$ is invariant under G: $\quad \mathcal{L}_\Phi(\Phi, \partial_\mu\Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu\Phi')$

examples:

self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu\Phi)^\dagger(\partial^\mu\Phi) - m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$$

(m = common mass, λ = coupling strength)

free fermion multiplet

$$\mathcal{L}_\Psi = \overline{\Psi}\mathrm{i}\partial\!\!\!/\Psi - m\overline{\Psi}\Psi$$

Non-Abelian gauge theory (Yang–Mills theory) (2)

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) \to \mathcal{L}_\Phi(\Phi, D_\mu \Phi) \quad \text{with} \quad D_\mu = \partial_\mu + \mathrm{i} g T^a A^a_\mu(x)$$

g = gauge coupling, T^a = generator of G in Φ representation

$A^a_\mu(x)$ = gauge fields, $a = 1, \ldots, N$

transformation property of gauge fields:

$\mathcal{L}_\Phi(\Phi, D_\mu \Phi)$ locally invariant if $D_\mu \Phi \;\to\; (D_\mu \Phi)' = D'_\mu \Phi' = U(\theta)(D_\mu \Phi)$

$\Rightarrow\; T^a A'^a_\mu \;=\; U T^a A^a_\mu U^\dagger - \frac{\mathrm{i}}{g} U(\partial_\mu U^\dagger),$ $\quad A^a_\mu A^{a,\mu}$ = not gauge invariant

infinitesimal form: $\delta A^a_\mu = g f^{abc} \delta\theta^b A^c_\mu + \partial_\mu \delta\theta^a$

covariant definition of field strength: $[D_\mu, D_\nu] = \mathrm{i} g T^a F^a_{\mu\nu}$

$\Rightarrow\; T^a F^a_{\mu\nu} \;\to\; T^a F'^a_{\mu\nu} = U T^a F^a_{\mu\nu} U^\dagger,$ $\quad F^a_{\mu\nu} F^{a,\mu\nu}$ = gauge invariant

explicit form: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$

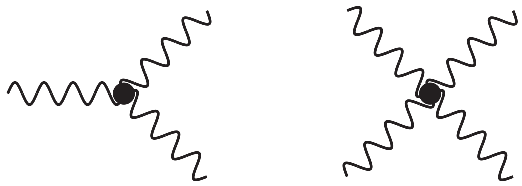
Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\mathrm{YM}} = \mathcal{L}_\Phi(\Phi, D_\mu \Phi) - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory) (3)

Remarks

- Lagrangian contains terms of order $(\partial A)A^2$, A^4 in F^2 part
  $\hookrightarrow$ cubic and quartic gauge-boson self-interactions



- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction $\to$ unification of interactions
- mass term $M^2(A^a_\mu A^{a,\mu})$ for gauge bosons forbidden by gauge invariance
  $\hookrightarrow$ gauge bosons of unbroken Yang–Mills theory are massless
- non-Abelian charges are quantized owing to $[T^a, T^b] = \mathrm{i} f^{abc} T^c$
  (Abelian charges are arbitrary)
- $G = \mathrm{SU}(3)_\mathrm{c}$ and fermion triplets $\Rightarrow$ Lagrangian of QCD

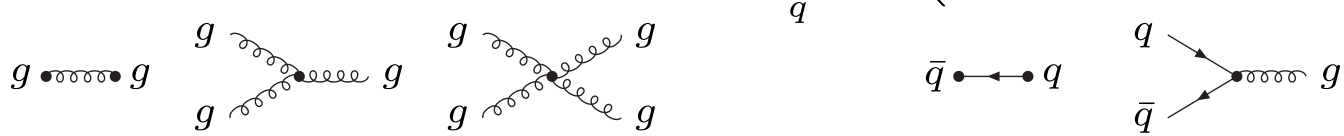
$$\mathcal{L}_\Psi = \overline{\Psi}\mathrm{i}\gamma^\mu D_\mu \Psi - m\overline{\Psi}\Psi - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}$$

Quantum chromodynamics

Gauge theory of strong interactions

- gauge group: SU(3)$_{\mathrm{c}}$, dimension $N = 8$
  structure constants f^{abc}, gauge coupling g_{s}, $\alpha_{\mathrm{s}} = \dfrac{g_{\mathrm{s}}^2}{4\pi}$
- gauge bosons: 8 massless gluons g with fields $A_\mu^a(x)$, $a = 1, \ldots, 8$
- matter fermions: quarks q (spin-$\frac{1}{2}$) with flavours $q = \mathrm{d, u, s, c, b, t}$
  in fundamental representation:
  $\psi_q(x) \;\equiv\; q(x) \;=\; (q_r(x), q_g(x), q_b(x))^{\mathrm{T}} \;=$ colour triplet
  $T^a = \dfrac{\lambda^a}{2}$, Gell-Mann matrices $\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, etc.
- Lagrangian:

$$\begin{aligned}
\mathcal{L}_{\mathrm{QCD}} &= -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \overline{\psi}_q (\mathrm{i}\not{D} - m_q) \psi_q \\
&= -\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_{\mathrm{s}} f^{abc} A^b_\mu A^c_\nu \right)^2 + \sum_q \overline{\psi}_q \left(\mathrm{i}\not{\partial} - g_{\mathrm{s}} \frac{\lambda^a}{2} \not{A}^a - m_q \right) \psi_q
\end{aligned}$$



