

The Standard Model of Particle Physics

Lecture I

Aspects and properties of the fundamental theory of particle interactions at the electroweak scale

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Outline of Lecture I

- Approaching the study of particles and their interactions:
 - global and local symmetry principles;
 - consequences of broken or hidden symmetries.
- Towards the Standard Model of Particle Physics:
 - main experimental evidence;
 - possible theoretical scenarios.
- The Standard Model of Particle Physics:
 - Lagrangian: building blocks and symmetries;
 - strong interactions: Quantum Chromodynamics;
 - electroweak interactions, the Glashow-Weinberg-Salam theory:
 - properties of charged and neutral currents;
 - breaking the electroweak symmetry: the SM Higgs mechanism.

Particles and forces are a realization of fundamental symmetries of nature

Very old story: Noether's theorem in *classical mechanics*

$$L(q_i, \dot{q}_i) \text{ such that } \frac{\partial L}{\partial q_i} = 0 \longrightarrow p_i = \frac{\partial L}{\partial \dot{q}_i} \text{ conserved}$$

to any symmetry of the Lagrangian is associated a conserved physical quantity:

- ▷ $q_i = x_i \longrightarrow p_i$ linear momentum;
- ▷ $q_i = \theta_i \longrightarrow p_i$ angular momentum.

Generalized to the case of a *relativistic quantum theory* at multiple levels:

- ▷ $q_i \rightarrow \phi_j(x)$ coordinates become “fields” \leftrightarrow “particles”
- ▷ $\mathcal{L}(\phi_j(x), \partial_\mu \phi_j(x))$ can be symmetric under many transformations.
- ▷ To any continuous symmetry of the Lagrangian we can associate a conserved current

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_j)} \delta \phi_j \text{ such that } \partial_\mu J^\mu = 0$$

The symmetries that make the world as we know it

...

- ▷ *translations*:
conservation of energy and momentum;
- ▷ *Lorentz transformations* (rotations and boosts):
conservation of angular momentum (orbital and spin);
- ▷ *discrete transformations* (P,T,C,CP,...):
conservation of corresponding quantum numbers;
- ▷ *global transformations of internal degrees of freedom* (ϕ_j “rotations”)
conservation of “isospin”-like quantum numbers;
- ▷ *local transformations of internal degrees of freedom* ($\phi_j(x)$ “rotations”):
define the interaction of fermion ($s=1/2$) and scalar ($s=0$) particles in terms of exchanged vector ($s=1$) massless particles \longrightarrow “forces”

Requiring different global and local symmetries defines a theory

AND

Keep in mind that they can be broken

From Global to Local: gauging a symmetry

Abelian case

A theory of free Fermi fields described by the Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$$

is invariant under a **global** $U(1)$ transformation ($\alpha = \text{constant}$ phase)

$$\psi(x) \rightarrow e^{i\alpha}\psi(x) \quad \text{such that} \quad \partial_\mu\psi(x) \rightarrow e^{i\alpha}\partial_\mu\psi(x)$$

and the corresponding Noether's current is conserved,

$$J^\mu = \bar{\psi}(x)\gamma^\mu\psi(x) \quad \rightarrow \quad \partial_\mu J^\mu = 0$$

The same is not true for a **local** $U(1)$ transformation ($\alpha = \alpha(x)$) since

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x) \quad \underline{\text{but}} \quad \partial_\mu\psi(x) \rightarrow e^{i\alpha(x)}\partial_\mu\psi(x) + i\partial_\mu\alpha(x)\psi(x)$$

Need to introduce a covariant derivative D_μ such that

$$D_\mu\psi(x) \rightarrow e^{i\alpha(x)}D_\mu\psi(x)$$

Only possibility: introduce a vector field $A_\mu(x)$ transforming as

$$A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{g} \partial_\mu \alpha(x)$$

and define a covariant derivative D_μ according to

$$D_\mu = \partial_\mu + igA_\mu(x)$$

modifying \mathcal{L} to accommodate D_μ and the gauge field $A_\mu(x)$ as

$$\mathcal{L} = \bar{\psi}(x)(i\not{D} - m)\psi(x) - \frac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x)$$

where the last term is the Maxwell Lagrangian for a vector field A^μ , i.e.

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) .$$

Requiring invariance under a local $U(1)$ symmetry has:

- promoted a free theory of fermions to an interacting one;
- defined univoquely the form of the interaction in terms of a new vector field $A^\mu(x)$:

$$\mathcal{L}_{int} = -g\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)$$

- no mass term $A^\mu A_\mu$ allowed by the symmetry → this is **QED**.

Non-abelian case: Yang-Mills theories

Consider the same Lagrangian density

$$\mathcal{L} = \bar{\psi}(x)(i\cancel{\partial} - m)\psi(x)$$

where $\psi(x) \rightarrow \psi_i(x)$ ($i = 1, \dots, n$) is a n -dimensional representation of a non-abelian compact Lie group (e.g. $SU(N)$).

