Electroweak Aspects of the Standard Model

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Lecture II: Electroweak Radiative Corrections

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Overview

1 Quantum field theories and higher perturbative orders

2 Electroweak Standard Model – radiative corrections

3 Radiative corrections to muon decay

Quantum field theories
and
higher perturbative orders

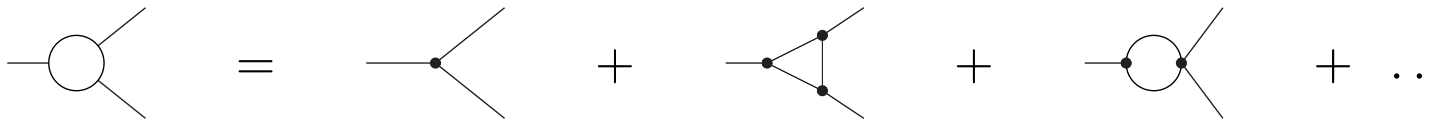
1.1 General procedure

Formulate theory:	Lagrangian
	$\Downarrow$
	quantization $\longrightarrow$ gauge fixing, Faddeev–Popov ghosts
	$\Downarrow$
Perturbative evaluation:	Feynman rules
	$\Downarrow$
	Feynman graphs
	$\Downarrow$
	loop integrals $\longrightarrow$ technical problem: divergences (UV, IR)
	$\Downarrow$
	regularization $\longrightarrow$ divergences mathematically meaningful
	$\Downarrow$
Define input parameters:	renormalization $\longrightarrow$ eliminates UV divergences
	$\Downarrow$
Theoretical predictions:	calculation of observables (cross sections, decay widths, etc.)
	$\hookrightarrow$ IR divergences cancel for sufficiently inclusive quantities (e.g. inclusion of photon bremsstrahlung)

1.2 Green functions, transition amplitudes, and observables

"Amputated" Green functions $G^{\phi_1\cdots\phi_n}_{\text{amp}}$:

calculated as sum of all connected Feynman diagrams with external n legs $\phi_1,\ldots,\phi_n$ with external propagators (and propagator corrections) omitted

$G^{\phi_1\phi_2\phi_3}_{\text{amp}} =$ 

Transition amplitude $\mathcal{M}_{fi}$ for $|i\rangle \rightarrow |f\rangle$:

calculated from amputated Green functions $G^{\phi_1\cdots\phi_n}_{\text{amp}}$ by "LSZ reduction":

- put external momenta to their mass shell, $p_i^2 = m_i^2$
- contract with wave functions of external particles (Dirac spinors, polarization vectors)

  Note: fields must be normalized: $R_{\phi_i} = 1$ (= residue of propagator pole), otherwise multiply by $\sqrt{R_{\phi_i}}$ for each external leg

Cross section for transition $|i\rangle \rightarrow |f\rangle$:

$$\sigma = \text{flux} \times \int \mathrm{dLIPS}\, |\mathcal{M}_{fi}|^2$$

“Vertex functions” $\Gamma^{\phi_1\cdots\phi_n}$ as irreducible building blocks:

- $\Gamma^{\phi_1\phi_2} \;\equiv\; -(G^{\phi_1\phi_2})^{-1} \;=\; -$(inverse propagator)

  example: scalar 2-point function

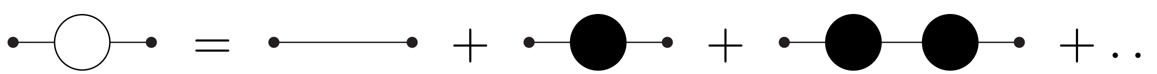
  $$\Gamma^{\phi\phi}(p) = \mathrm{i}(p^2 - m^2) + \mathrm{i}\Sigma(p^2),$$

  $\Sigma =$ self-energy = sum of 1PI graphs

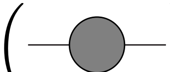
  1PI = 1-particle-irreducible
  (graph cannot be disconnected by cutting *one* line)

  $$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2 - m^2} + \frac{\mathrm{i}}{p^2 - m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2 - m^2} + \dots$$

  (Dyson series)

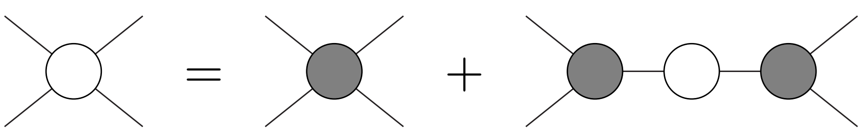


  $$= \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} = -\left(\Gamma^{\phi\phi}(p)\right)^{-1} = -\left(\quad \right)^{-1}$$