Lagrangian of the Standard Model

Choice of electroweak gauge group

- Why unification of weak and elmg. interaction ?
  - similarity: spin-1 fields couple to matter currents formed by spin-$\frac{1}{2}$ fields
  - elmg. coupling of charged $\mathrm{W}^{\pm}$ bosons
  - unitarity of theory with elmg. charged massive gauge bosons requires unification
- minimal choice of gauge group: $\mathrm{SU}(2)_{\mathrm{I}} \times \mathrm{U}(1)_{\mathrm{Y}}$
  - $\mathrm{SU}(2)_{\mathrm{I}}$ $\to$ weak isospin group with gauge bosons $\mathrm{W}^+, \mathrm{W}^-, \mathrm{W}^0$
    generators I_{w}^a, $a = 1, 2, 3,$ gauge coupling g_2
  - $\mathrm{U}(1)_{\mathrm{Y}}$ $\to$ weak hypercharge group with gauge boson B
    generator Y_{w}, gauge coupling g_1

  W^0 and B carry identical elmg. and spin quantum numbers
  $\hookrightarrow$ two neutral gauge bosons γ, Z as mixed states

  experiment: 1973 discovery of neutral weak currents at CERN
  $\hookrightarrow$ indirect confirmation of Z exchange
  1983 discovery of $\mathrm{W}^{\pm}$ and Z bosons at CERN

Fermion multiplet structure

Distinguish between left-/right-handed parts of fermions: $\psi^{\mathrm{L}} = \omega_- \psi$, $\psi^{\mathrm{R}} = \omega_+ \psi$

- ψ^{L} couple to $\mathrm{W}^\pm$ $\rightarrow$ group ψ^{L} into SU(2)$_{\mathrm{I}}$ doublets, weak isospin $I_{\mathrm{w}}^a = \frac{\sigma^a}{2}$
- ψ^{R} do not couple to $\mathrm{W}^\pm$ $\rightarrow$ ψ^{R} are SU(2)$_{\mathrm{I}}$ singlets, weak isospin $I_{\mathrm{w}}^a = 0$
- $\psi^{\mathrm{L/R}}$ couple to γ in the same way
  $\hookrightarrow$ adjust coupling to $U(1)_{\mathrm{Y}}$ (i.e. fix weak hypercharges $Y_{\mathrm{w}}^{\mathrm{L/R}}$ for $\psi^{\mathrm{L/R}}$)
  such that elmg. coupling results: $\mathcal{L}_{\mathrm{int,QED}} = -\sum_f Q_f e \overline{\psi_f} \not{A} \psi_f$

fermion content of the SM:
(ignoring right-handed neutrinos)

					I_{w}^3	Q
leptons:	$\Psi_{L,i}^{\mathrm{L}} =$	$\begin{pmatrix} \nu_{\mathrm{e}}^{\mathrm{L}} \\ \mathrm{e}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \nu_{\mu}^{\mathrm{L}} \\ \mu^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \nu_{\tau}^{\mathrm{L}} \\ \tau^{\mathrm{L}} \end{pmatrix}$,	$+\frac{1}{2}$ $-\frac{1}{2}$	0 -1
	$\psi_{l,i}^{\mathrm{R}} =$	e^{R},	μ^{R},	τ^{R},	0	-1
quarks: (each quark exists in 3 colours!)	$\Psi_{Q,i}^{\mathrm{L}} =$	$\begin{pmatrix} \mathrm{u}^{\mathrm{L}} \\ \mathrm{d}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \mathrm{c}^{\mathrm{L}} \\ \mathrm{s}^{\mathrm{L}} \end{pmatrix}$,	$\begin{pmatrix} \mathrm{t}^{\mathrm{L}} \\ \mathrm{b}^{\mathrm{L}} \end{pmatrix}$,	$+\frac{1}{2}$ $-\frac{1}{2}$	$+\frac{2}{3}$ $-\frac{1}{3}$
	$\psi_{u,i}^{\mathrm{R}} =$	u^{R},	c^{R},	t^{R},	0	$+\frac{2}{3}$
	$\psi_{d,i}^{\mathrm{R}} =$	d^{R},	s^{R},	b^{R},	0	$-\frac{1}{3}$