\mathcal{L} is invariant under the **global transformation** $U(\alpha)$

$$\psi(x) \rightarrow \psi'(x) = U(\alpha)\psi(x) \quad , \quad U(\alpha) = e^{i\alpha^a T^a} = 1 + i\alpha^a T^a + O(\alpha^2)$$

where T^a ($(a = 1, \dots, d_{adj})$) are the generators of the group infinitesimal transformations with algebra,

$$[T^a, T^b] = i f^{abc} T^c$$

and the corresponding Noether's current are conserved. However, requiring \mathcal{L} to be invariant under the corresponding **local transformation** $U(x)$

$$U(x) = 1 + i\alpha^a(x)T^a + O(\alpha^2)$$

brings us to replace ∂_μ by a covariant derivative

$$D_\mu = \partial_\mu - igA_\mu^a(x)T^a$$

in terms of vector fields $A_\mu^a(x)$ that transform as

$$A_\mu^a(x) \rightarrow A_\mu^a(x) + \frac{1}{g} \partial_\mu \alpha^a(x) + f^{abc} A_\mu^b(x) \alpha^c(x)$$

such that

$$\begin{aligned} D_\mu &\rightarrow U(x) D_\mu U^{-1}(x) \\ D_\mu \psi(x) &\rightarrow U(x) D_\mu U^{-1}(x) U(x) \psi = U(x) D_\mu \psi(x) \\ F_{\mu\nu} \equiv \frac{i}{g} [D_\mu, D_\nu] &\rightarrow U(x) F_{\mu\nu} U^{-1}(x) \end{aligned}$$

The invariant form of \mathcal{L} or Yang Mills Lagrangian will then be

$$\mathcal{L}_{YM} = \mathcal{L}(\psi, D_\mu \psi) - \frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} = \bar{\psi} (i \not{D} - m) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

where $F_{\mu\nu} = F_{\mu\nu}^a T^a$ and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

We notice that:

- as in the abelian case:

- mass terms of the form $A^{a,\mu} A_\mu^a$ are forbidden by symmetry: gauge bosons are massless
- the form of the interaction between fermions and gauge bosons is fixed by symmetry to be

$$\mathcal{L}_{int} = -g\bar{\psi}(x)\gamma_\mu T^a \psi(x) A^{a,\mu}(x)$$

- at difference from the abelian case:

- gauge bosons carry a group charge and therefore ...
- gauge bosons have self-interaction.

Spontaneous Breaking of a Gauge Symmetry

Abelian Higgs mechanism: one vector field $A^\mu(x)$ and one complex scalar field $\phi(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi$$

where

$$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

and $(D^\mu = \partial^\mu + igA^\mu)$

$$\mathcal{L}_\phi = (D^\mu \phi)^* D_\mu \phi - V(\phi) = (D^\mu \phi)^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

\mathcal{L} invariant under local phase transformation, or local $U(1)$ symmetry:

$$\begin{aligned}\phi(x) &\rightarrow e^{i\alpha(x)}\phi(x) \\ A^\mu(x) &\rightarrow A^\mu(x) + \frac{1}{g}\partial^\mu\alpha(x)\end{aligned}$$

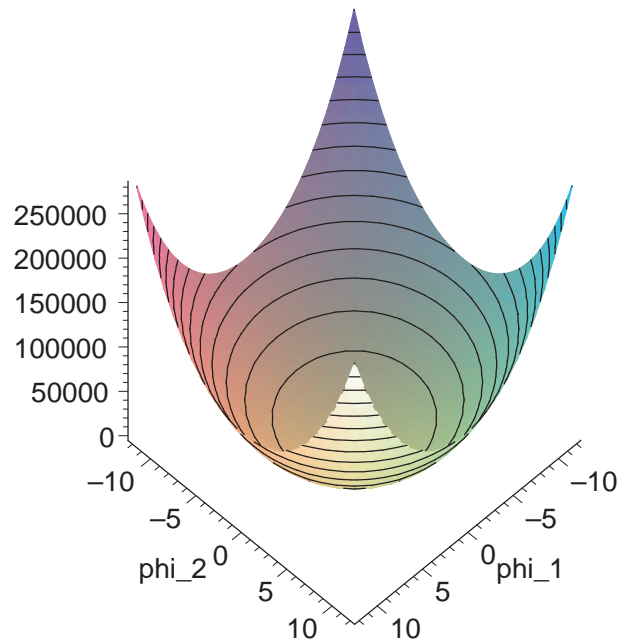
Mass term for A^μ breaks the $U(1)$ gauge invariance.

Can we build a gauge invariant massive theory? Yes.

