



Electroweak Aspects of the Standard Model



Lecture III: Weak Gauge Bosons

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Overview

- 1 **Electroweak precision physics at e^+e^- colliders**
- 2 **Cross sections at hadron colliders**
- 3 **W- and Z-boson production at hadron colliders**



Electroweak precision physics at e^+e^- colliders



1.1 Brief overview

Features of the electroweak Standard Model

- **Higgs boson not yet found**, particle content verified otherwise
- **No really significant contradictions** of GSW model with experiment

- Input parameters:

$$\alpha = \frac{e^2}{4\pi} \approx 1/137, \quad M_W \approx 80 \text{ GeV}, \quad M_Z \approx 91 \text{ GeV}, \quad M_H \gtrsim 100 \text{ GeV}, \quad m_f, \quad V$$

- **GSW model = consistent quantum field theory**

- ◇ matrix elements respect unitarity
- ◇ renormalizability

⇒ evaluation of higher perturbative orders possible
(and phenomenologically necessary !)

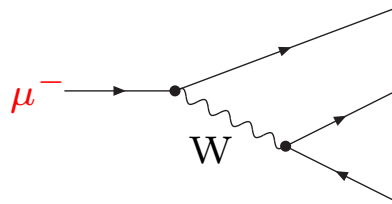


Important electroweak experiments

- **Muon decay:**

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$$

determination of the **Fermi constant**

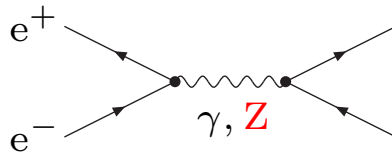


$$G_\mu = \frac{\pi\alpha M_Z^2}{\sqrt{2}M_W^2(M_Z^2 - M_W^2)} + \dots$$

- **Z production (LEP1/SLC):**

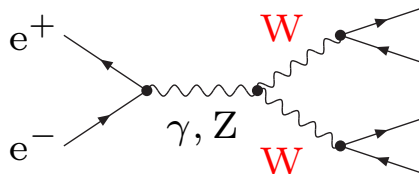
$$e^+e^- \rightarrow Z \rightarrow f\bar{f}$$

various precision measurements at the Z resonance: $M_Z, \Gamma_Z, \sigma_{\text{had}}, A_{\text{FB}}, A_{\text{LR}}, \text{etc.}$



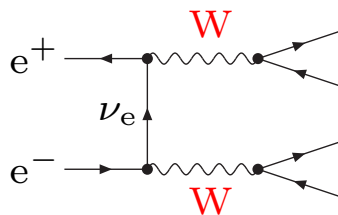
⇒ **good knowledge of the $Zf\bar{f}$ sector**

- **W-pair production (LEP2/ILC):** $e^+e^- \rightarrow WW \rightarrow 4f(+\gamma)$



– measurement of M_W

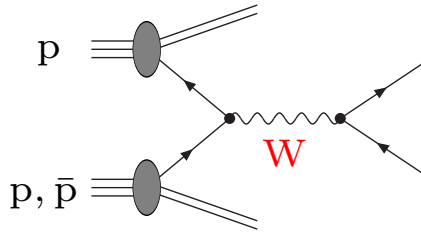
– $\gamma WW/ZWW$ couplings



– quartic couplings: $\gamma\gamma WW, \gamma ZWW$
(via $e^+e^- \rightarrow WW + \gamma$)

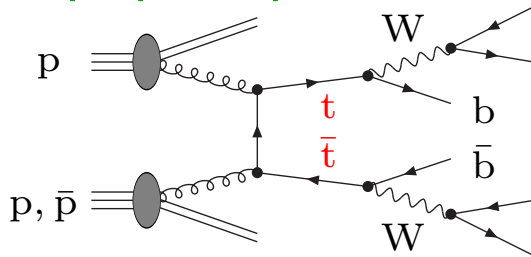
Important electroweak experiments (continued)

- **W production** (Tevatron/LHC): $pp, p\bar{p} \rightarrow W \rightarrow l\nu_l(+\gamma)$



- measurement of M_W
- bounds on γWW coupling

- **top-quark production** (Tevatron/LHC): $pp, p\bar{p} \rightarrow t\bar{t} \rightarrow 6f$



- measurement of m_t

Theoretical predictions

parametrized by $\alpha(M_Z)$, M_W , M_Z , m_t , m_f , $\alpha_s(M_Z)$ and M_H

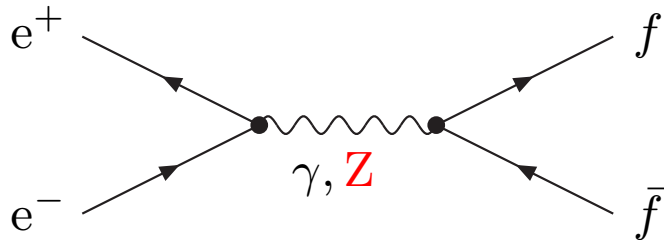
↪ global fit of SM to data yields bounds on M_H

But: high precision necessary,
since M_H sensitivity weak $\sim \frac{\alpha}{\pi} \log(M_H/M_W)$

(in contrast to top-loops where sensitivity $\sim G_\mu m_t^2$)

1.2 Z-boson physics at LEP1 and SLC

Precision study of the Z line shape

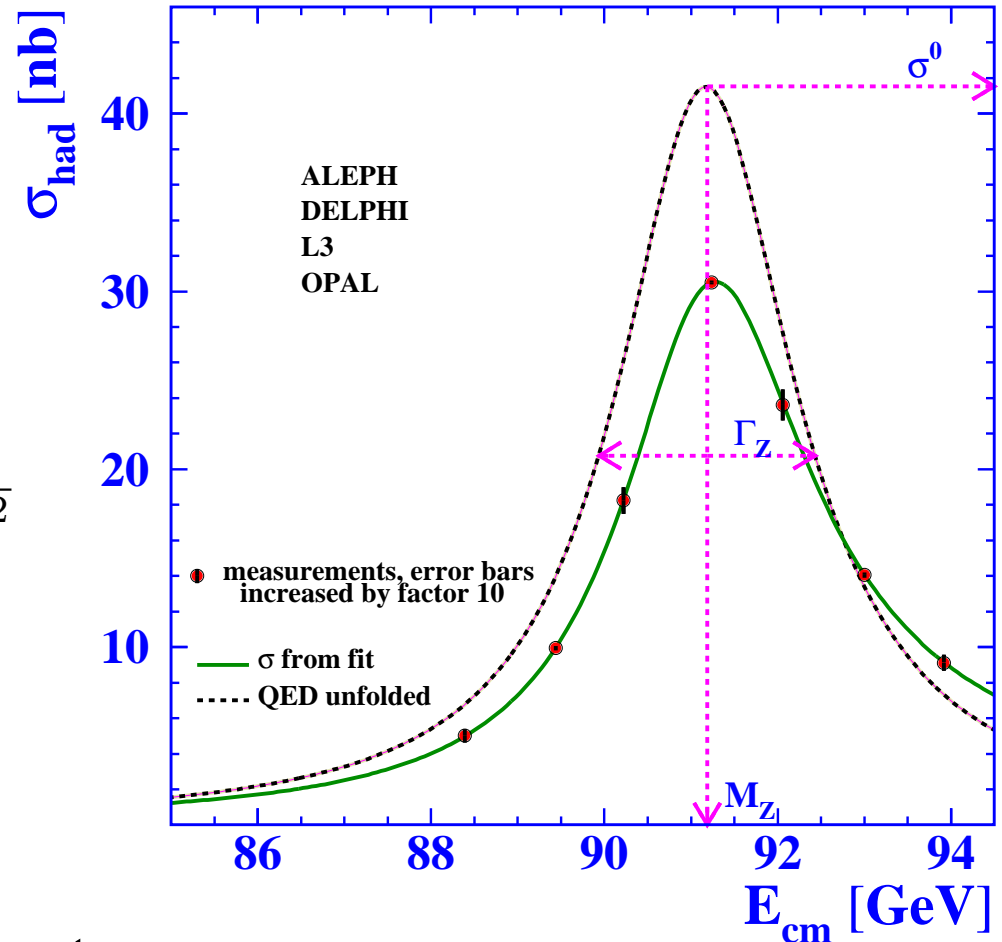


Unfolded resonance:

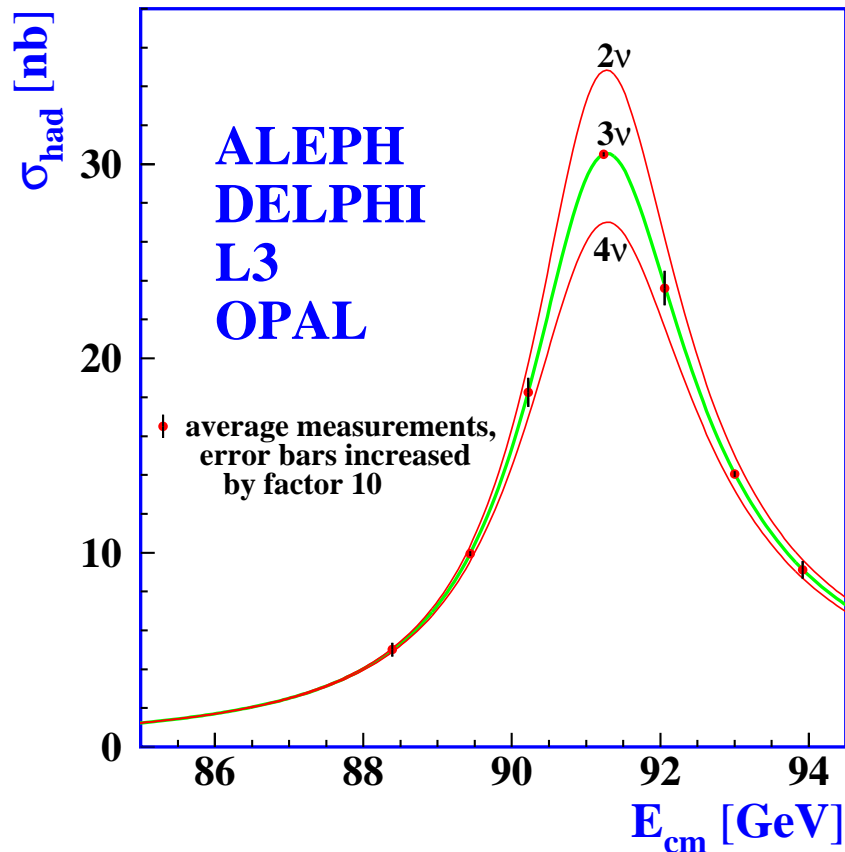
$$\sigma_{\text{res}}(s) = \sigma^0 \frac{s \Gamma_Z^2}{\left| s - M_Z^2 + i M_Z \Gamma_Z \frac{s}{M_Z^2} \right|^2}$$

Resonance observables:

- **Z mass** and **width**: M_Z, Γ_Z
- **peak cross section**: σ_{had}^0
- various asymmetries: $A_{\text{FB}}, A_{\text{LR}}$, etc.
- ratios of decay widths: $R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l}$, etc.