Fermion Lagrangian and minimal substitution

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\mathrm{ferm}} = \sum_f \mathrm{i}\overline{\psi_f}\partial\!\!\!/\psi_f$$

$$= \sum_i (\mathrm{i}\overline{\Psi^{\mathrm{L}}_{L,i}}\partial\!\!\!/\Psi^{\mathrm{L}}_{L,i} + \mathrm{i}\overline{\Psi^{\mathrm{L}}_{Q,i}}\partial\!\!\!/\Psi^{\mathrm{L}}_{Q,i} + \mathrm{i}\overline{\psi^{\mathrm{R}}_{l,i}}\partial\!\!\!/\psi^{\mathrm{R}}_{l,i} + \mathrm{i}\overline{\psi^{\mathrm{R}}_{u,i}}\partial\!\!\!/\psi^{\mathrm{R}}_{u,i} + \mathrm{i}\overline{\psi^{\mathrm{R}}_{d,i}}\partial\!\!\!/\psi^{\mathrm{R}}_{d,i})$$

minimal substitution: $\partial_\mu \;\to\; D_\mu$

$$D_\mu = \partial_\mu - \mathrm{i}g_2 I^a_{\mathrm{w}} W^a_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y_{\mathrm{w}} B_\mu$$

$$D^{\mathrm{L}}_\mu = \partial_\mu - \frac{\mathrm{i}g_2}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu \\ W^-_\mu & 0 \end{pmatrix} + \frac{\mathrm{i}}{2}\begin{pmatrix} -g_2 W^3_\mu + g_1 Y^{\mathrm{L}}_{\mathrm{w}} B_\mu & 0 \\ 0 & g_2 W^3_\mu + g_1 Y^{\mathrm{L}}_{\mathrm{w}} B_\mu \end{pmatrix}$$

$$D^{\mathrm{R}}_\mu = \partial_\mu + \mathrm{i}g_1 \tfrac{1}{2} Y^{\mathrm{R}}_{\mathrm{w}} B_\mu$$

charge eigenstates: $W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp \mathrm{i}W^2_\mu), \quad W^3_\mu, \quad B_\mu$

$$QW^\pm_\mu = I^3_{\mathrm{w}} W^\pm_\mu = \pm W^\pm_\mu, \qquad QW^3_\mu = QB_\mu = 0$$

with $Q = I^3_{\mathrm{w}} + Y_{\mathrm{w}}/2$ (see below)

Photon identification

"Weinberg rotation": $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_\mathrm{w} & s_\mathrm{w} \\ -s_\mathrm{w} & c_\mathrm{w} \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix},$

$c_\mathrm{w} = \cos\theta_\mathrm{w}, \; s_\mathrm{w} = \sin\theta_\mathrm{w},$ θ_w = Weinberg angle = electroweak mixing angle

$$D^\mathrm{L}_\mu\Big|_{A_\mu} = \frac{\mathrm{i}}{2} A_\mu \begin{pmatrix} g_2 s_\mathrm{w} + g_1 c_\mathrm{w} Y^\mathrm{L}_\mathrm{w} & 0 \\ 0 & -g_2 s_\mathrm{w} + g_1 c_\mathrm{w} Y^\mathrm{L}_\mathrm{w} \end{pmatrix} \stackrel{!}{=} \mathrm{i}eA_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

$$D^\mathrm{R}_\mu\Big|_{A_\mu} = \frac{\mathrm{i}}{2} A_\mu g_1 c_\mathrm{w} Y^\mathrm{R}_\mathrm{w} \stackrel{!}{=} \mathrm{i}eA_\mu Q$$

- charge difference in doublet $Q_1 - Q_2 = 1$ $\rightarrow$ $g_2 = \frac{e}{s_\mathrm{w}}$
- normalize $Y^{\mathrm{L/R}}_\mathrm{w}$ such that $g_1 = \frac{e}{c_\mathrm{w}}$

  $\hookrightarrow$ Y_w fixed by "Gell-Mann–Nishijima relation": $Q = I^3_\mathrm{w} + \frac{Y_\mathrm{w}}{2}$

parameter relations: $e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \qquad \tan\theta_\mathrm{w} = \frac{g_1}{g_2}$

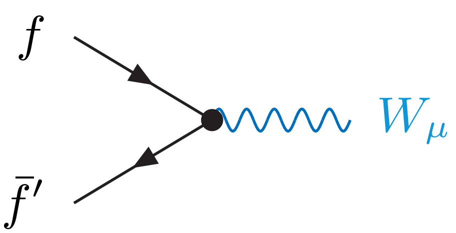
$$c_\mathrm{w} = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \qquad s_\mathrm{w} = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}$$