Consider the potential of the scalar field:

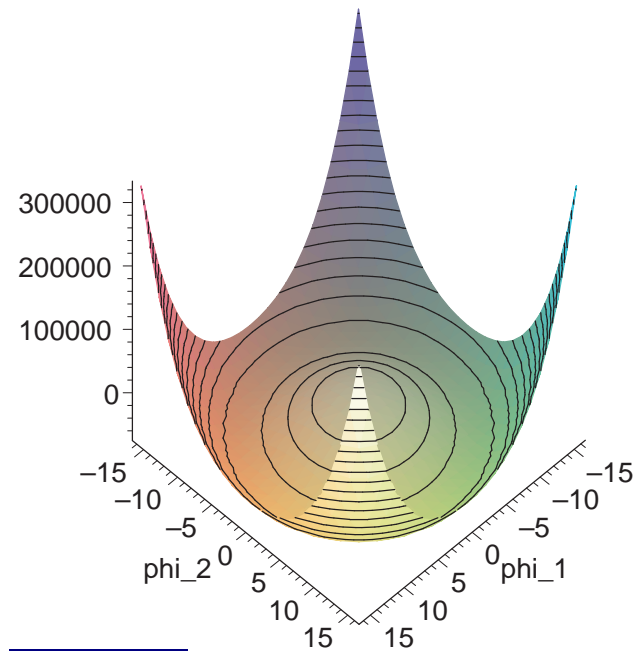
$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

where $\lambda > 0$ (to be bounded from below), and observe that:



$\mu^2 > 0$ → unique minimum:

$$\phi^* \phi = 0$$



$\mu^2 < 0$ → degeneracy of minima:

$$\phi^* \phi = \frac{-\mu^2}{2\lambda}$$

- $\mu^2 > 0 \longrightarrow$ electrodynamics of a massless photon and a massive scalar field of mass μ ($g = -e$).
- $\mu^2 < 0 \longrightarrow$ when we choose a minimum, the original $U(1)$ symmetry is spontaneously broken or hidden.

$$\phi_0 = \left(-\frac{\mu^2}{2\lambda} \right)^{1/2} = \frac{v}{\sqrt{2}} \longrightarrow \phi(x) = \phi_0 + \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x))$$

\Downarrow

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g^2v^2A^\mu A_\mu}_{\text{massive vector field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_1)^2 + \mu^2\phi_1^2}_{\text{massive scalar field}} + \underbrace{\frac{1}{2}(\partial^\mu\phi_2)^2 + gvA_\mu\partial^\mu\phi_2 + \dots}_{\text{Goldstone boson}}$$

Side remark: The ϕ_2 field actually generates the correct transverse structure for the mass term of the (now massive) A^μ field propagator:

$$\langle A^\mu(k)A^\nu(-k) \rangle = \frac{-i}{k^2 - m_A^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) + \dots$$

More convenient parameterization (unitary gauge):

$$\phi(x) = \frac{e^{i\frac{\chi(x)}{v}}}{\sqrt{2}}(v + H(x)) \xrightarrow{U(1)} \frac{1}{\sqrt{2}}(v + H(x))$$

The $\chi(x)$ degree of freedom (Goldstone boson) is rotated away using gauge invariance, while the original Lagrangian becomes:

$$\mathcal{L} = \mathcal{L}_A + \frac{g^2 v^2}{2} A^\mu A_\mu + \frac{1}{2} (\partial^\mu H \partial_\mu H + 2\mu^2 H^2) + \dots$$

which describes now the dynamics of a system made of:

- a massive vector field A^μ with $m_A^2 = g^2 v^2$;
- a real scalar field H of mass $m_H^2 = -2\mu^2 = 2\lambda v^2$: the Higgs field.

⇓

Total number of degrees of freedom is balanced

Non-Abelian Higgs mechanism: several vector fields $A_\mu^a(x)$ and several (real) scalar field $\phi_i(x)$:

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi \quad , \quad \mathcal{L}_\phi = \frac{1}{2}(D^\mu \phi)^2 - V(\phi) \quad , \quad V(\phi) = \mu^2 \phi^2 + \frac{\lambda}{2} \phi^4$$

($\mu^2 < 0$, $\lambda > 0$) invariant under a non-Abelian symmetry group G :

$$\phi_i \longrightarrow (1 + i\alpha^a t^a)_{ij} \phi_j \xrightarrow{t^a = iT^a} (1 - \alpha^a T^a)_{ij} \phi_j$$

(s.t. $D_\mu = \partial_\mu + gA_\mu^a T^a$). In analogy to the Abelian case:

$$\begin{aligned} \frac{1}{2}(D_\mu \phi)^2 &\longrightarrow \dots + \frac{1}{2}g^2 (T^a \phi)_i (T^b \phi)_i A_\mu^a A^{b\mu} + \dots \\ \xrightarrow{\phi_{min} = \phi_0} &\dots + \frac{1}{2}g^2 \underbrace{(T^a \phi_0)_i (T^b \phi_0)_i}_{m_{ab}^2} A_\mu^a A^{b\mu} + \dots = \end{aligned}$$