Number of light neutrinos



$$\Gamma_Z = \Gamma_{\text{had}} + \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{inv}}$$

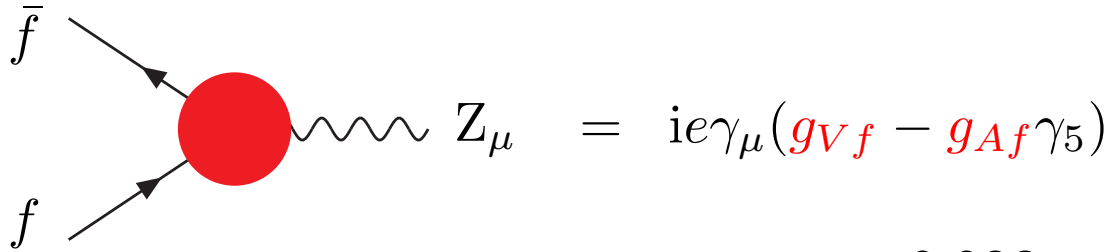
- Γ_Z measured from Z line shape
- Γ_{had} and $\Gamma_{l=e,\mu,\tau}$ from

$$R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l} \quad \text{and} \quad \sigma_{\text{had}}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}$$

Fit of Γ_Z , R_l , and σ_{had}^0 yields invisible Z-decay width: $\Gamma_{\text{inv}} = N_\nu \Gamma_{Z \rightarrow \nu\bar{\nu}}^{\text{theory}}$

$$\hookrightarrow N_\nu = 2.9840 \pm 0.0082$$

Effective Z-boson–fermion couplings



Leptonic couplings from LEP1
asymmetry measurements, e.g.:

$$A_{\text{FB}}^{0,f} = \frac{\sigma_{f,F}^0 - \sigma_{f,B}^0}{\sigma_{f,F}^0 + \sigma_{f,B}^0} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

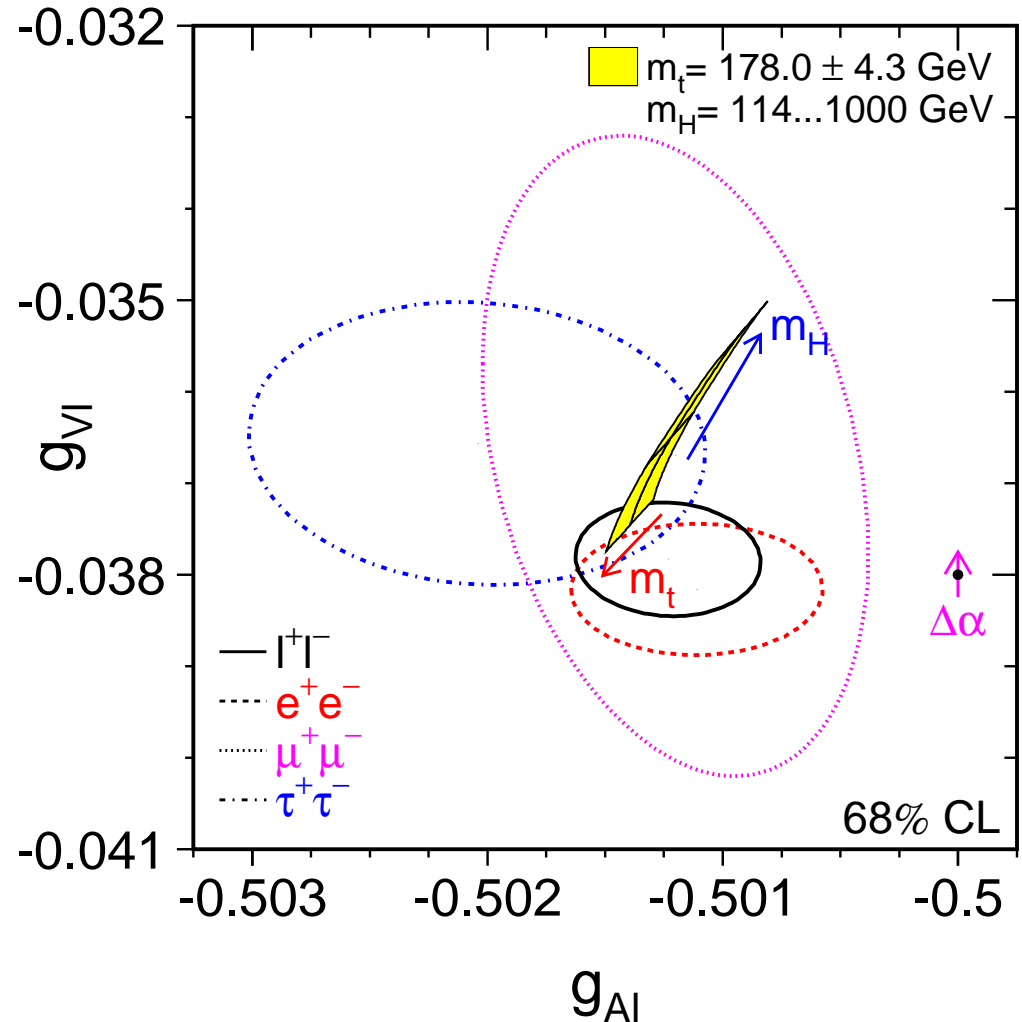
(F/B = For/Backward hemisphere)

$$\text{with } \mathcal{A}_f = \frac{2g_V f g_A f}{g_V f^2 + g_A f^2}$$

Good agreement with SM

- lepton universality confirmed
- constraints on m_t and M_H

LEPEWWG '05



Translation of effective couplings into effective weak mixing angle

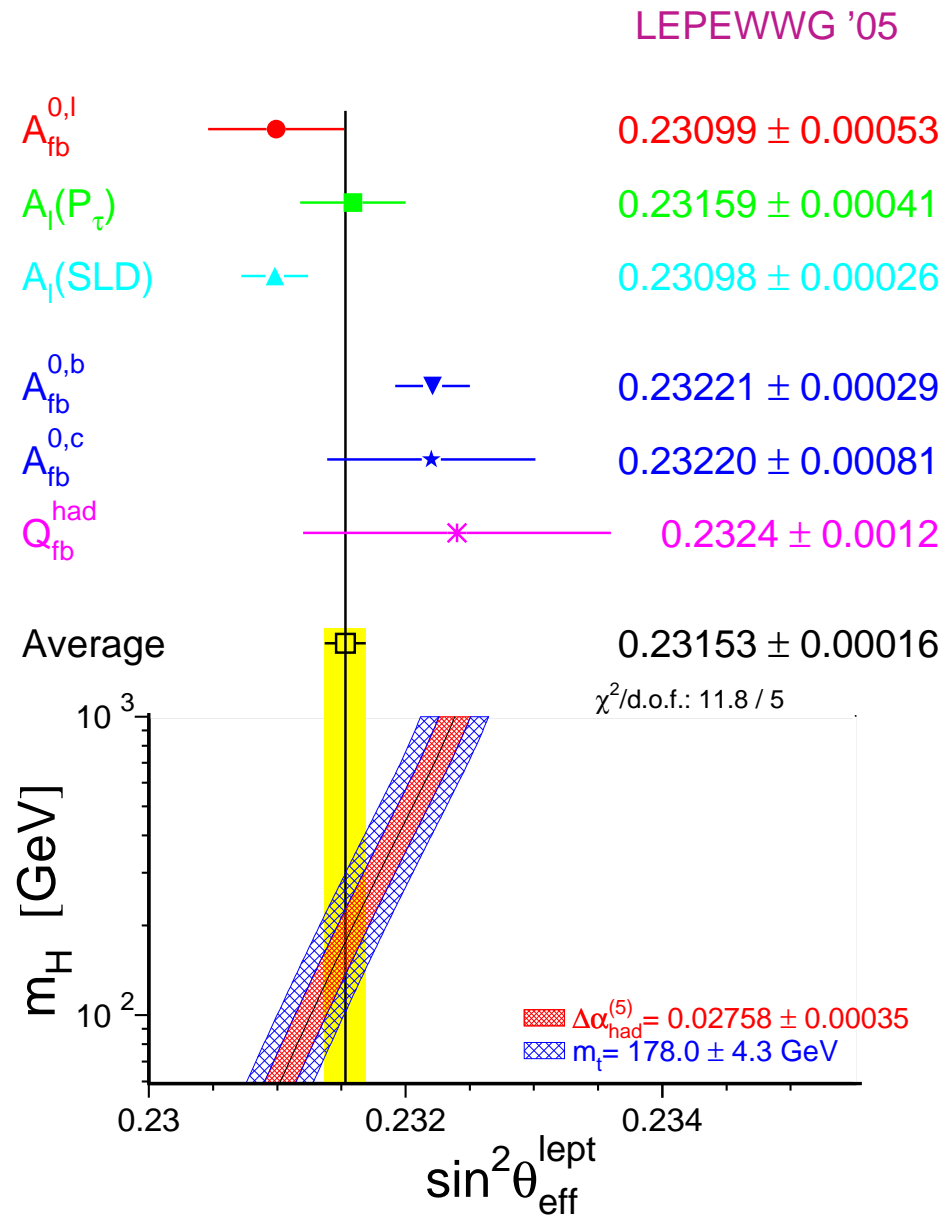
$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \frac{1}{4} \left(1 - \text{Re} \left\{ \frac{g_{Vl}}{g_{Al}} \right\} \right)$$

Important features:

- high sensitivity to M_H
- combination of very different observables
- $\sim 3\sigma$ difference between $A_{\text{FB}}^{0,b}$ (LEP) and $A_{\text{LR}}^{0,l}$ (SLD)

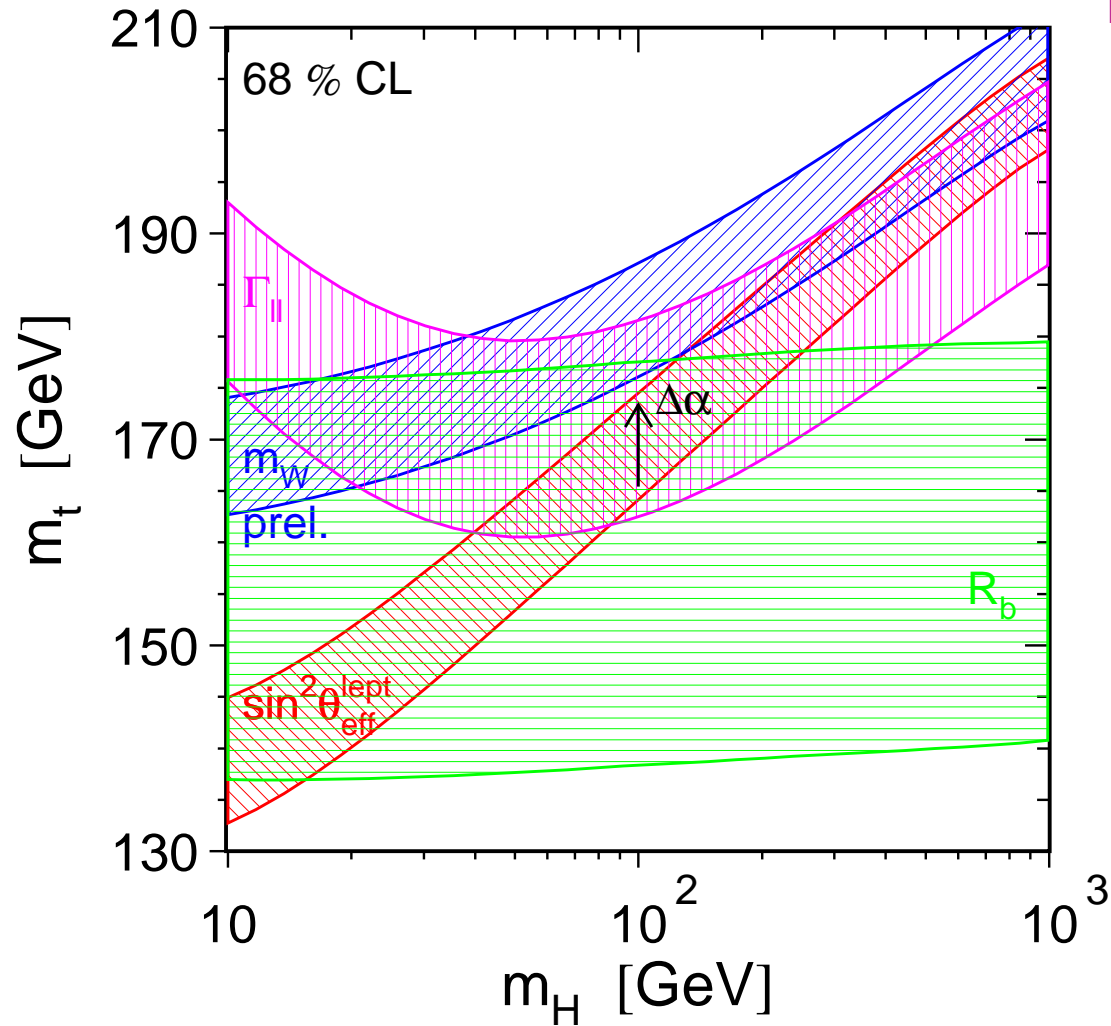
with the initial-state pol. asymmetry

$$A_{\text{LR}}^{0,l} = \frac{\sigma_L^0 - \sigma_R^0}{\sigma_L^0 + \sigma_R^0} \frac{1}{\langle |\mathcal{P}_e| \rangle}$$