Fermion–gauge-boson interaction:

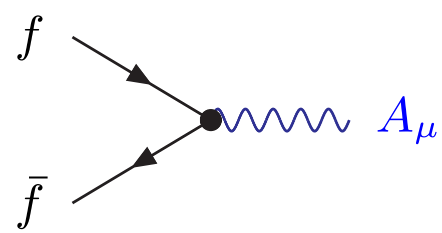
$$\mathcal{L}_{\mathrm{ferm,YM}} = \sum_F \left(\frac{e}{\sqrt{2} s_{\mathrm{w}}} \overline{\Psi^{\mathrm{L}}_F} \begin{pmatrix} 0 & \not{W}^+ \\ \not{W}^- & 0 \end{pmatrix} \Psi^{\mathrm{L}}_F + \frac{e}{2 c_{\mathrm{w}} s_{\mathrm{w}}} \overline{\Psi^{\mathrm{L}}_F} \sigma^3 \not{Z} \Psi^{\mathrm{L}}_F \right)$$

$$- \sum_f \left(e \frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} Q_f \overline{\psi_f} \not{Z} \psi_f + e Q_f \overline{\psi_f} \not{A} \psi_f \right) \qquad (f\text{=all fermions, } F\text{= all doublets})$$

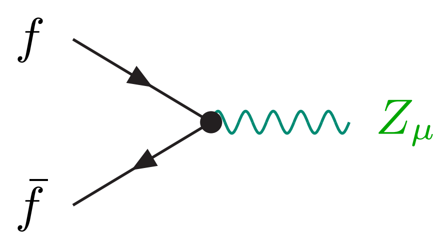
Feynman rules:



$$\frac{\mathrm{i}e}{\sqrt{2} s_{\mathrm{w}}} \gamma_\mu \omega_-$$



$$-\mathrm{i} Q_f e \gamma_\mu$$



$$\mathrm{i}e\gamma_\mu (g_f^+ \omega_+ + g_f^- \omega_-) = \mathrm{i}e\gamma_\mu (v_f - a_f \gamma_5)$$

with $$g_f^+ = -\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} Q_f, \quad g_f^- = -\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} Q_f + \frac{I^3_{\mathrm{w},f}}{c_{\mathrm{w}} s_{\mathrm{w}}}$$

$$v_f = -\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}} Q_f + \frac{I^3_{\mathrm{w},f}}{2 c_{\mathrm{w}} s_{\mathrm{w}}}, \quad a_f = \frac{I^3_{\mathrm{w},f}}{2 c_{\mathrm{w}} s_{\mathrm{w}}}$$

Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

field-strength tensors:

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

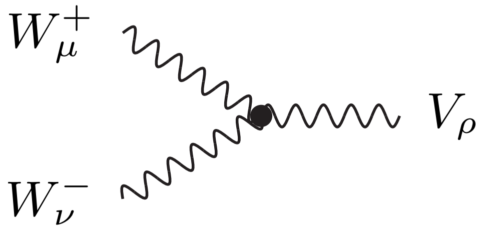
Yang–Mills Lagrangian in terms of "physical" fields:

$$\begin{aligned}
\mathcal{L}_{\mathrm{YM}} = &-\frac{1}{4} \left|\partial_\mu A_\nu - \partial_\nu A_\mu - \mathrm{i}e(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\right|^2 \\
&- \frac{1}{4} \left|\partial_\mu Z_\nu - \partial_\nu Z_\mu + \mathrm{i}e\frac{c_\mathrm{w}}{s_\mathrm{w}}(W^-_\mu W^+_\nu - W^-_\nu W^+_\mu)\right|^2 \\
&- \frac{1}{2} \left|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu - \mathrm{i}e(W^+_\mu A_\nu - W^+_\nu A_\mu) + \mathrm{i}e\frac{c_\mathrm{w}}{s_\mathrm{w}}(W^+_\mu Z_\nu - W^+_\nu Z_\mu)\right|^2
\end{aligned}$$

$\hookrightarrow$ triple gauge-boson couplings AW^+W^-, ZW^+W^-
quartic gauge-boson couplings AAW^+W^-, AZW^+W^-, ZZW^+W^-, $W^+W^-W^+W^-$

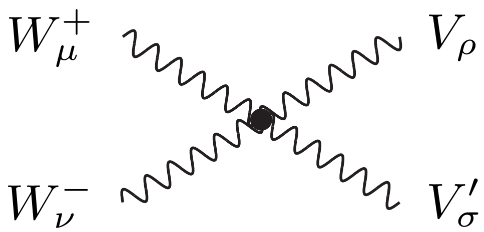
Feynman rules for gauge-boson self-interactions