- $\Gamma^{\phi_1\cdots\phi_n} \;\equiv\; G_{\mathrm{amp}}^{\phi_1\cdots\phi_n}\Big|_{\text{only 1PI graphs}}$

  example:



  $+$ two permutations

  $G_{\mathrm{amp}}^{\phi\phi\phi\phi}$ $\quad \Gamma^{\phi\phi\phi\phi}$ $\quad \Gamma^{\phi\phi\phi}G^{\phi\phi}\Gamma^{\phi\phi\phi}$

1.3 Loop integrals and regularization

Regularization of divergences

Observation: loop integrals involve divergences

- UV divergences for $q \to \infty$, e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{(q^2 - m_0^2)(q^2 - m_1^2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to \infty \qquad \to \text{logarithmic divergence}$$

- IR divergences for $q \to q_0$, e.g.:

$$\int \mathrm{d}^4 q \, \frac{1}{q^2(q^2 + 2qp_1)(q^2 + 2qp_2)} \sim \int \frac{\mathrm{d}q}{q} \text{ for } q \to 0 \qquad \to \text{logarithmic divergence}$$

"Regularization": extension of theory by free parameter δ such that

- integrals (and thus the theory) become finite, i.e. well defined
- original theory is obtained as limiting case $\delta \to \delta_0$

$\hookrightarrow$ fix input parameters x_i of regularized theory ($\delta \neq \delta_0$) by experiment

$\Rightarrow$ observables must have finite limit $\delta \to \delta_0$ as functions of x_i

(independent of regularization scheme)

Convenient regularization schemes:

- Dimensional regularization: switch to $D \neq 4$ space-time dimensions
  - ◇ regularizes UV (and IR) divergences, respects gauge invariance, easy use
  - ◇ prescription: (μ = arbitrary reference mass, drops out in observables)

    $\int \mathrm{d}^4 q \;\to\; (2\pi\mu)^{4-D} \int \mathrm{d}^D q$ and D-dim. momenta, metric, Dirac algebra

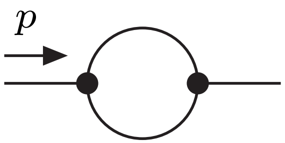
    and analytic continuation to complex D !
  - ◇ divergences appear as poles $\dfrac{1}{4-D}$ in results

    $\hookrightarrow$ define $\Delta \equiv \dfrac{2}{4-D} - \gamma_{\mathrm{E}} + \ln(4\pi) = \dfrac{2}{4-D} + \text{const.}$

- IR regularization by infinitesimal photon mass m_γ
  and (if relevant) by small fermion mass m_f
  - ◇ prescription: photon propagator pole $\dfrac{1}{q^2} \;\to\; \dfrac{1}{q^2 - m_\gamma^2}$
  - ◇ divergences appear as $\ln(m_\gamma)$ and $\ln(m_f)$ terms

Standard 1-loop integrals:

- 2-point integrals:

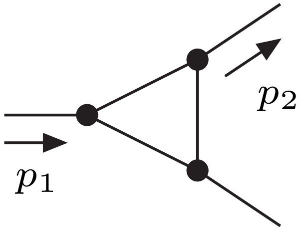


$$B_{0,\mu,\mu\nu,\ldots}(p,m_0,m_1) \;=\; \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q\,\frac{1,q_\mu,q_\mu q_\nu,\ldots}{(q^2-m_0^2+\mathrm{i}0)[(q+p)^2-m_1^2+\mathrm{i}0]}$$

scalar integral $B_0 =$ logarithmically UV divergent $= \Delta +$ finite,

vector integral $B_\mu = -\frac{1}{2}p_\mu\Delta +$ finite, etc.

- 3-point integrals:



$$C_{0,\mu,\mu\nu,\ldots}(p_1,p_2,m_0,m_1,m_2)$$
$$= \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int \mathrm{d}^D q\,\frac{1,q_\mu,q_\mu q_\nu,\ldots}{(q^2-m_0^2+\mathrm{i}0)[(q+p_1)^2-m_1^2+\mathrm{i}0][(q+p_2)^2-m_2^2+\mathrm{i}0]}$$

C_0, $C_\mu =$ UV finite,

$C_{\mu\nu} =$ logarithmically UV divergent $= \frac{1}{4}g_{\mu\nu}\Delta +$ finite, etc.

- 4-point integrals: $D_{\ldots}$ functions, etc.