$\boxed{T^a \phi_0 \neq 0}$ \longrightarrow massive vector boson + (Goldstone boson)

$\boxed{T^a \phi_0 = 0}$ \longrightarrow massless vector boson + massive scalar field

Classical \longrightarrow Quantum : $V(\phi) \longrightarrow V_{eff}(\varphi_{cl})$

The stable vacuum configurations of the theory are now determined by the extrema of the Effective Potential:

$$V_{eff}(\varphi_{cl}) = -\frac{1}{VT}\Gamma_{eff}[\phi_{cl}] \quad , \quad \phi_{cl} = \text{constant} = \varphi_{cl}$$

where

$$\Gamma_{eff}[\phi_{cl}] = W[J] - \int d^4y J(y)\phi_{cl}(y) \quad , \quad \phi_{cl}(x) = \frac{\delta W[J]}{\delta J(x)} = \langle 0|\phi(x)|0\rangle_J$$

$W[J] \longrightarrow$ generating functional of connected correlation functions

$\Gamma_{eff}[\phi_{cl}] \longrightarrow$ generating functional of 1PI connected correlation functions

$V_{eff}(\varphi_{cl})$ can be organized as a loop expansion (expansion in \hbar), s.t.:

$$V_{eff}(\varphi_{cl}) = V(\varphi_{cl}) + \text{loop effects}$$

SSB \longrightarrow non trivial vacuum configurations

Gauge fixing : the R_ξ gauges. Consider the abelian case:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^*D_\mu\phi - V(\phi)$$

upon SSB:

$$\phi(x) = \frac{1}{\sqrt{2}}((v + \phi_1(x)) + i\phi_2(x))$$

\Downarrow

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}(\partial^\mu\phi_1 + gA^\mu\phi_2)^2 + \frac{1}{2}(\partial^\mu\phi_2 - gA^\mu(v + \phi_1))^2 - V(\phi)$$

Quantizing using the gauge fixing condition:

$$G = \frac{1}{\sqrt{\xi}}(\partial_\mu A^\mu + \xi g v \phi_2)$$

in the generating functional

$$Z = C \int \mathcal{D}A \mathcal{D}\phi_1 \mathcal{D}\phi_2 \exp \left[\int d^4x \left(\mathcal{L} - \frac{1}{2}G^2 \right) \right] \det \left(\frac{\delta G}{\delta \alpha} \right)$$

($\alpha \longrightarrow$ gauge transformation parameter)

$$\begin{aligned}
\mathcal{L} - \frac{1}{2}G^2 &= -\frac{1}{2}A_\mu \left(-g^{\mu\nu} \partial^2 + \left(1 - \frac{1}{\xi}\right) \partial^\mu \partial^\nu - (gv)^2 g^{\mu\nu} \right) A_\nu \\
&\quad + \frac{1}{2}(\partial_\mu \phi_1)^2 - \frac{1}{2}m_{\phi_1}^2 \phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{\xi}{2}(gv)^2 \phi_2^2 + \dots \\
\mathcal{L}_{ghost} &= \bar{c} \left[-\partial^2 - \xi(gv)^2 \left(1 + \frac{\phi_1}{v}\right) \right] c
\end{aligned}$$

such that:

$$\langle A^\mu(k) A^\nu(-k) \rangle = \frac{-i}{k^2 - m_A^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) + \frac{-i\xi}{k^2 - \xi m_A^2} \left(\frac{k^\mu k^\nu}{k^2} \right)$$

$$\langle \phi_1(k) \phi_1(-k) \rangle = \frac{-i}{k^2 - m_{\phi_1}^2}$$

$$\langle \phi_2(k) \phi_2(-k) \rangle = \langle c(k) \bar{c}(-k) \rangle = \frac{-i}{k^2 - \xi m_A^2}$$

Goldstone boson ϕ_2 , \iff longitudinal gauge bosons

Towards the Standard Model of particle physics

Translating experimental evidence into the right gauge symmetry group.

- Electromagnetic interactions \rightarrow QED
 - ▷ well established example of abelian gauge theory
 - ▷ extremely successful quantum implementation of field theories
 - ▷ useful but very simple template
- Strong interactions \rightarrow QCD
 - ▷ evidence for strong force in hadronic interactions
 - ▷ Gell-Mann-Nishijima quark model interprets hadron spectroscopy
 - ▷ need for extra three-fold quantum number (color)
(ex.: hadronic spectroscopy, $e^+e^- \rightarrow$ hadrons, ...)
 - ▷ natural to introduce the gauge group $\rightarrow SU(3)_C$
 - ▷ DIS experiments: confirm parton model based on $SU(3)_C$
 - ▷ ... and much more!
- Weak interactions \rightarrow most puzzling ...
 - ▷ discovered in neutron β -decay: $n \rightarrow p + e^- + \bar{\nu}_e$
 - ▷ new force: small rates/long lifetimes
 - ▷ universal: same strength for both hadronic and leptonic processes
($n \rightarrow pe^- \nu_e$, $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$, $\mu^- \rightarrow e^- \bar{\nu}_e + \nu_\mu$, ...)