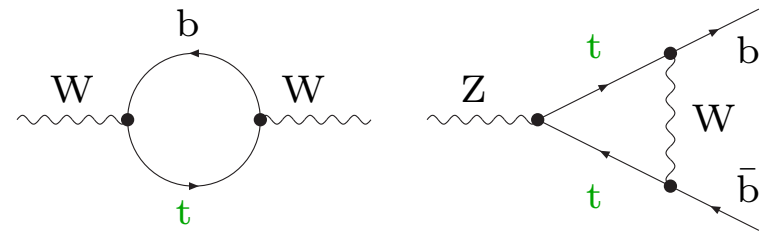


Observables most sensitive to m_t and M_H

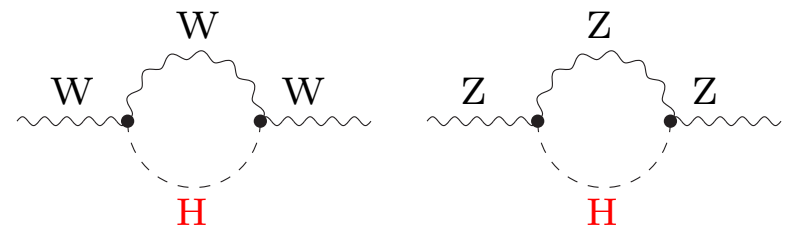
LEPEWWG '05



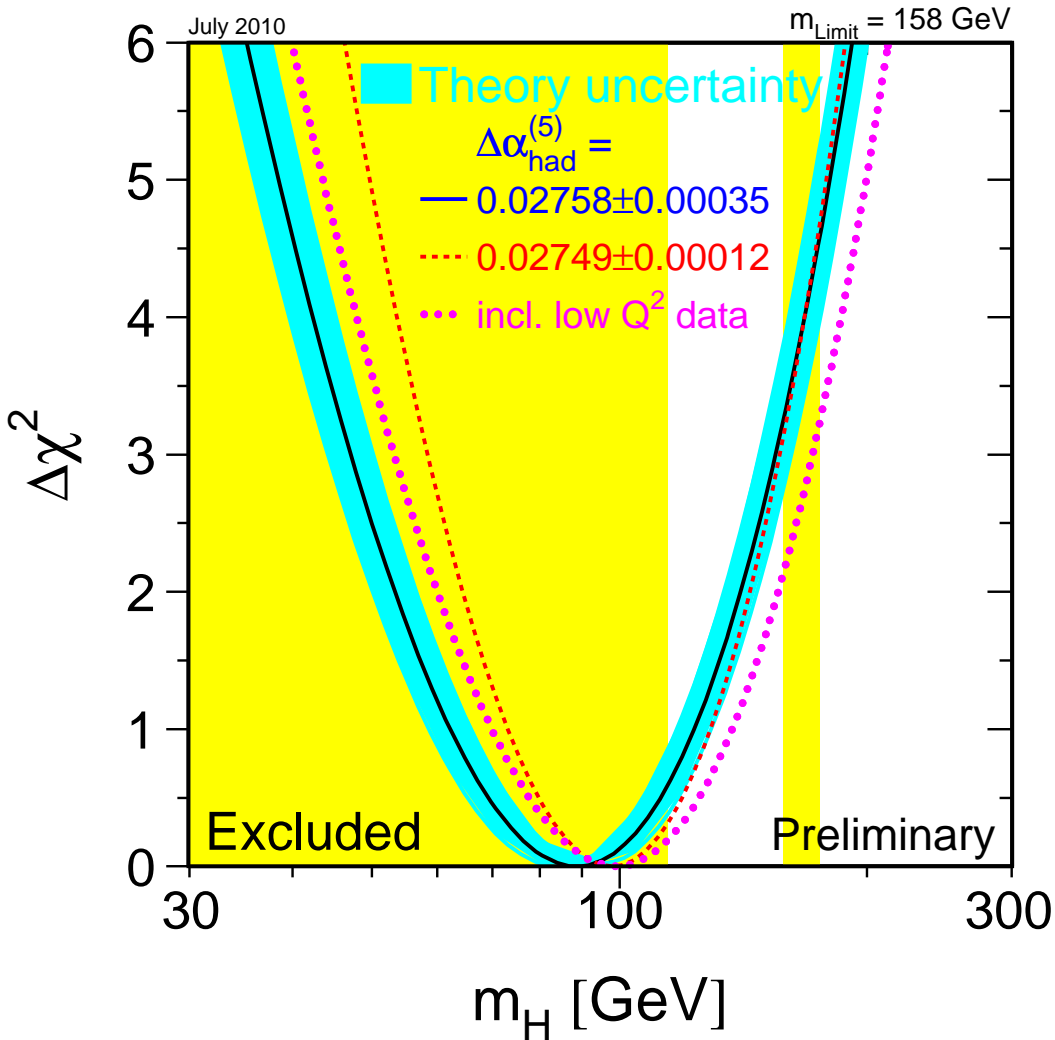
main sensitivity to m_t via



main sensitivity to M_H via



Bounds on M_H (95% C.L.)

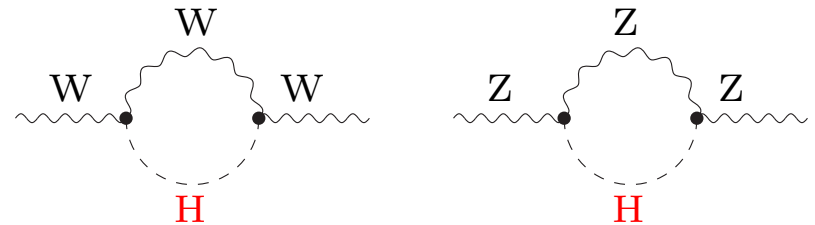


– $M_H > 114.4 \text{ GeV}$ (LEPHIGGS '02)

$e^+e^- \not\rightarrow ZH$ at LEP2

– $M_H < 158 \text{ GeV}$ (LEPEWWG '10)

fit to precision data,
i.e. via quantum corrections



Sensitivity via “high-precision observables”: $m_t, M_W, \sin^2 \theta_{\text{eff}}^{\text{lept}}$, etc.

↪ precise measurement is possible at future ILC !

⇒ stronger bounds on M_H

1.3 W-boson physics at LEP2

W-pair production $e^+e^- \rightarrow WW \rightarrow 4f(+\gamma)$

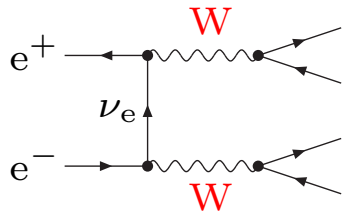


diagram dominates near W-pair threshold

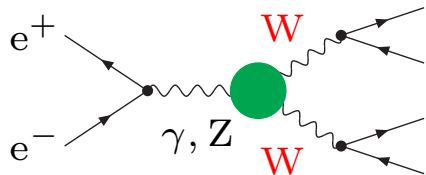


diagram contains $\gamma WW/ZWW$ couplings

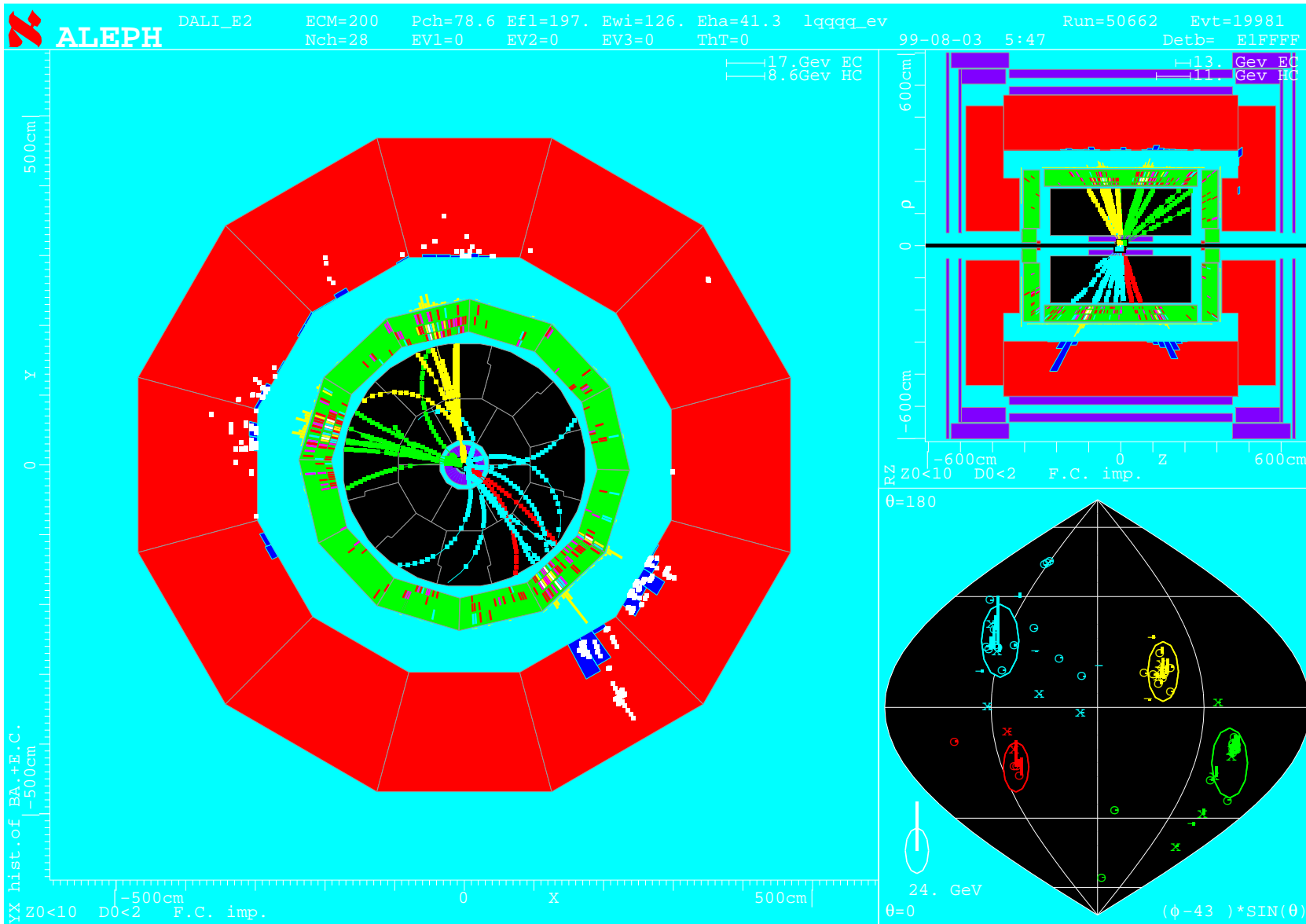
Physics issues:

- test of non-abelian structure of triple gauge-boson couplings (TGCs)
 - \hookrightarrow constraint on non-standard $\gamma WW/ZWW$ couplings
- precision measurement of W-pair cross section
- precision measurement of W mass M_W
- first bounds on non-standard quartic gauge-boson couplings (QGCs)

\Rightarrow Theoretical requirement:

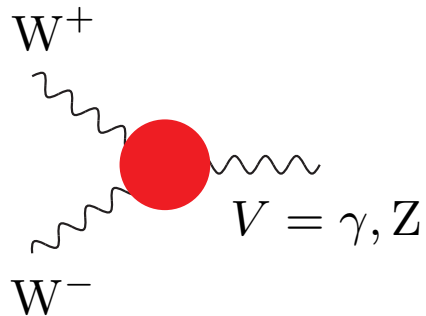
precise understanding of $2 \rightarrow 4$ process (0.5% level for cross section)

A typical 4-jet event observed at ALEPH



Made on 3-Aug-1999 14:42:48 by lancon with DALLI_E2.
 Filename: DC050662_019981_990803_1442.ps

General parametrization (C- and P-conserving):



$$\mathcal{L}_{VWW} = -ie g_{VWW} \left\{ g_1^V (W_{\mu\nu}^+ W^{-,\mu} V^\nu - W^{-,\mu\nu} W_\mu^+ V_\nu) \right. \\ \left. + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\rho\mu}^+ W_\nu^- V^{\nu\rho} \right\}$$

Meaning for static W^+ bosons:

$$Q_W = e g_1^\gamma = \text{electric charge } (= e \text{ by charge conservation})$$

$$\mu_W = \frac{e}{2M_W} (g_1^\gamma + \kappa_\gamma + \lambda_\gamma) = \text{magnetic dipole moment}$$

$$q_W = -\frac{e}{M_W^2} (\kappa_\gamma - \lambda_\gamma) = \text{electric quadrupole moment}$$

Standard Model values:

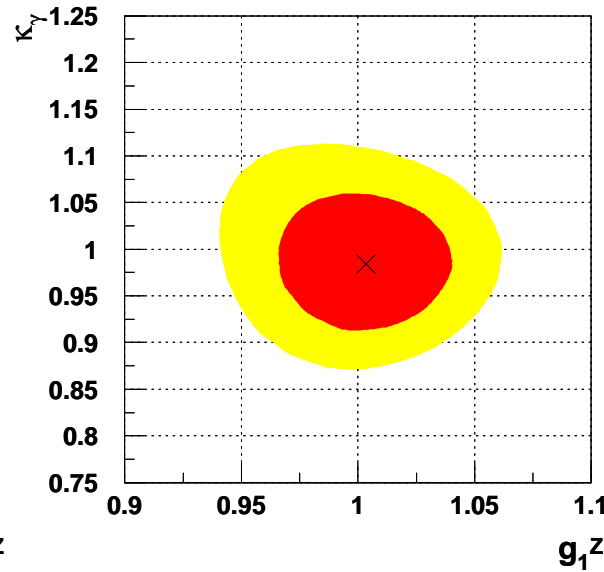
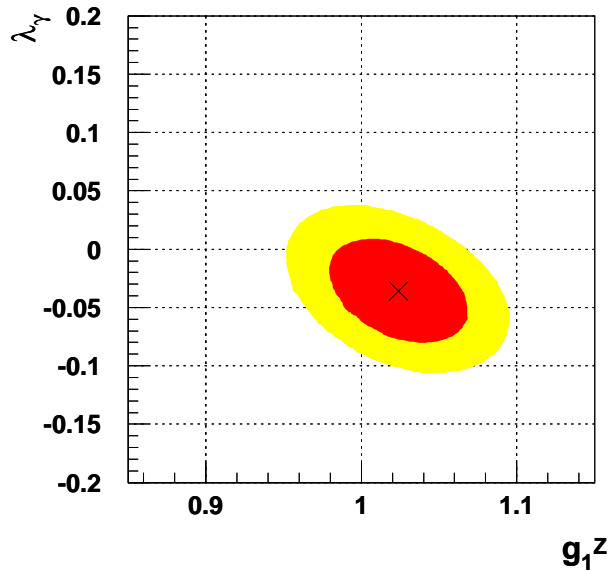
$$g_1^V = \kappa_V = 1, \quad \lambda_V = 0$$

Restriction to $SU(2) \times U(1)$ -symmetric dim-6 operators:

$$\kappa_Z = g_1^Z - (\kappa_\gamma - 1) \tan^2 \theta_W, \quad \lambda_Z = \lambda_\gamma$$

LEP2 constraints on charged TGCs

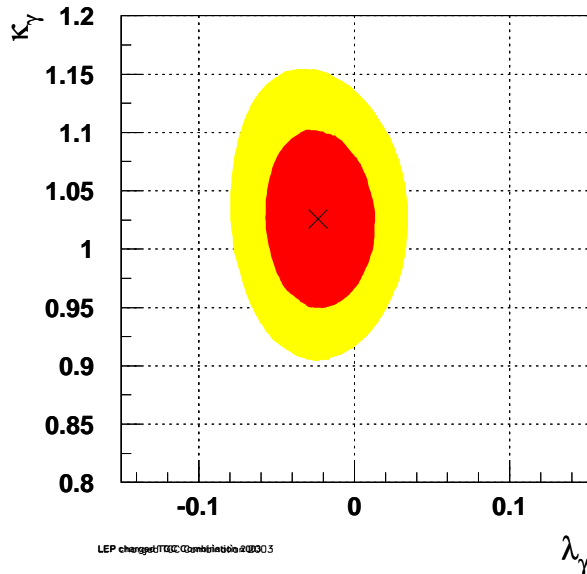
LEPEWWG '04



$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$

$$\Delta \kappa_\gamma = -0.016^{+0.042}_{-0.047}$$

$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$



LEP Preliminary

- 95% c.l.
- 68% c.l.
- × 2d fit result

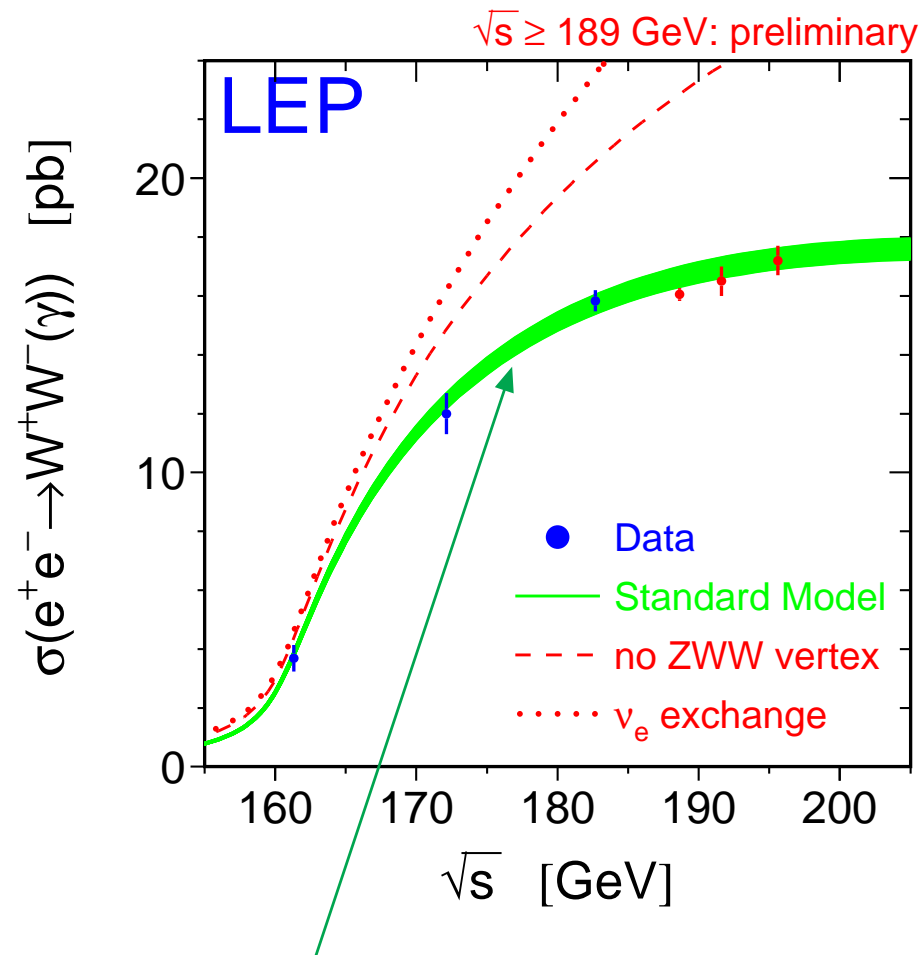
Standard Model values verified at the level of 2–4%

Note: TGC bounds $\sim \mathcal{O}(\text{EW corrections})$

Total WW cross section at LEP2

Status of 1999: (LEPEWWG '99)

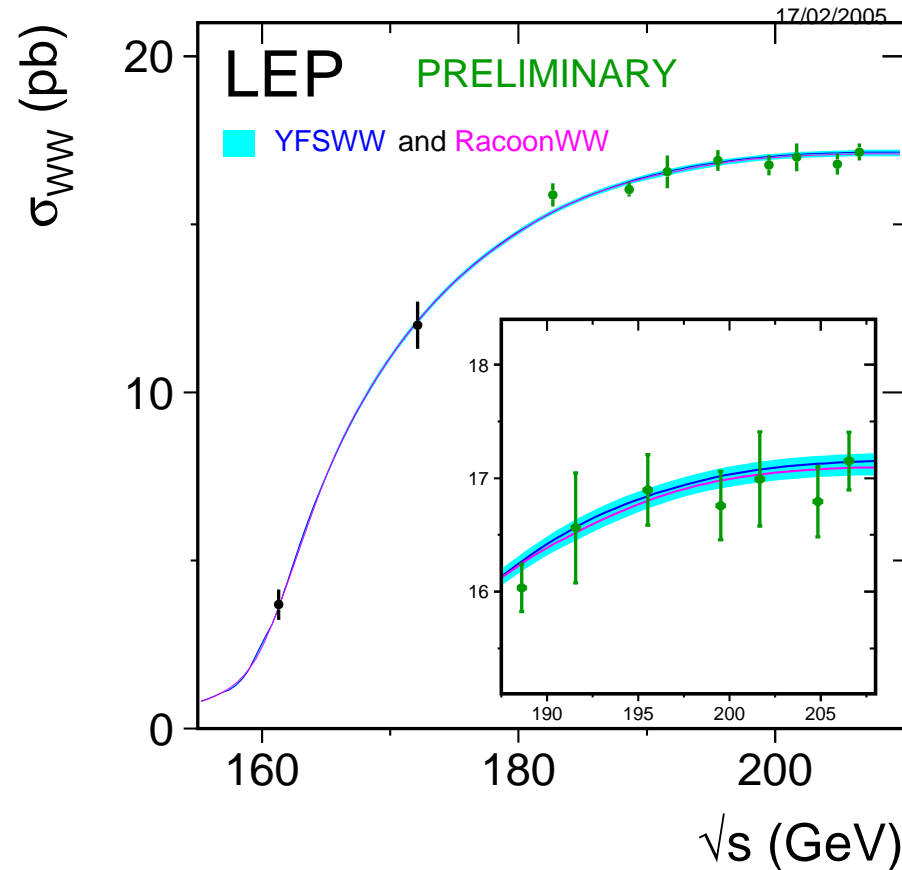
Final result: (LEPEWWG '05)



GENTLE (Bardin et al.)

only universal EW corrections

↪ theoretical uncertainty $\sim \pm 2\%$



YFSWW (Jadach et al.) / RacoonWW (Denner et al.)

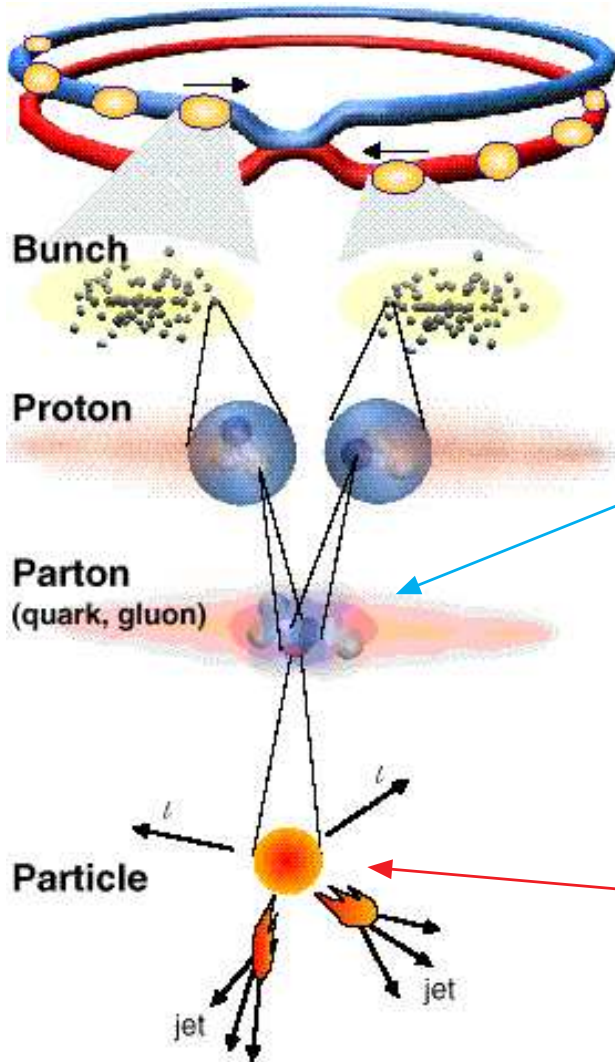
non-universal corrections included

↪ th. uncertainty $\sim \pm 0.5\%$ for $\sqrt{s} > 170$ GeV

Cross sections at hadron colliders



Inelastic hadronic collisions:



Parton content of the proton:

valence quarks uud ,

sea quarks $u, d, c, s, (+b,)$

gluons g (+photons γ)