(fields and momenta incoming)



$$\mathrm{i}e\,C_{WWV}\Big[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu\Big]$$

with $C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_\mathrm{w}}{s_\mathrm{w}}$



$$\mathrm{i}e^2 C_{WWVV'}\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho}\Big]$$

with $C_{WW\gamma\gamma} = -1, \qquad C_{WW\gamma Z} = \frac{c_\mathrm{w}}{s_\mathrm{w}}$

$$C_{WWZZ} = -\frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}, \quad C_{WWWW} = \frac{1}{s_\mathrm{w}^2}$$

Higgs mechanism

Introduction of gauge-boson masses

Consistency of theory (unitarity, renormalizability)
$\Leftarrow$ gauge symmetry of Lagrangian

explicit gauge-boson mass terms violate gauge invariance

solution: spontaneous symmetry breaking (SSB) (hidden symmetry)

- invariant Lagrangian $\Rightarrow$ unitarity, renormalizability
- non-invariant ground state $\Rightarrow$ gauge-boson masses

Standard Model: Higgs mechanism

introduce scalar field with non-vanishing vacuum expectation value (vev) that couples to gauge bosons

(non-zero vev of fields with spin violates Lorentz invariance)

idea: spontaneous breakdown $\mathrm{SU(2)_I \times U(1)_Y \rightarrow U(1)_{elmg}}$
$\hookrightarrow$ masses for $\mathrm{W}^{\pm}$ and Z bosons, but γ remains massless

note: choice of scalar extension of massless model involves freedom
Standard Model corresponds to minimal choice

Higgs sector and SSB in SM

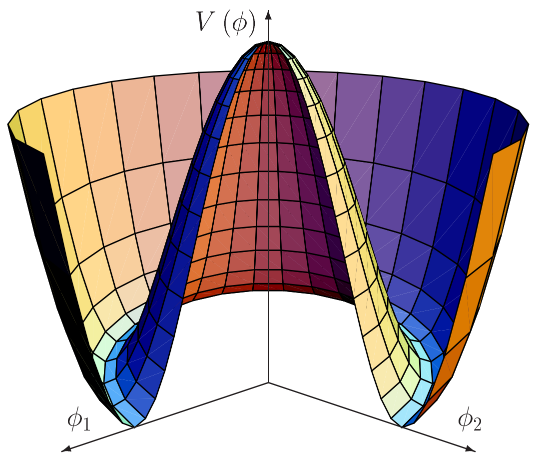
GSW model:

minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\mathrm{w}^\Phi = 1$

scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2, \quad \mu^2, \lambda > 0,$$

$= \mathrm{SU(2)}_\mathrm{I} \times \mathrm{U(1)}_\mathrm{Y}$ symmetric



$V(\Phi) =$ minimal for $\quad |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$

ground state Φ_0 (= vacuum expectation value of Φ) not unique

choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant $\Rightarrow$ spontaneous symmetry breaking

elmg. gauge invariance unbroken, since $\quad Q\Phi_0 = \left(I_\mathrm{w}^3 + \frac{Y_\mathrm{w}}{2}\right)\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\Phi_0 = 0$

field excitations in Φ:
(expansion about v)

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}\left(v + H(x) + \mathrm{i}\chi(x)\right) \end{pmatrix}$$

Higgs mechanism

spontaneous breaking of a global symmetry

Goldstone theorem: for each spontaneously broken symmetry exists
one massless scalar boson
= Goldstone boson (excitation along minimum of potential)

spontaneous breaking of a local symmetry (gauge symmetry)

Higgs mechanism: degrees of freedom of Goldstone bosons are
transmuted into longitudinal degrees of freedom of massless gauge
bosons

(would-be) Goldstone bosons are unphysical degrees of freedom:
gauge-dependent masses, absent in unitary gauge

Standard Model

need three (real) longitudinal degrees of freedom for massive $\mathrm{Z}, \mathrm{W}^{\pm}$
$\hookrightarrow$ three components of scalar field(s) are transmuted
other components appear as physical scalar fields

complex Higgs doublet (4 d.o.f) $\Rightarrow$ one physical scalar field

Higgs Lagrangian of the Electroweak SM

Gauge-invariant Lagrangian of Higgs sector: $\quad (\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_\mathrm{H} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi) \qquad \text{with} \;\; D_\mu = \partial_\mu - \mathrm{i}g_2\frac{\sigma^a}{2}W_\mu^a + \mathrm{i}\frac{g_1}{2}B_\mu$$

$$= (\partial_\mu\phi^+)(\partial^\mu\phi^-) - \frac{\mathrm{i}ev}{2s_\mathrm{w}}(W_\mu^+\partial^\mu\phi^- - W_\mu^-\partial^\mu\phi^+) + \frac{e^2v^2}{4s_\mathrm{w}^2}W_\mu^+W^{-,\mu}$$

$$+ \frac{1}{2}(\partial\chi)^2 + \frac{ev}{2c_\mathrm{w}s_\mathrm{w}}Z_\mu\partial^\mu\chi + \frac{e^2v^2}{4c_\mathrm{w}^2s_\mathrm{w}^2}Z^2 + \frac{1}{2}(\partial H)^2 - \mu^2H^2$$