- ▷ violates parity (P)
- ▷ charged currents only affect left-handed particles (right-handed antiparticles)
- ▷ neutral currents not of electromagnetic nature
- ▷ First description: **Fermi Theory** (1934)

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} (\bar{p}\gamma_\mu(1 - \gamma_5)n)(\bar{e}\gamma^\mu(1 - \gamma_5)\nu_e)$$

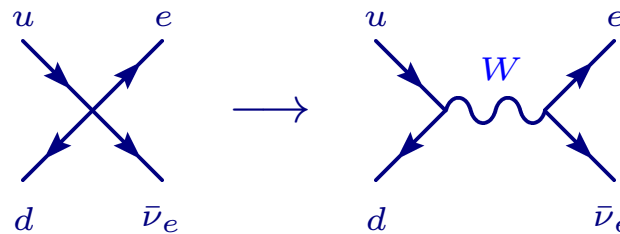
$G_F \rightarrow$ Fermi constant, $[G_F] = m^{-2}$ (in units of $c = \hbar = 1$).

- ▷ Easily accomodates a massive intermediate vector boson

$$\mathcal{L}_{IVB} = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^- + \text{h.c.}$$

with (in a proper quark-based notation)

$$J_\mu^- = \bar{u}\gamma_\mu \frac{1 - \gamma_5}{2} d + \bar{\nu}_e\gamma^\mu \frac{1 - \gamma_5}{2} e$$



provided that,

$$q^2 \ll M_W^2 \longrightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

- ▷ Promote it to a gauge theory: natural candidate $SU(2)_L$, but if $T^{1,2}$ can generate the charged currents ($T^\pm = (T^1 \pm iT^2)$), T^3 cannot be the electromagnetic charge (Q) ($T^3 = \sigma^3/2$'s eigenvalues do not match charges in $SU(2)$ doublets)
- ▷ Need extra $U(1)_Y$, such that $Y = T^3 - Q$!
- ▷ Need massive gauge bosons \rightarrow SSB

\Downarrow

$$SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$$

$$\mathcal{L}_{SM} = \mathcal{L}_{QCD} + \mathcal{L}_{EW}$$

where

$$\mathcal{L}_{EW} = \mathcal{L}_{EW}^{\text{ferm}} + \mathcal{L}_{EW}^{\text{gauge}} + \mathcal{L}_{EW}^{SSM} + \mathcal{L}_{EW}^{Yukawa}$$

Strong interactions: Quantum Chromodynamics

Exact Yang-Mills theory based on $SU(3)_C$ (quark fields only):

$$\mathcal{L}_{QCD} = \sum_i \bar{Q}_i (i\not{D} - m_i) Q_i - \frac{1}{4} F^{a,\mu\nu} F_{\mu\nu}^a$$

with

$$D_\mu = \partial_\mu - ig A_\mu^a T^a$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$

- $Q_i \rightarrow (i = 1, \dots, 6 \rightarrow u, d, s, c, b, t)$ fundamental representation of $SU(3) \rightarrow$ triplets:

$$Q_i = \begin{pmatrix} Q_i \\ Q_i \\ Q_i \end{pmatrix}$$

- $A_\mu^a \rightarrow$ adjoint representation of $SU(3) \rightarrow N^2 - 1 = 8$ massless gluons
 $T^a \rightarrow SU(3)$ generators (Gell-Mann's matrices)

Electromagnetic and weak interactions: unified into Glashow-Weinberg-Salam theory

Spontaneously broken Yang-Mills theory based on $SU(2)_L \times U(1)_Y$.

- $SU(2)_L \rightarrow$ weak isospin group, gauge coupling g :
 - ▷ three generators: $T^i = \sigma^i/2$ ($\sigma^i =$ Pauli matrices, $i = 1, 2, 3$)
 - ▷ three gauge bosons: W_1^μ , W_2^μ , and W_3^μ
 - ▷ $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ fields are doublets of $SU(2)$
 - ▷ $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$ fields are singlets of $SU(2)$
 - ▷ mass terms not allowed by gauge symmetry
- $U(1)_Y \rightarrow$ weak hypercharge group ($Q = T_3 + Y$), gauge coupling g' :
 - ▷ one generator \rightarrow each field has a Y charge
 - ▷ one gauge boson: B^μ

Example: first generation

$$L_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}_{Y=-1/2} \quad (\nu_{eR})_{Y=0} \quad (e_R)_{Y=-1}$$
$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}_{Y=1/6} \quad (u_R)_{Y=2/3} \quad (d_R)_{Y=-1/3}$$

Three fermionic generations, summary of gauge quantum numbers:

				<u>$SU(3)_C$</u>	<u>$SU(2)_L$</u>	<u>$U(1)_Y$</u>	<u>$U(1)_Q$</u>
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$	0 -1
$e_R^i =$	e_R	μ_R	τ_R	1	1	-1	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

where a minimal extension to include ν_R^i has been allowed (notice however that it has zero charge under the entire SM gauge group!)