“Parton distribution functions” (PDF) $f_{i/p}(x, Q)$

determine fraction x of the p momentum

carried by parton i at “factorization scale” Q

= non-perturbative input (from exp.),

but **process independent**

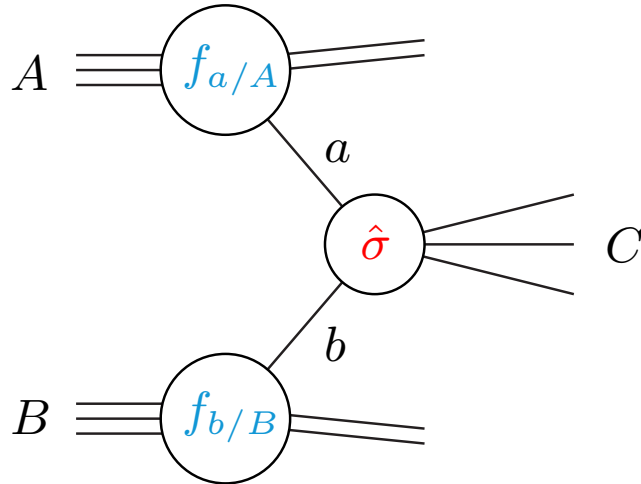
Hard interaction of partons

↪ perturbative QCD applicable,

model for hard interactions

(apart from QCD/QED) enters only here

Parton model description of hadronic collisions



hadronic momenta: p_A, p_B

hadronic CM energy: $\sqrt{s} = E_A + E_B$

partonic momenta: $p_a = x_A p_A, p_b = x_B p_B$

partonic CM energy: $\sqrt{\hat{s}} = \sqrt{x_A x_B s}$

Hadronic cross section for $AB \rightarrow C + X$: ($X =$ any hadronic remnant/activity)

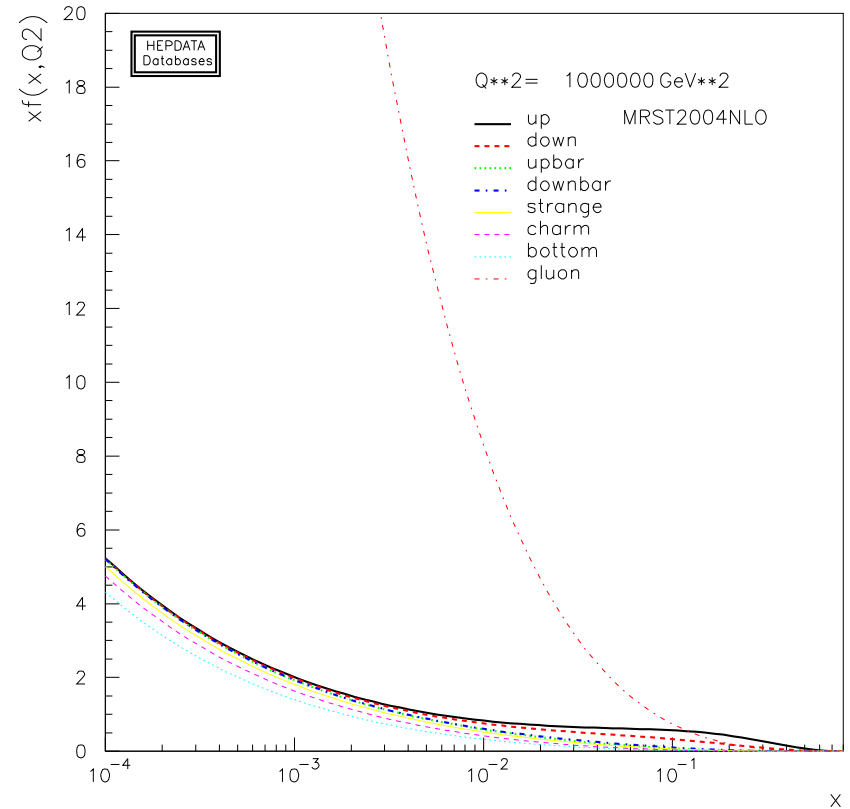
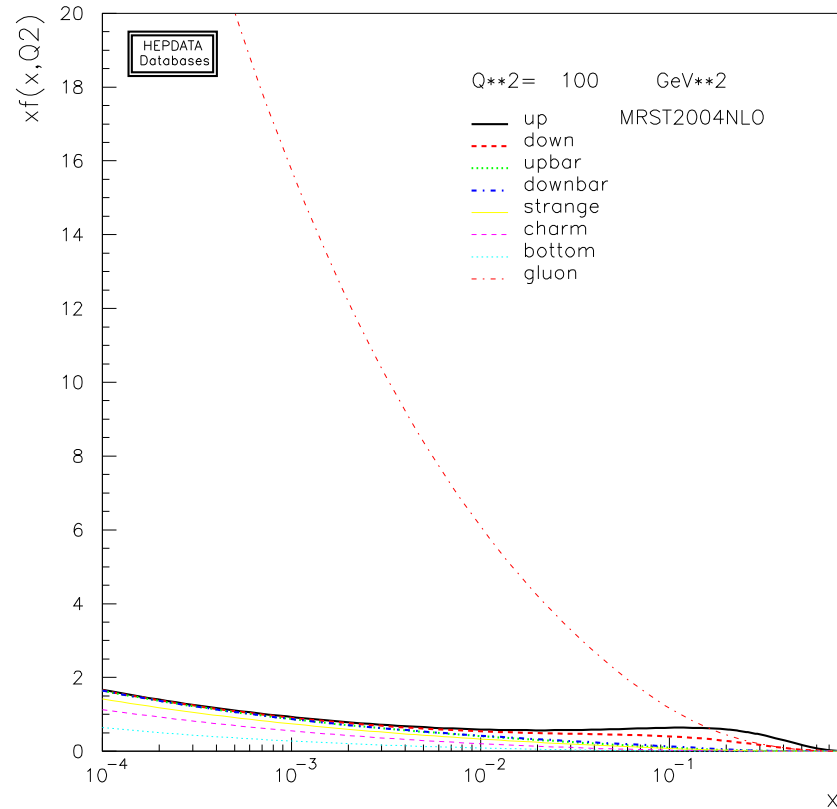
$$\sigma_{AB \rightarrow C+X}(s) = \int_0^1 dx_A \int_0^1 dx_B \sum_{a,b} f_{a/A}(x_A, Q) f_{b/B}(x_B, Q) \hat{\sigma}_{ab \rightarrow C}(\hat{s}, Q)$$

Factorization scale Q separates **soft** from **hard** contributions.

- Q dependence of PDFs $f_{a/A}(x_A, Q)$ ruled by DGLAP evolution equations
- Q dependence of hard scattering cross section $\hat{\sigma}_{ab \rightarrow C}(\hat{s}, Q)$ universal
- Q drops out in “all-order” calculations for $\sigma_{AB \rightarrow C+X}(s)$

↪ residual Q dependence in finite-order predictions reflects theoretical uncertainty

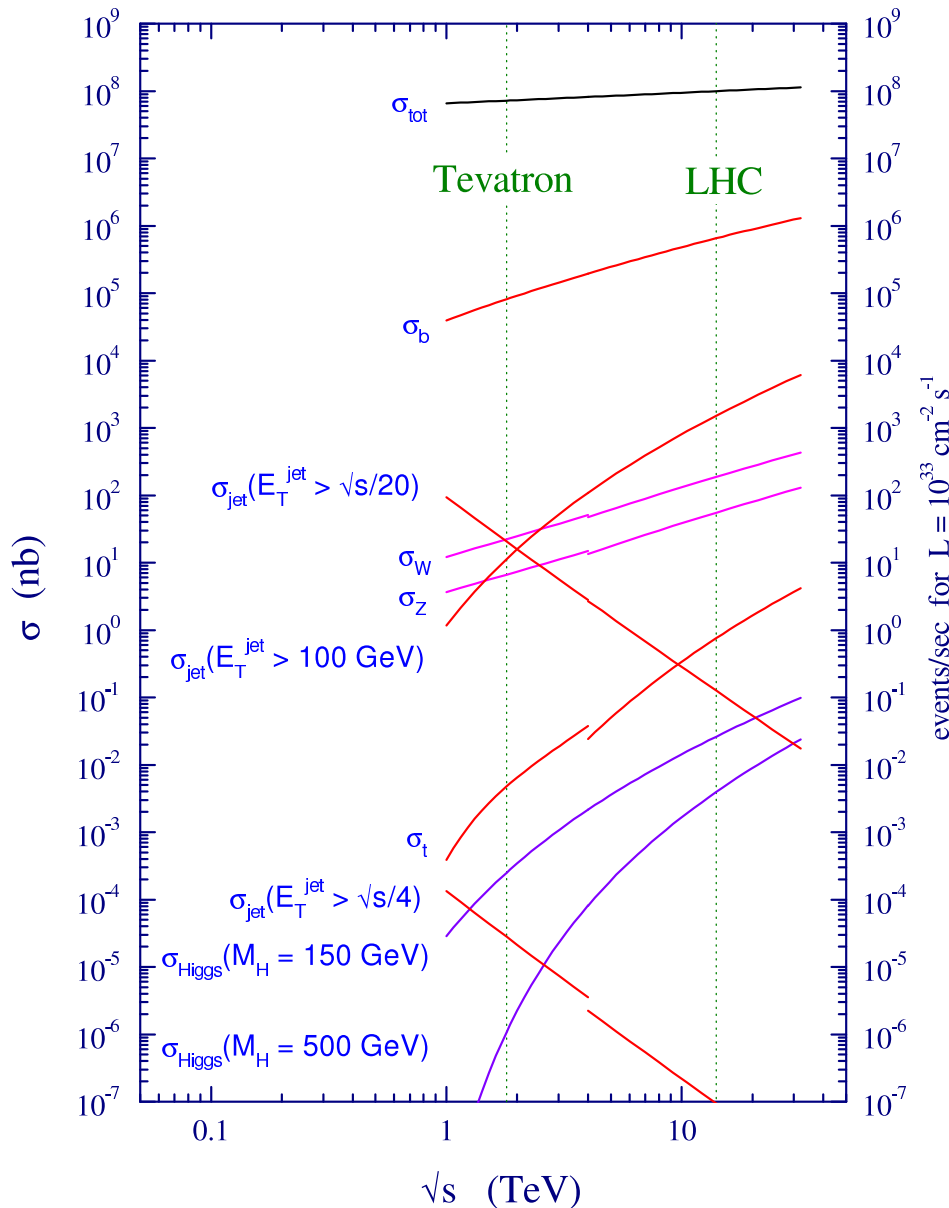
Parton distribution functions



- DGLAP evolution to larger Q shifts PDFs to lower x
 \hookrightarrow enhancement of sea-quark and gluon PDFs
- Processes with both gg and $q\bar{q}$ channels (e.g. $t\bar{t}$ production)
 Tevatron: $q\bar{q}$ often dominates by $\sim 90\%$
LHC: gg often much more important than $q\bar{q}$

Some Standard Model cross sections at the Tevatron ($p\bar{p}$) and the LHC (pp)

proton - (anti)proton cross sections



$$\# \text{ events} = \sigma \times \text{luminosity}$$

$$\text{design luminosity: } 100 \text{ fb}^{-1} \text{ a}^{-1} / \text{exp.}$$

2 experiments in 5 years:

$$\sigma \sim 1 \text{ pb} \rightarrow \sim 10^6 \text{ events}$$

↪ precision physics
(systematics dominates uncertainty)

$$\sigma \sim 1 \text{ fb} \rightarrow \sim 10^3 \text{ events}$$

↪ good prospects for searches
(statistics dominates uncertainty)

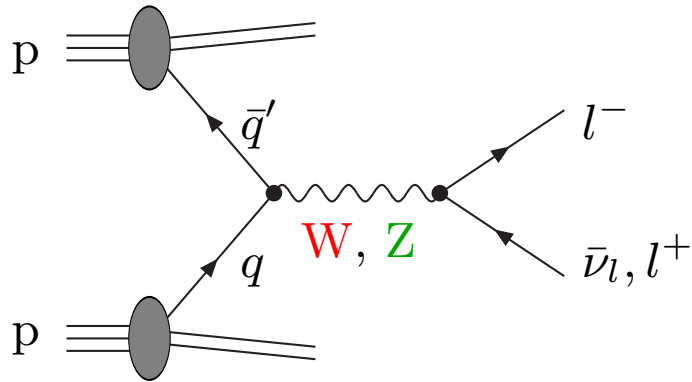
BUT:

inclusive cross sections reduced
by branching ratios, event selection,
experimental efficiencies, etc.