$+$ (interaction terms)

implications:

- gauge-boson masses: $\quad M_\mathrm{W} = \frac{ev}{2s_\mathrm{w}}, \quad M_\mathrm{Z} = \frac{ev}{2c_\mathrm{w}s_\mathrm{w}} = \frac{M_\mathrm{W}}{c_\mathrm{w}}$
- ρ-parameter: $\rho \equiv \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2c_\mathrm{w}^2} = 1$ (for Higgs doublets and singlets!)
- photon remains massless owing to unbroken $\mathrm{U}(1)_\mathrm{em}$ invariance
- physical Higgs boson H: $\quad M_\mathrm{H} = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons $\phi^\pm$, χ
  unphysical degrees of freedom (gauge dependent, absent in unitary gauge)

Higgs-boson interactions

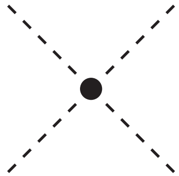
gauge-boson–Higgs-boson couplings

gauge couplings

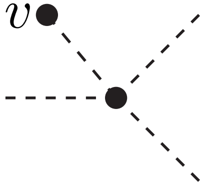
from spontaneous symmetry breaking

$\hookrightarrow$ gauge-boson masses

Higgs-boson self-couplings



$\propto \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}$ from Higgs potential



$\propto \frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}}$ from spontaneous symmetry breaking

Fermion masses and Yukawa couplings

Ordinary Dirac mass terms $m_f\overline{\psi_f}\psi_f = m_f(\overline{\psi^{\rm L}_f}\psi^{\rm R}_f + \overline{\psi^{\rm R}_f}\psi^{\rm L}_f)$ not gauge invariant

$\hookrightarrow$ introduce fermion masses by (gauge-invariant) Yukawa interaction

Lagrangian for Yukawa couplings: (needs Higgs doublets with $Y_{\rm w} = \pm 1$)

$$\mathcal{L}_{\rm Yuk} = -\overline{\Psi'^{\rm L}_L}G_l\psi'^{\rm R}_l\Phi - \overline{\Psi'^{\rm L}_Q}G_u\psi'^{\rm R}_u\tilde{\Phi} - \overline{\Psi'^{\rm L}_Q}G_d\psi'^{\rm R}_d\Phi + \text{h.c.}$$

- $G_l, G_u, G_d = 3\times 3$ matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = {\rm i}\sigma^2\Phi^* = \begin{pmatrix}\phi^{0*}\\ -\phi^-\end{pmatrix}$ = charge conjugate Higgs doublet, $Y^{\tilde{\Phi}}_{\rm w} = -1$

fermion mass terms:

mass terms = bilinear terms in $\mathcal{L}_{\rm Yuk}$, obtained by setting $\Phi \to \Phi_0 = v/\sqrt{2}$:

$$\mathcal{L}_{m_f} = -\frac{v}{\sqrt{2}}\overline{\psi'^{\rm L}_l}G_l\psi'^{\rm R}_l - \frac{v}{\sqrt{2}}\overline{\psi'^{\rm L}_u}G_u\psi'^{\rm R}_u - \frac{v}{\sqrt{2}}\overline{\psi'^{\rm L}_d}G_d\psi'^{\rm R}_d + \text{h.c.}$$

$\hookrightarrow$ diagonalization by unitary field transformations $(f = l, u, d)$

$\psi^{\rm L/R}_f \equiv U^{\rm L/R}_f\psi'^{\rm L/R}_f$ such that $\frac{v}{\sqrt{2}}U^{\rm L}_f G_f (U^{\rm R}_f)^\dagger = {\rm diag}(m_f)$

$\Rightarrow$ standard form: $\mathcal{L}_{m_f} = -m_f\overline{\psi^{\rm L}_f}\psi^{\rm R}_f + \text{h.c.} = -m_f\overline{\psi_f}\psi_f$

Quark mixing

- ψ'_f correspond to eigenstates of the gauge interaction
- ψ_f correspond to mass eigenstates,
  for massless neutrinos define $\psi_\nu^{\mathrm{L}} \equiv U_l^{\mathrm{L}} \psi_\nu^{\mathrm{L}}$ $\rightarrow$ no lepton-flavour change

Yukawa and gauge interactions in terms of mass eigenstates:

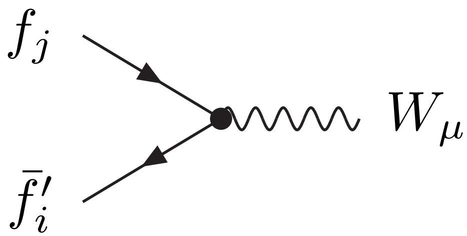
$$\mathcal{L}_{\mathrm{Yuk}} = -\frac{\sqrt{2}m_l}{v}\left(\phi^+\overline{\psi^{\mathrm{L}}_{\nu_l}}\psi^{\mathrm{R}}_l + \phi^-\overline{\psi^{\mathrm{R}}_l}\psi^{\mathrm{L}}_{\nu_l}\right) + \frac{\sqrt{2}m_u}{v}\left(\phi^+\overline{\psi^{\mathrm{R}}_u}\mathbf{V}\psi^{\mathrm{L}}_d + \phi^-\overline{\psi^{\mathrm{L}}_d}\mathbf{V}^\dagger\psi^{\mathrm{R}}_u\right)$$
$$-\frac{\sqrt{2}m_d}{v}\left(\phi^+\overline{\psi^{\mathrm{L}}_u}\mathbf{V}\psi^{\mathrm{R}}_d + \phi^-\overline{\psi^{\mathrm{R}}_d}\mathbf{V}^\dagger\psi^{\mathrm{L}}_u\right) - \frac{m_f}{v}\mathrm{i}\,\mathrm{sgn}(I^3_{\mathrm{w},f})\chi\overline{\psi_f}\gamma_5\psi_f$$
$$-\frac{m_f}{v}(v+H)\overline{\psi_f}\psi_f,$$

$$\mathcal{L}_{\mathrm{ferm,YM}} = \frac{e}{\sqrt{2}s_{\mathrm{w}}}\overline{\Psi^{\mathrm{L}}_L}\begin{pmatrix} 0 & W\!\!\!/^+ \\ W\!\!\!/^- & 0 \end{pmatrix}\psi^{\mathrm{L}}_L + \frac{e}{\sqrt{2}s_{\mathrm{w}}}\overline{\Psi^{\mathrm{L}}_Q}\begin{pmatrix} 0 & \mathbf{V}W\!\!\!/^+ \\ \mathbf{V}^\dagger W\!\!\!/^- & 0 \end{pmatrix}\psi^{\mathrm{L}}_Q$$
$$+\frac{e}{2c_{\mathrm{w}}s_{\mathrm{w}}}\overline{\Psi^{\mathrm{L}}_F}\sigma^3 Z\!\!\!/\Psi^{\mathrm{L}}_F - e\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}}Q_f\overline{\psi_f}Z\!\!\!/\psi_f - eQ_f\overline{\psi_f}A\!\!\!/\psi_f$$

- only charged-current coupling of quarks modified by $\mathbf{V} = U^{\mathrm{L}}_u (U^{\mathrm{L}}_d)^\dagger$ = unitary
  (Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- Higgs–fermion coupling strength = m_f/v

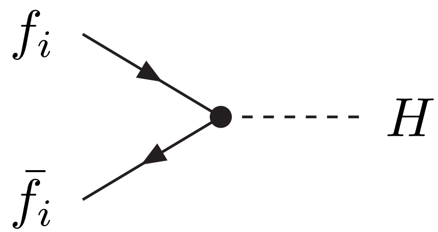
Feynman rules for fermion interactions

quark-mixing matrix $\mathbf{V}_{ij}$ in $\mathrm{W}ff'$ couplings

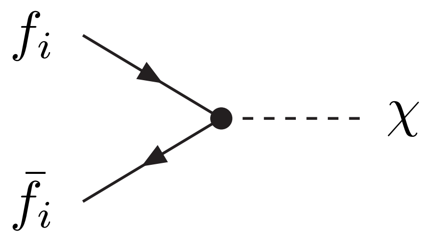


$$\frac{\mathrm{i}e}{\sqrt{2}s_\mathrm{w}}\mathbf{V}_{ij}\gamma_\mu\omega_-$$

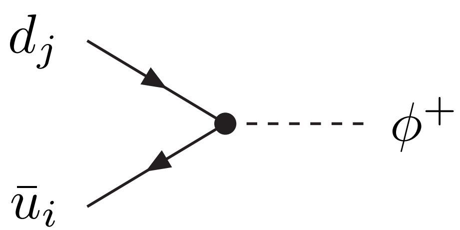
Higgs-boson–fermion couplings (Yukawa couplings)



$$-\frac{\mathrm{i}e}{2s_\mathrm{w}}\frac{m_f}{M_\mathrm{W}}$$



$$-\frac{e}{2s_\mathrm{w}}2I^3_{\mathrm{w},f}\frac{m_f}{M_\mathrm{W}}\gamma_5$$



$$-\frac{\mathrm{i}e}{\sqrt{2}s_\mathrm{w}}\mathbf{V}_{ij}\left(\frac{m_{u,i}}{M_\mathrm{W}}\omega_- - \frac{m_{d,j}}{M_\mathrm{W}}\omega_+\right)$$