Lagrangian of fermion fields

For each generation (here specialized to the first generation):

$$\mathcal{L}_{EW}^{\text{ferm}} = \bar{L}_L(i\not{D})L_L + \bar{e}_R(i\not{D})e_R + \bar{\nu}_{eR}(i\not{D})\nu_{eR} + \bar{Q}_L(i\not{D})Q_L + \bar{u}_R(i\not{D})u_R + \bar{d}_R(i\not{D})d_R$$

where in each term the covariant derivative is given by

$$D_\mu = \partial_\mu - igW_\mu^i T^i - ig' \frac{1}{2} Y B_\mu$$

and $T^i = \sigma^i/2$ for L-fields, while $T^i = 0$ for R-fields ($i = 1, 2, 3$), i.e.

$$D_{\mu,L} = \partial_\mu - \frac{ig}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} gW_\mu^3 - g'Y B_\mu & 0 \\ 0 & -gW_\mu^3 - g'Y B_\mu \end{pmatrix}$$

$$D_{\mu,R} = \partial_\mu + ig' \frac{1}{2} Y B_\mu$$

with

$$W^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2)$$

$\mathcal{L}_{EW}^{\text{ferm}}$ can then be written as

$$\mathcal{L}_{EW}^{\text{ferm}} = \mathcal{L}_{kin}^{\text{ferm}} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

where

$$\mathcal{L}_{kin}^{\text{ferm}} = \bar{L}_L(i\partial)L_L + \bar{e}_R(i\partial)e_R + \dots$$

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} W_\mu^+ \bar{\nu}_{eL} \gamma^\mu e_L + W_\mu^- \bar{e}_L \gamma^\mu \nu_{eL} + \dots$$

$$\begin{aligned} \mathcal{L}_{NC} &= \frac{g}{2} W_\mu^3 [\bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \bar{e}_L \gamma^\mu e_L] + \frac{g'}{2} B_\mu [Y(L)(\bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{e}_L \gamma^\mu e_L) \\ &+ Y(e_R) \bar{\nu}_{eR} \gamma^\mu \nu_{eR} + Y(e_R) \bar{e}_R \gamma^\mu e_R] + \dots \end{aligned}$$

where

$W^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2) \rightarrow$ mediators of **Charged Currents**

W_μ^3 and $B_\mu \rightarrow$ mediators of **Neutral Currents**.

↓

However neither W_μ^3 nor B_μ can be identified with the photon field A_μ , because they couple to neutral fields.

Rotate W_μ^3 and B_μ introducing a weak mixing angle (θ_W)

$$\begin{aligned} W_\mu^3 &= \sin \theta_W A_\mu + \cos \theta_W Z_\mu \\ B_\mu &= \cos \theta_W A_\mu - \sin \theta_W Z_\mu \end{aligned}$$

such that the kinetic terms are still diagonal and the neutral current lagrangian becomes

$$\mathcal{L}_{NC} = \bar{\psi} \gamma^\mu \left(g \sin \theta_W T^3 + g' \cos \theta_W \frac{Y}{2} \right) \psi A_\mu + \bar{\psi} \gamma^\mu \left(g \cos \theta_W T^3 - g' \sin \theta_W \frac{Y}{2} \right) \psi Z_\mu$$

for $\psi^T = (\nu_{eL}, e_L, \nu_{eR}, e_R, \dots)$. One can then identify ($Q \rightarrow$ e.m. charge)

$$eQ = g \sin \theta_W T^3 + g' \cos \theta_W \frac{Y}{2}$$

and, e.g., from the leptonic doublet L_L derive that

$$\begin{cases} \frac{g}{2} \sin \theta_W - \frac{g'}{2} \cos \theta_W = 0 \\ -\frac{g}{2} \sin \theta_W - \frac{g'}{2} \cos \theta_W = -e \end{cases} \longrightarrow g \sin \theta_W = g' \cos \theta_W = e$$

$$\begin{aligned}
 & \begin{array}{c} i \\ \nearrow \\ \nearrow \\ j \end{array} \text{---} A^\mu = -ieQ_f \gamma^\mu \\
 & \begin{array}{c} j \\ i \\ \nearrow \\ \nearrow \\ j \end{array} \text{---} W^\mu = \frac{ie}{2\sqrt{2}s_w} \gamma^\mu (1 - \gamma_5) \\
 & \begin{array}{c} j \\ i \\ \nearrow \\ \nearrow \\ j \end{array} \text{---} Z^\mu = ie\gamma^\mu (v_f - a_f \gamma_5)
 \end{aligned}$$

where

$$\begin{aligned}
 v_f &= -\frac{s_w}{c_w} Q_f + \frac{T_f^3}{2s_w c_w} \\
 a_f &= \frac{T_f^3}{2s_w c_w}
 \end{aligned}$$