W- and Z-boson production at hadron colliders



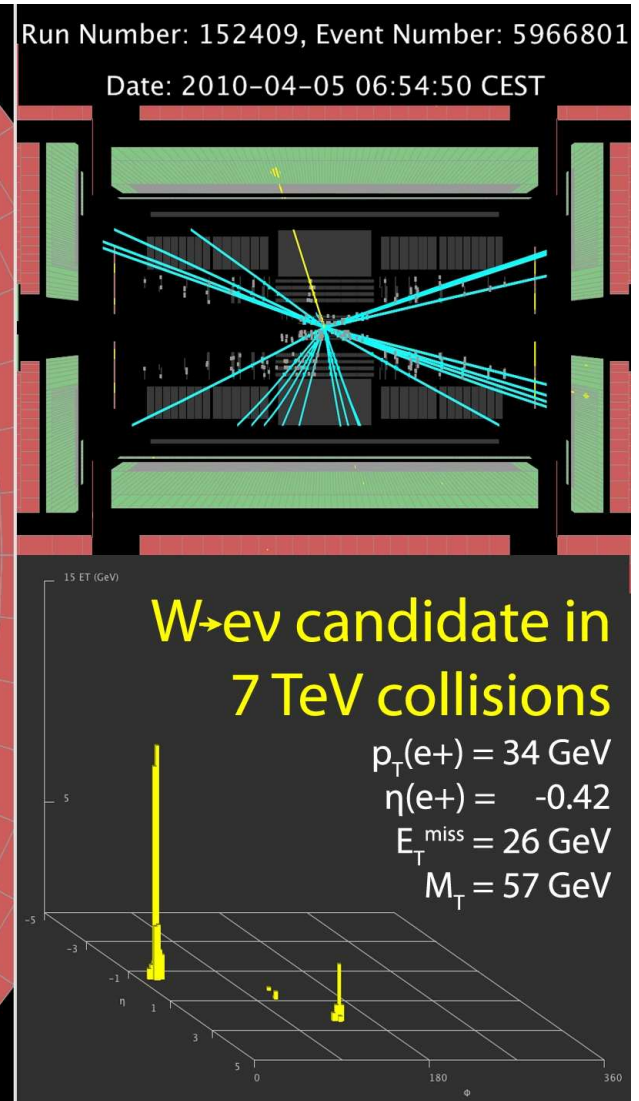
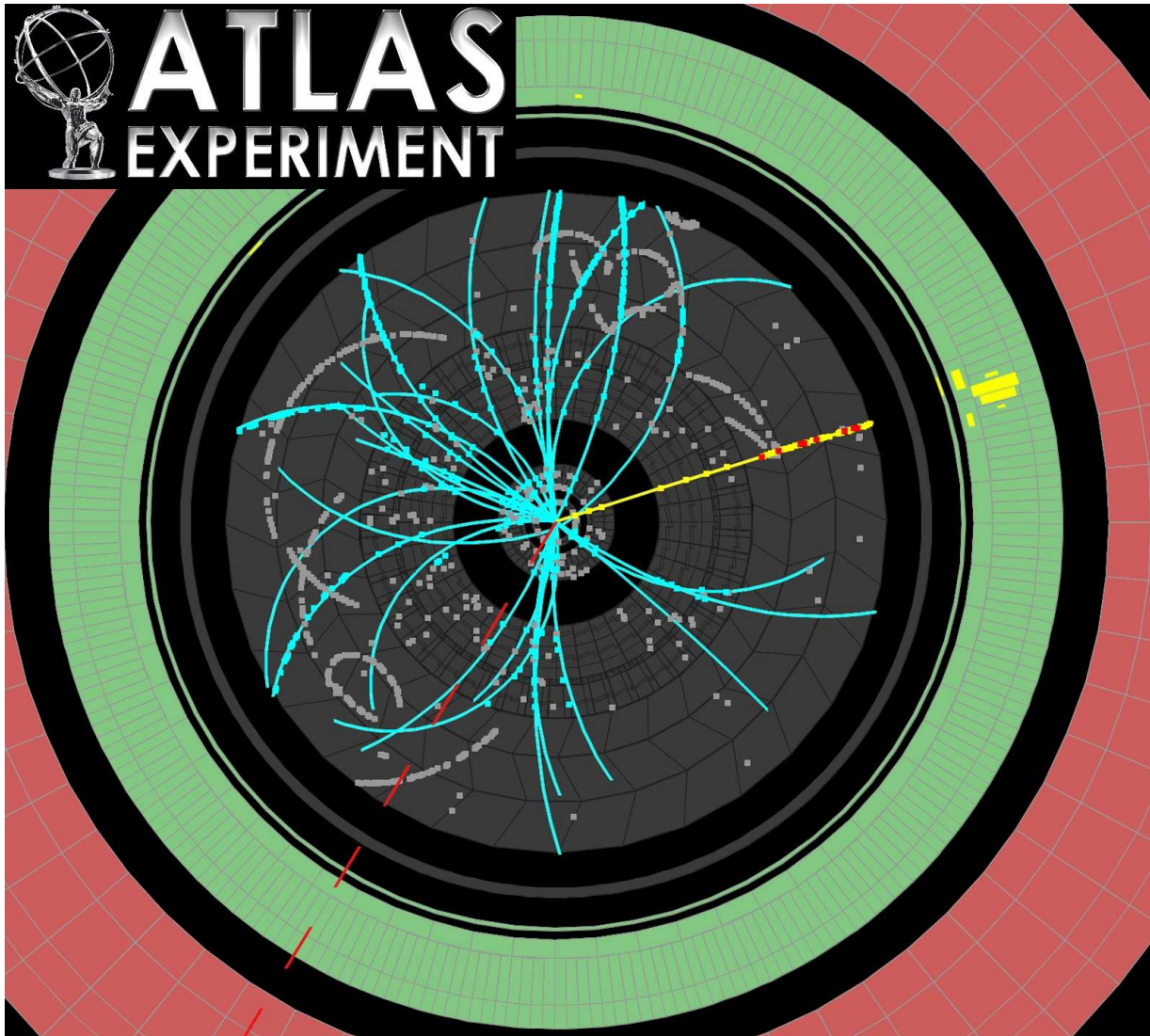
W- and Z-boson production at hadron colliders



Physics goals:

- M_Z → detector calibration by comparing with LEP1 result
- $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ → comparison with results of LEP1 and SLC
- M_W → improvement to $\Delta M_W \sim 15 \text{ MeV}$, strengthen EW precision tests
(W/Z shape comparisons even sensitive to $\Delta M_W \sim 7 \text{ MeV}$ at LHC)
Besson et al. '08
- decay widths Γ_Z and Γ_W from M_{ll} or $M_{T,l\nu_l}$ tails
- search for Z' and W' at high M_{ll} or $M_{T,l\nu_l}$
- information on PDFs or parton-parton luminosities

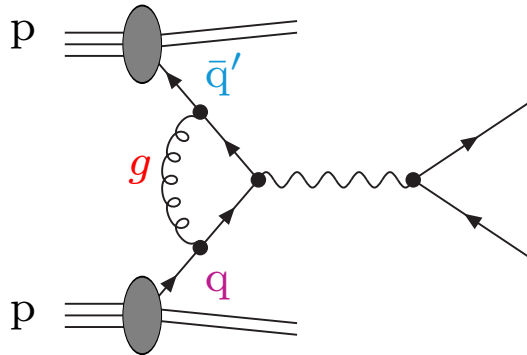
A W-boson event at the LHC



W/Z production in the QCD-improved parton model

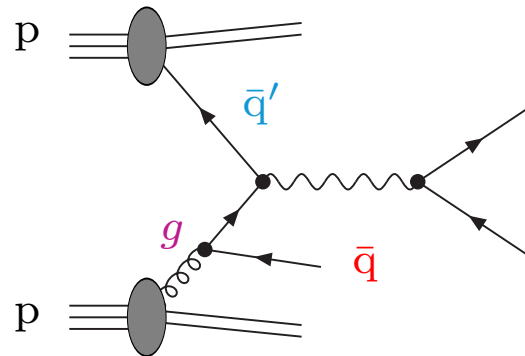
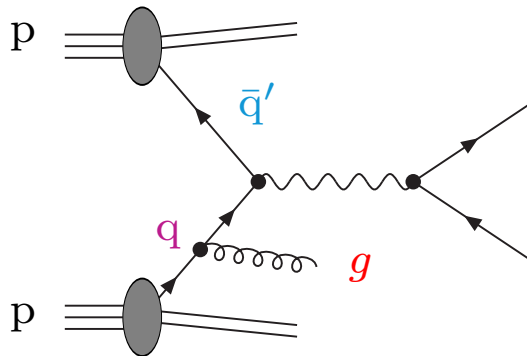
↪ inclusion of higher-order corrections in α_s :

- **virtual corrections:** diagrams with loops



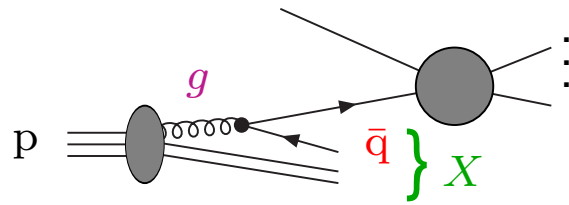
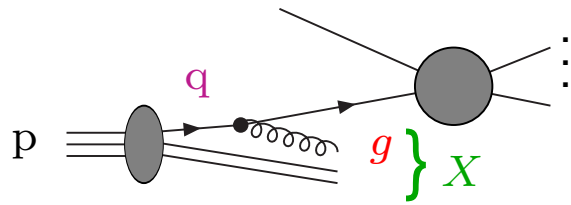
- ◇ UV divergences
↪ absorbed by renormalization ($\alpha_s(\mu_{\text{ren}})$, etc.)
- ◇ soft IR divergences
- ◇ collinear divergences

- **real corrections:** diagrams with additional external partons



- ◇ soft IR divergences → compensate soft IR divergences of virtual corrections
- ◇ collinear divergences → not completely compensated

Collinear divergences in initial state and factorization



g, \bar{q} have transverse momenta $\mathbf{k}_T \rightarrow 0$

- process-independent divergence in $\int d\mathbf{k}_T^2$
- g, \bar{q} are part of **proton remnant X**

\hookrightarrow **singularity belongs to proton** \Rightarrow absorption into **renormalized parton densities**

$$q(x, Q) = q(x) + \left\{ \text{divergent part from } \int_0^{Q^2} d\mathbf{k}_T^2 \right\}, \text{ etc.}$$

Q = “factorization scale”; Q dependence ruled by DGLAP evolution

pp cross section:

$$\sigma_{pp} = \sum_{f_1 f_2} \int_0^1 dx_1 f_1(x_1, Q) \int_0^1 dx_2 f_2(x_2, Q) \hat{\sigma}_{f_1 f_2}(x_1 x_2 s, Q)$$

$$(f_1 f_2 = \bar{q}' q, q \bar{q}', g q, g \bar{q}', \bar{q}' g, q g, g g)$$

Note: incomplete compensation of Q dependence in finite perturbative orders

\hookrightarrow measure of missing corrections

QCD corrections to W/Z production:

- **NNLO QCD** corrections
 - ◇ total cross sections Hamberg, v.Neerven, Matsuura '91; v.Neerven, Zijlstra '92
Harlander, Kilgore '02
 - ◇ W/Z rapidity distributions Anastasiou et al. '03
 - ◇ fully differential Melnikov, Petriello '06; Catani et al. '09
- **soft-gluon resummation** (partially combined with γ emission) Balazs, Yuan '97; Landry et al. '02
Cao, Yuan '04

EW corrections to W/Z production:

- **NLO EW correction** to W production Baur, Keller, Wackerath '98; S.D., Krämer '02
Baur, Wackerath '04; Arbuzov et al. '05
Carloni Calame et al. '06
- **NLO EW correction** to Z production Baur, Keller, Sakumoto '97; Baur, Wackerath '99
Brein, Hollik, Schappacher '99; Arbuzov et al. '06
S.D., Huber '09
- **multi-photon radiation** via leading logs Baur, Stelzer '99; Carloni Calame et al. '03
Placzek, Jadach '04; Breusing et al. '07

Corrections beyond the SM:

- **NLO SUSY corrections** in the MSSM Breusing et al. '07; S.D., Huber '09
↪ negligible near W/Z resonances ⇒ standard candles !