Features of the CKM mixing

- $\mathbf{V}$ = 3-dim. generalization of Cabibbo matrix U_C
- $\mathbf{V}$ is parametrized by 4 free parameters: 3 real angles, 1 complex phase
  $\hookrightarrow$ complex phase is the only source of CP violation in SM

  counting:

  $$\begin{pmatrix}\text{\#real d.o.f.}\\ \text{in } \mathbf{V}\end{pmatrix} - \begin{pmatrix}\text{\#unitarity}\\ \text{relations}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ u\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diffs. of}\\ d\text{-type quarks}\end{pmatrix} - \begin{pmatrix}\text{\#phase diff. between}\\ u\text{- and } d\text{-type quarks}\end{pmatrix}$$

  $$= 18 - 9 - 2 - 2 - 1 = 4$$

- no flavour-changing neutral currents in lowest order,
  flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
  ("Glashow–Iliopoulos–Maiani mechanism")

Lagrangian of Electroweak Standard Model

Sum of all contributions (classical Lagrangian)

$$\mathcal{L}_{\mathrm{class}} = \mathcal{L}_{\mathrm{ferm,YM}} + \mathcal{L}_{\mathrm{YM}} + \mathcal{L}_{\mathrm{H}} + \mathcal{L}_{\mathrm{Yuk}} \quad + \mathcal{L}_{\mathrm{QCD}}$$

contains all possible terms that

- can be build from fields of Standard Model
- are gauge-invariant
- are renormalizable (dimension ≤ 4)

these restrictions imply perturbative baryon-number conservation and lepton-number conservation

addition of right-handed neutrinos $\nu_{\mathrm{e}}^{\mathrm{R}}, \nu_{\mu}^{\mathrm{R}}, \nu_{\tau}^{\mathrm{R}}$:
$\Rightarrow$ neutrino masses, mixing matrix in lepton sector

without lepton-number violation $\Rightarrow$ analogously as in quark sector

right-handed neutrinos allow for lepton-number violation
$\Rightarrow$ Majorana neutrinos $\Rightarrow$ new phenomena

Summary

Physical parameters of the Electroweak SM

- gauge sector

  $$g_1, g_2 \to e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}, \quad \cos\theta_\mathrm{w} = \frac{g_2}{\sqrt{g_1^2+g_2^2}} = \frac{M_\mathrm{W}}{M_\mathrm{Z}}$$

  elementary charge, weak mixing angle (2 parameters)

- Higgs sector

  $$\lambda, \mu \to M_\mathrm{H} = \sqrt{2}\mu, \quad M_\mathrm{W} = \frac{g_2}{2}v \quad (v = \frac{2\mu}{\sqrt{\lambda}})$$

  Higgs-boson mass, W-boson mass (2 parameters)

  $$M_\mathrm{Z} = \frac{\sqrt{g_2^2+g_1^2}}{2}v$$ Z-boson mass, weak mixing angle

- flavour sector

  $$G_{l,ij}, G_{u,ij}, G_{d,ij} \to m_{f,i} = \frac{1}{\sqrt{2}} \sum_{k,m} U^\mathrm{L}_{f,ik} G_{f,km} U^{\mathrm{R}\dagger}_{f,mi} v, \quad \mathbf{V} = U^\mathrm{L}_u U^{\mathrm{L}\dagger}_d$$

  fermion masses, quark-mixing matrix (9+4 parameters)

  with right-handed neutrinos: 12+8 parameters
  (3 neutrino masses, 4 parameters of lepton-mixing matrix)

  + 2 extra phases for Majorana neutrinos

Features of the electroweak Standard Model

- particle content verified (apart from Higgs boson)

  recent evidence for Higgs boson with $M_\mathrm{H} \approx 125\,\mathrm{GeV}$

  ATLAS,CMS '12

- GSW model describes experimental observations remarkably well (per-mille level!)

- Input parameters:

  $$\alpha = \frac{e^2}{4\pi} \approx 1/137, \;\; M_\mathrm{W} \approx 80\,\mathrm{GeV}, \;\; M_\mathrm{Z} \approx 91\,\mathrm{GeV}, \;\; M_\mathrm{H}, m_f, V$$

- GSW model = consistent quantum field theory
  - matrix elements respect unitarity
  - renormalizability

  $\Rightarrow$ evaluation of higher perturbative orders possible (and phenomenologically necessary !)

- observation of neutrino oscillations requires right-handed neutrinos, only relevant for neutrino physics

Literature

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