Lagrangian of gauge fields

$$\mathcal{L}_{EW}^{\text{gauge}} = -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

where

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc}W_\mu^b W_\nu^c \end{aligned}$$

in terms of physical fields:

$$\mathcal{L}_{EW}^{\text{gauge}} = \mathcal{L}_{kin}^{\text{gauge}} + \mathcal{L}_{EW}^{3V} + \mathcal{L}_{EW}^{4V}$$

where

$$\begin{aligned} \mathcal{L}_{kin}^{\text{gauge}} &= -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\ &\quad - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ \mathcal{L}_{EW}^{3V} &= (3\text{-gauge-boson vertices involving } ZW^+W^- \text{ and } AW^+W^-) \\ \mathcal{L}_{EW}^{4V} &= (4\text{-gauge-boson vertices involving } ZZW^+W^-, AAW^+W^-, \\ &\quad AZW^+W^-, \text{ and } W^+W^-W^+W^-) \end{aligned}$$

$$\begin{array}{c} k \\ \text{wavy line} \\ \mu \quad \nu \end{array} = \frac{-i}{k^2 - M_V^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_V^2} \right)$$

$$\begin{array}{c} W_\mu^+ \\ \text{wavy line} \\ \text{wavy line} \\ V_\rho \end{array} = ieC_V [g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu]$$

$$\begin{array}{c} W_\mu^- \\ W_\mu^+ \\ \text{wavy line} \\ \text{wavy line} \\ W_\nu^- \\ V'_\sigma \end{array} = ie^2 C_{VV'} (2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$$

where

$$C_\gamma = 1, \quad C_Z = -\frac{c_W}{s_W}$$

and

$$C_{\gamma\gamma} = -1, \quad C_{ZZ} = -\frac{c_W^2}{s_W^2}, \quad C_{\gamma Z} = \frac{c_W}{s_W}, \quad C_{WW} = \frac{1}{s_W^2}$$

The Higgs sector of the Standard Model: $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_Q$

Introduce one **complex scalar doublet** of $SU(2)_L$ with $Y = 1/2$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longleftrightarrow \mathcal{L}_{EW}^{SSB} = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

where $D_\mu \phi = (\partial_\mu - igW_\mu^a T^a - ig'Y_\phi B_\mu)$, ($T^a = \sigma^a/2$, $a=1, 2, 3$).

The SM symmetry is spontaneously broken when $\langle \phi \rangle$ is chosen to be (e.g.):

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{with} \quad v = \left(\frac{-\mu^2}{\lambda} \right)^{1/2} \quad (\mu^2 < 0, \lambda > 0)$$

The **gauge boson mass terms** arise from:

$$\begin{aligned} (D^\mu \phi)^\dagger D_\mu \phi &\longrightarrow \dots + \frac{1}{8} (0 \ v) (gW_\mu^a \sigma^a + g' B_\mu) (gW^{b\mu} \sigma^b + g' B^\mu) \begin{pmatrix} 0 \\ v \end{pmatrix} + \dots \\ &\longrightarrow \dots + \frac{1}{2} \frac{v^2}{4} [g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-gW_\mu^3 + g' B_\mu)^2] + \dots \end{aligned}$$

And correspond to the weak gauge bosons:

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(W_{\mu}^1 \mp iW_{\mu}^2) \longrightarrow \boxed{M_W = g\frac{v}{2}}$$

$$Z_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(gW_{\mu}^3 - g'B_{\mu}) \longrightarrow \boxed{M_Z = \sqrt{g^2 + g'^2}\frac{v}{2}}$$

while the linear combination orthogonal to Z_{μ} remains massless and corresponds to the photon field:

$$A_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}(g'W_{\mu}^3 + gB_{\mu}) \longrightarrow \boxed{M_A = 0}$$

Notice: using the definition of the weak mixing angle, θ_w :