Structure of EW corrections: example of W production

Born amplitude:

$$\mathcal{M}_0 = \frac{e^2}{2s_W^2} [\bar{v}_d \gamma^\mu \frac{1}{2}(1 - \gamma_5) u_u] \frac{1}{\hat{s} - M_W^2 + \underbrace{iM_W \Gamma_W}} [\bar{u}_{\nu_l} \gamma_\mu \frac{1}{2}(1 - \gamma_5) v_l]$$

results from (partial) Dyson summation of W self-energy

Virtual corrections:

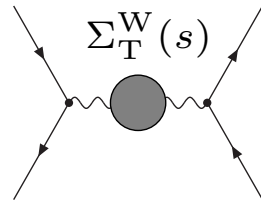
consistent gauge-invariant treatment of W resonance required

↪ solutions: pole scheme, factorization schemes, complex-mass scheme

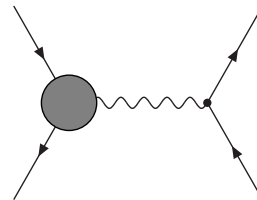
see S.D., Huber '09 and refs. therein

$$\mathcal{M}_1 = \delta^{\text{virt}} \mathcal{M}_0$$

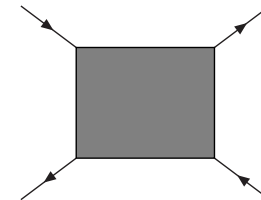
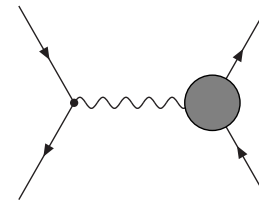
$$\delta^{\text{virt}} = \delta_{WW}(\hat{s}) + \delta_{Wdu}(\hat{s}) + \delta_{W\nu_l l}(\hat{s}) + \delta_{\text{box}}(\hat{s}, \hat{t})$$



W self-energy



Wud and Wlνl vertex corrections



box diagrams

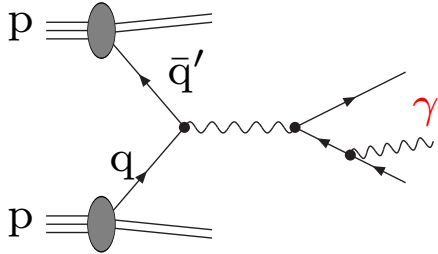
Note: δ^{virt} is not analytic at $\hat{s} \rightarrow M_W^2$,

but contains terms $\propto \alpha \ln(\hat{s} - M_W^2 + iM_W \Gamma_W)$

Real photon corrections:

consistent treatment of W resonance with correct IR (soft and collinear) limits

- final-state radiation:

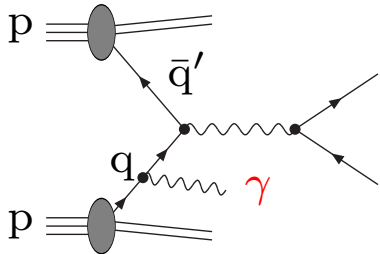


$$\sigma_{\text{FSR}} = \sigma_0 \int_0^1 dz \frac{Q_l^2 \alpha}{2\pi} \ln\left(\frac{M^2}{m_l^2}\right) \left(\frac{1+z^2}{1-z}\right)_+ \Theta_{\text{cut}}(k_l = z\hat{k}_l) + \dots$$

↪ large mass-singular correction if $\int dz$ restricted

Note: singularity vanishes after “photon recombination”
due to full z integration (inclusiveness → KLN theorem)

- initial-state radiation:

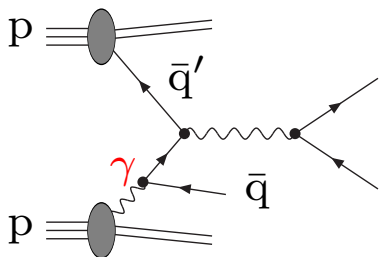


Mass singularity absorbed via PDF redefinition as in QCD

↪ remaining ISR corrections not enhanced
and depend on factorization scale Q

But: QED-corrected PDFs required (corrections $\sim 1\%$)
only old set MRST2004qed available → update necessary

- photon-induced processes:



PDF redefinition necessarily includes photon PDF

↪ γq processes deliver real corrections,
effects typically $\lesssim 1\%$

γ PDF measurable via $\gamma\gamma \rightarrow l^+l^-$ via special cuts ?

Observables for W/Z production at hadron colliders

Momentum/energy measurements:

- $p_{T,l}$ = lepton transverse momenta
- $p_{T,\text{miss}}$ = missing transverse momentum (or energy)
- $y_l = \frac{1}{2} \ln \left(\frac{p_l^0 + p_l^3}{p_l^0 - p_l^3} \right)$ = lepton rapidities

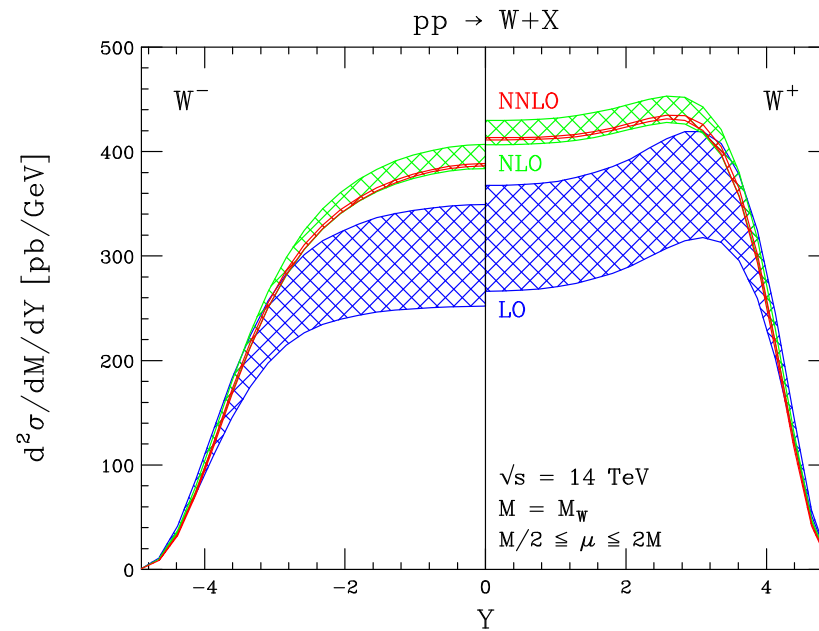
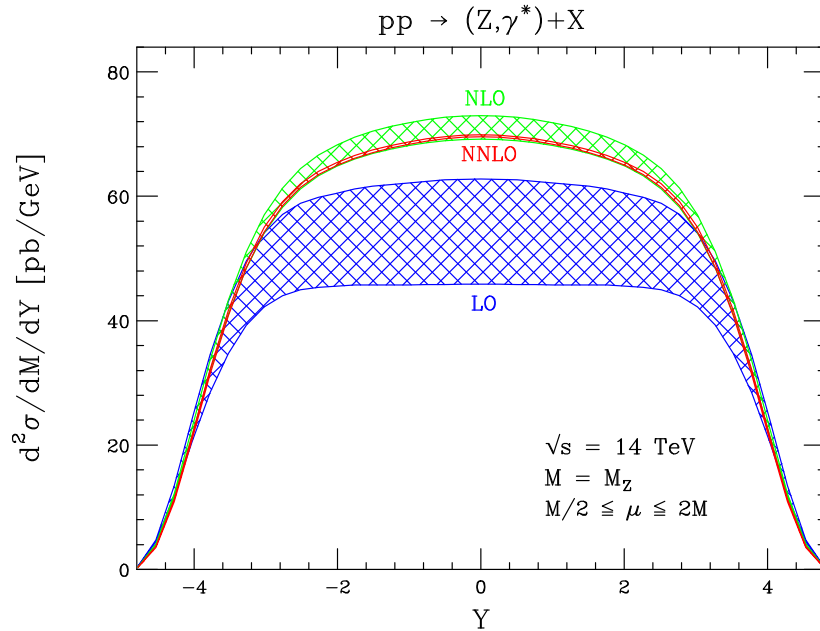
“Derived” observables:

- $M_{ll} = \sqrt{(p_{l+} + p_{l-})^2}$ = dilepton invariant mass for Z production
↪ important for calibration
- $M_{T,\nu l} = \sqrt{2p_{T,l}p_{T,\text{miss}}(1 - \cos \phi_{\nu l})}$ = transverse invariant mass for W production
↪ measurement of M_W
- $p_{T,W/Z} = p_{T,\nu/l} + p_{T,l'}$ = transverse W/Z momenta
↪ induced by QCD radiation (jets)
- various asymmetries



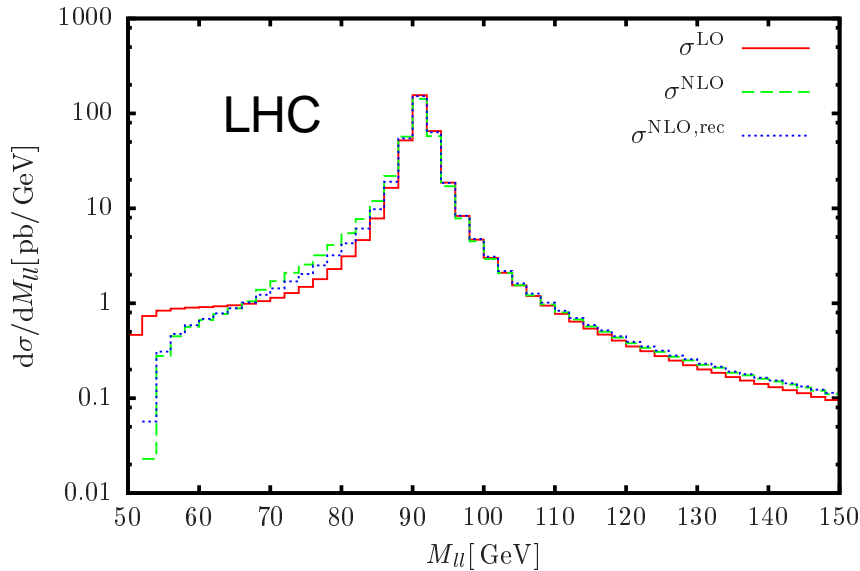
W/Z rapidity distributions

Anastasiou et al. '03



- **LHC:**
 - ◇ all rapidity distributions are forward–backward (FB) symmetric (see plot)
 - ◇ FB asymmetry defined wrt boost direction of Z boson
 - $\hookrightarrow \sin^2 \theta_{\text{eff}}^{\text{lept}}$ measurable with precision competitive to LEP/SLC ?
- **Tevatron:**
 - ◇ FB asymmetry defined wrt p direction $\rightarrow \sin^2 \theta_{\text{eff}}^{\text{lept}}$ in progress
 - ◇ W (charged lepton) rapidity constrains PDF ratio $u(x)/d(x)$
- QCD predictions show nice convergence in **LO** \rightarrow **NLO** \rightarrow **NNLO**
- EW corrections at the level of few % (not shown)

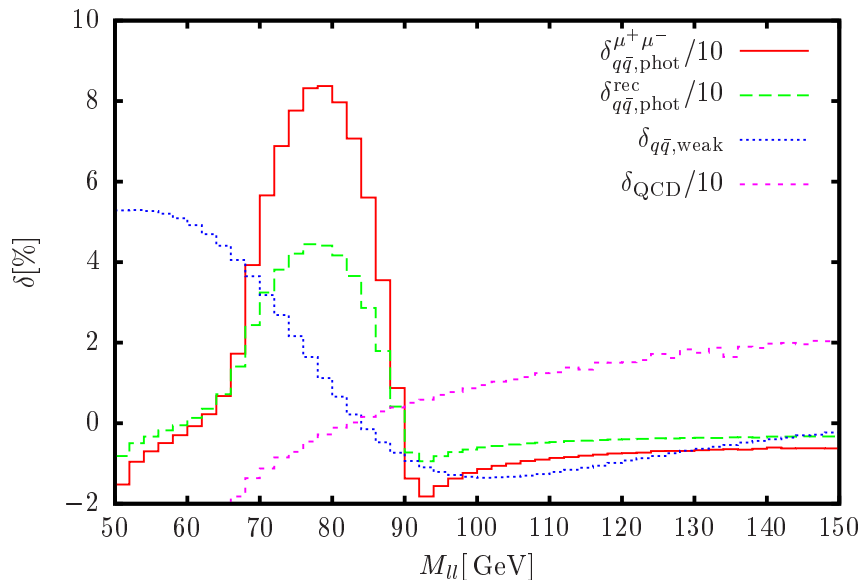
Dilepton invariant-mass distribution (at intermediate M_{ll} values)



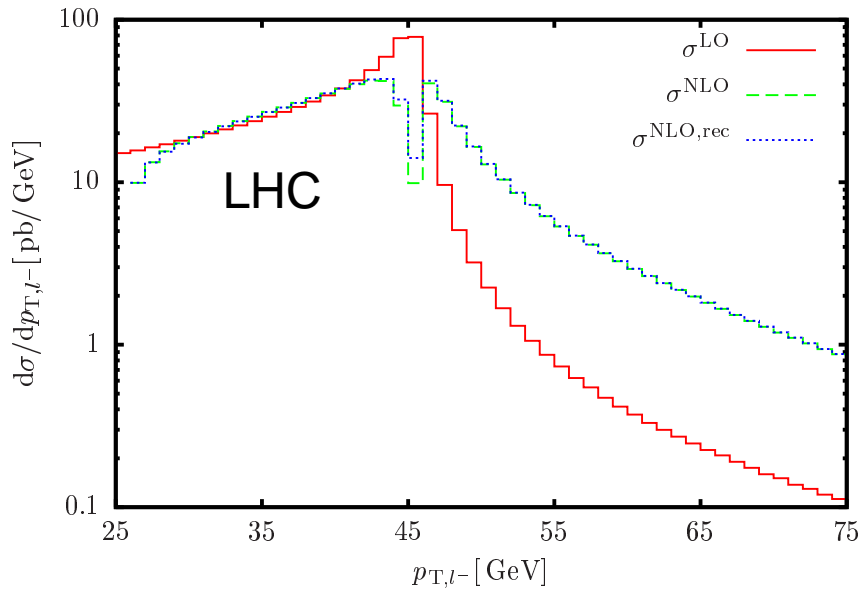
S.D., Huber '09

- **QCD** corrections of $\mathcal{O}(20\%)$, but very little resonance distortion
- **photonic** corrections very large due to FSR (“inverse radiative tail”)
 - ↪ resonance distortion
 - ◇ **“bare muons”**: enhanced by mass sing. $\propto \alpha \ln m_\mu$
 - ◇ **“photon recombination”** (e^\pm case): mass sing. canceled due to KLN
- **weak** corrections (loops with W’s, Z’s, fermions) at the order of $\lesssim 5\%$

Relative corrections:



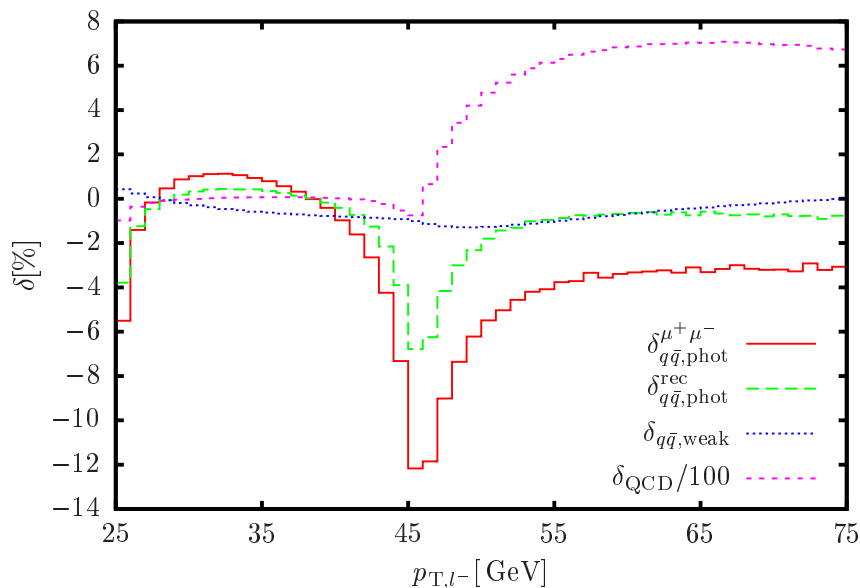
Lepton transverse-momentum distribution for Z-boson production



S.D., Huber '09

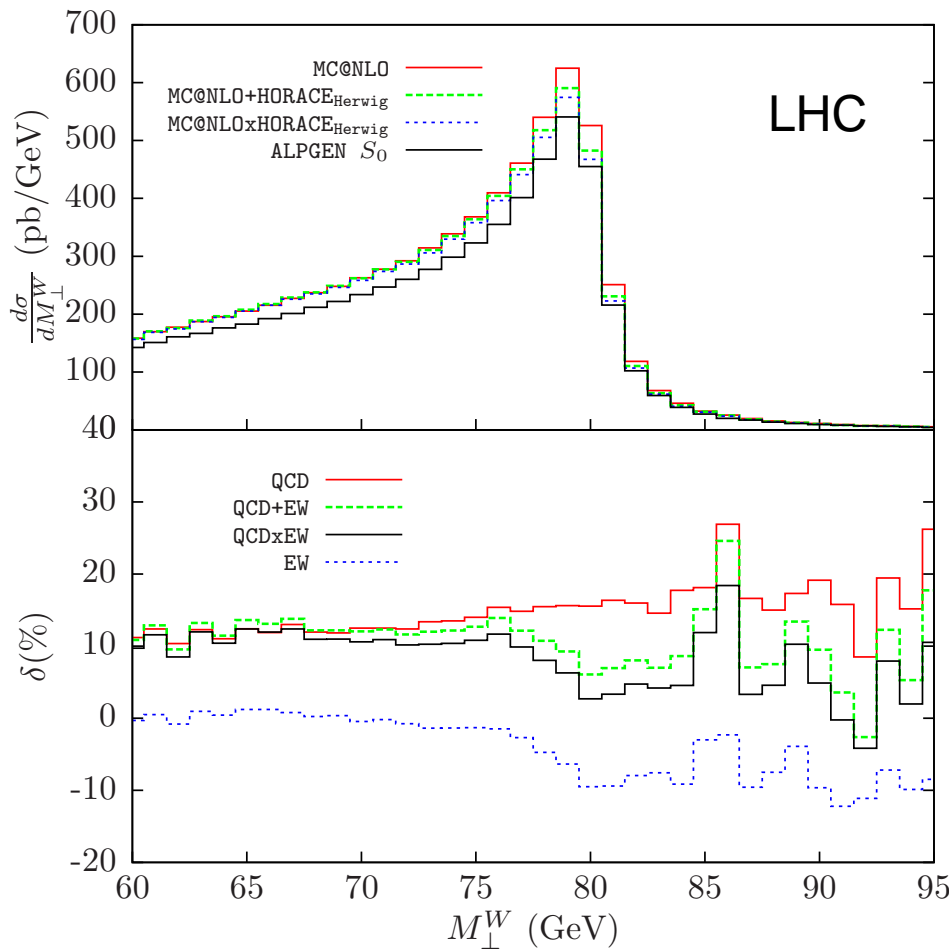
- “Jacobian peak” near $p_{T,l} \approx M_Z/2$, resonant Z’s contribute only to $p_{T,l} \lesssim M_Z/2$
- patterns of corrections similar for Z and W production

Relative corrections:



- size of **photonic** and **weak** corrections qualitatively similar to M_{ll} distribution
- **QCD** corrections extremely huge for $p_{T,l} \gtrsim M_Z/2$ because of jet ISR: LO process becomes Z+jet prod.
 \hookrightarrow no good perturbative control of QCD
 $\Rightarrow M_W$ measured better from $M_{T,\nu l}$!

Transverse-mass distribution for W production



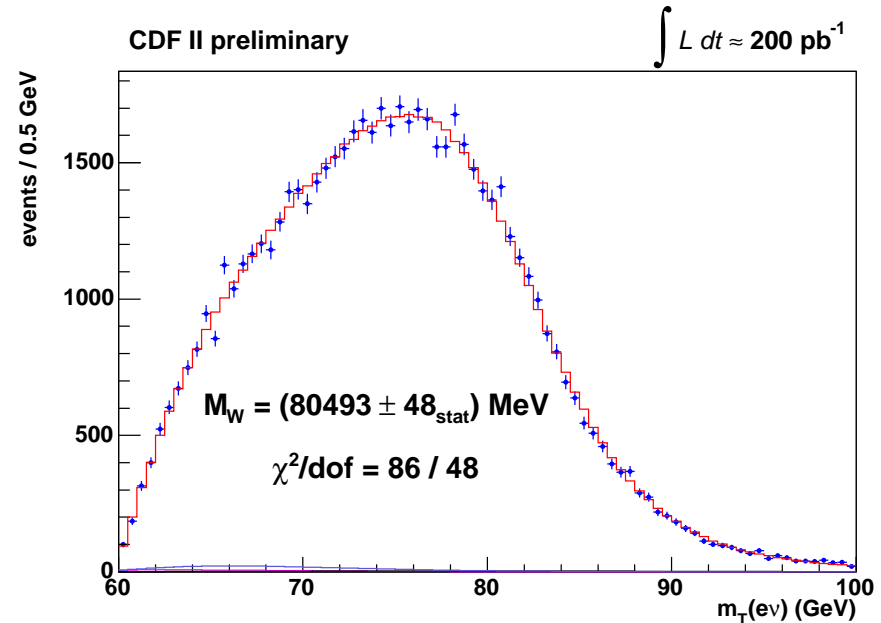
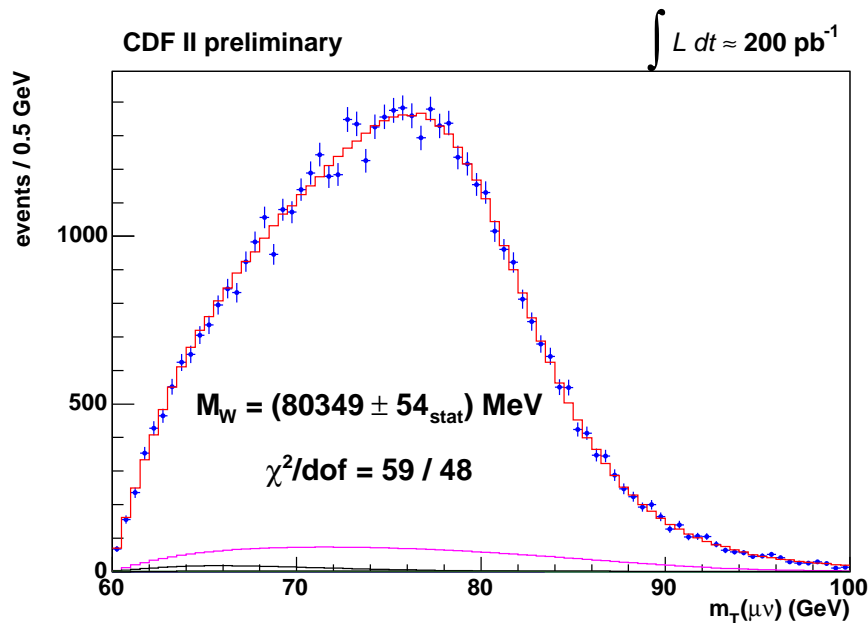
Balossini et al. '09

- **QCD** corrections moderate and flat near the peak
- **EW** corrections (mainly FSR) shift peak by about $\mathcal{O}(100 \text{ MeV})$
- different ways of combining QCD and EW corrections
 $(1+\delta_{\text{QCD}}+\delta_{\text{EW}})$ vs. $(1+\delta_{\text{QCD}})\times(1+\delta_{\text{EW}})$
 differ at %-level
 $\hookrightarrow \mathcal{O}(\alpha\alpha_s)$ calculation needed ...

W-mass reconstruction from “Jacobian peak” near $M_{\perp}^W \equiv M_{T,\nu l} \approx M_W$:

- resonant W bosons contribute only for $M_{T,\nu l} < M_{\nu l} \approx M_W$
- the tail $M_{T,\nu l} \gtrsim M_W$ can be used to fit the W width Γ_W

Fits of M_W to W transverse mass at the Tevatron



Theory prediction based on QCD resummations (improved by some EW corrections)

Result from CDF Run II: $M_W = 80.413 \pm 0.048 \text{ GeV}$

Result from D0 Run II: $M_W = 80.402 \pm 0.043 \text{ GeV}$

CDF/D0 combination 2009: $M_W = 80.420 \pm 0.031 \text{ GeV}$ (from Fermilab homepage)

Result from LEP: $M_W = 80.376 \pm 0.033 \text{ GeV}$

World average 2009: $M_W = 80.399 \pm 0.023 \text{ GeV}$