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

the W and Z masses are related by: $\boxed{M_W = M_Z \cos \theta_w}$

The scalar sector becomes more transparent in the unitary gauge:

$$\phi(x) = \frac{e^{\frac{i}{v}\vec{\chi}(x)\cdot\vec{\tau}}}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \xrightarrow{SU(2)} \phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

after which the Lagrangian becomes

$$\mathcal{L} = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4$$

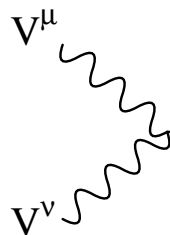
Three degrees of freedom, the $\chi^a(x)$ Goldstone bosons, have been reabsorbed into the longitudinal components of the W_μ^\pm and Z_μ weak gauge bosons. One real scalar field remains:

the Higgs boson, H, with mass $M_H^2 = -2\mu^2 = 2\lambda v^2$

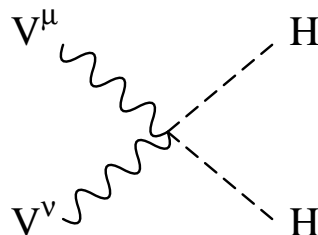
and self-couplings:

$$\begin{array}{c}
 \text{H} \\
 \text{H}
 \end{array}
 \begin{array}{c}
 \diagdown \\
 \diagup
 \end{array}
 \text{H} = -3i \frac{M_H^2}{v}
 \qquad
 \begin{array}{c}
 \text{H} \\
 \text{H}
 \end{array}
 \begin{array}{c}
 \diagdown \quad \diagup \\
 \diagup \quad \diagdown
 \end{array}
 \text{H} = -3i \frac{M_H^2}{v^2}$$

From $(D^\mu\phi)^\dagger D_\mu\phi \longrightarrow$ Higgs-Gauge boson couplings:



$$= 2i \frac{M_V^2}{v} g^{\mu\nu}$$



$$= 2i \frac{M_V^2}{v^2} g^{\mu\nu}$$

Notice: The entire Higgs sector depends on only **two** parameters, e.g.

M_H and v

v measured in μ -decay:

$$v = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV}$$

\longrightarrow

SM Higgs Physics depends on M_H

Higgs boson couplings to quarks and leptons

The gauge symmetry of the SM also forbids fermion mass terms ($m_{Q_i} Q_L^i u_R^i, \dots$), but all fermions are massive.

⇓

Fermion masses are generated via gauge invariant Yukawa couplings:

$$\mathcal{L}_{EW}^{Yukawa} = -\Gamma_u^{ij} \bar{Q}_L^i \phi^c u_R^j - \Gamma_d^{ij} \bar{Q}_L^i \phi d_R^j - \Gamma_e^{ij} \bar{L}_L^i \phi l_R^j + \text{h.c.}$$

such that, upon spontaneous symmetry breaking:

$$\begin{aligned} \mathcal{L}_{EW}^{Yukawa} &= -\Gamma_u^{ij} \bar{u}_L^i \frac{v+H}{\sqrt{2}} u_R^j - \Gamma_d^{ij} \bar{d}_L^i \frac{v+H}{\sqrt{2}} d_R^j - \Gamma_e^{ij} \bar{l}_L^i \frac{v+H}{\sqrt{2}} l_R^j + \text{h.c.} \\ &= -\sum_{f,i,j} \bar{f}_L^i M_f^{ij} f_R^j \left(1 + \frac{H}{v}\right) + \text{h.c.} \end{aligned}$$

where

$$M_f^{ij} = \Gamma_f^{ij} \frac{v}{\sqrt{2}}$$

is a non-diagonal mass matrix.

Upon diagonalization (by unitary transformation U_L and U_R)

$$M_D = (U_L^f)^\dagger M_f U_R^f$$

and defining mass eigenstates:

$$f_L'^i = (U_L^f)_{ij} f_L^j \quad \text{and} \quad f_R'^i = (U_R^f)_{ij} f_R^j$$

the fermion masses are extracted as

$$\begin{aligned} \mathcal{L}_{EW}^{Yukawa} &= \sum_{f,i,j} \bar{f}_L'^i [(U_L^f)^\dagger M_f U_R^f] f_R'^j \left(1 + \frac{H}{v}\right) + \text{h.c.} \\ &= \sum_{f,i,j} m_f (\bar{f}_L' f_R' + \bar{f}_R' f_L') \left(1 + \frac{H}{v}\right) \end{aligned}$$

$$\begin{array}{c} f \\ \searrow \\ \text{---} \\ \nearrow \\ \bar{f} \end{array} \text{---} H = -i \frac{m_f}{v} = -i y_t$$

In terms of the new mass eigenstates the quark part of \mathcal{L}_{CC} now reads

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \bar{u}'_L{}^i [(U_L^u)^\dagger U_R^d] \gamma^\mu d_L^j + \text{h.c.}$$

where

$$V_{CKM} = (U_L^u)^\dagger U_R^d$$

is the Cabibbo-Kobayashi-Maskawa matrix, origin of flavor mixing in the SM.



see G. Buchalla's lectures at this school