Features of one-loop integrals:

- sign of infinitesimally small imaginary part $\mathrm{i}0$ in mass terms reflects causality
- general results for 1-loop integrals known
  (complicated but straightforward calculation)
  - ◇ momentum integrals can be carried out after "Feynman parametrization"
    $\hookrightarrow$ $(n-1)$-dimensional integrals for n-point functions
  - ◇ B functions $\to$ can be expressed in terms of log's
  - ◇ C, D, etc. $\to$ involve dilogarithms $\mathrm{Li}_2(x) = -\int_0^x \frac{\mathrm{d}t}{t} \ln(1-t)$
- tensor integrals can be decomposed into Lorentz covariants:

$$B^\mu = p^\mu B_1, \qquad B^{\mu\nu} = g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11},$$

$$C^\mu = p_1^\mu C_1 + p_2^\mu C_2, \quad C^{\mu\nu} = p_1^\mu p_1^\nu C_{11} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + g^{\mu\nu} C_{00}, \quad \text{etc.}$$

$\hookrightarrow$ tensor coefficients B_1, B_{ij}, C_i, etc. can be obtained as
linear combinations of scalar integrals B_0, C_0, etc.
(e.g. by "Passarino–Veltman reduction")

1.4 Renormalization

Propagators and 2-point functions:

Structure of one-loop self-energies (scalar case as example):

$$\Sigma(p^2) \;=\; C_1\, p^2\, \Delta \;+\; C_2\, \Delta \;+\; \Sigma_{\mathrm{finite}}(p^2) \;=\; \text{UV divergent}$$

Behaviour of propagator near pole for free propagation:

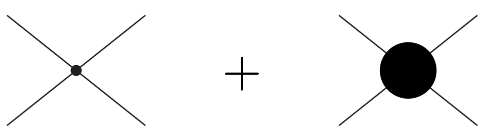
$$G^{\phi\phi}(p^2) \;=\; \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(p^2)} \;\underset{p^2\to m^2}{\widetilde{}}\; \frac{1}{1+\Sigma'(m^2)}\, \frac{\mathrm{i}}{p^2 - m^2 + \Sigma(m^2)}$$

$\hookrightarrow$ higher-order corrections change location and residue of propagator pole

Interaction vertices:

Example: scalar 4-point interaction $\mathcal{L}_{\phi^4} = \lambda\phi^4/4!$

$$\Gamma^{\phi\phi\phi\phi}(p_1,p_2,p_3) = \quad \mathrm{i}\lambda \quad + \quad \mathrm{i}\Lambda\,(p_1,p_2,p_3)$$



momentum-dependent one-loop correction:

$$\Lambda\,(p_1,p_2,p_3) \;=\; C_3\Delta \;+\; \Lambda_{\mathrm{finite}}\,(p_1,p_2,p_3) \;=\; \text{UV divergent}$$

$\hookrightarrow$ higher-order corrections change coupling strengths

Structure of UV divergences:

- Renormalizable field theories:

  UV divergences in vertex functions have analytical form of elementary vertex structures (directly related to $\mathcal{L}$)

  $\hookrightarrow$ idea: absorb divergences in free parameters

  $\Rightarrow$ Reparametrization of theory (=renormalization)

  Different types of renormalizable theories:

  ◇ theories with unrelated couplings of non-negative mass dimensions

  $\hookrightarrow$ renormalizability proven by power counting and "BPHZ procedure"

  ◇ gauge theories (couplings unified by gauge invariance)

  $\hookrightarrow$ renormalizability non-trivial consequence of gauge symmetry ‘t Hooft ’71

- Non-renormalizable field theories:

  e.g. theories with couplings of negative mass dimensions (cf. Fermi model)

  operators of higher and higher mass dimensions needed to absorb UV divergences

  $\hookrightarrow$ infinitely many free parameters, much less predictive power

Practical procedure for renormalization:

consider original ("bare") parameters and fields as preliminary
(denoted with subscripts "0" in the following)

$\hookrightarrow$ switch to new "renormalized" parameters and fields that obey certain conditions

Propagators and 2-point functions:

- mass renormalization: $m_0^2 = m^2 + \delta m^2$,
  $m^2 \stackrel{!}{=}$ location of propagator pole $=$ "physical mass" $\quad \to \quad \delta m^2 = \Sigma(m^2)$
- wave-function ren.: rescale fields $\phi_0 = \sqrt{Z_\phi}\phi$, $\quad G^{\phi\phi} = Z_\phi^{-1} G^{\phi_0\phi_0}$
  fix $Z_\phi = 1 + \delta Z_\phi$ such that residue of $G^{\phi\phi}$ at $p^2 = m^2$ equals 1
  $\hookrightarrow \quad \delta Z_\phi = -\Sigma'(m^2)$

$\Rightarrow$ Renormalized propagator $G^{\phi\phi}$ is UV finite:

$$G^{\phi\phi}(p^2) = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma_{\mathrm{ren}}(p^2)},$$

$$\Sigma_{\mathrm{ren}}(p^2) = \Sigma(p^2) - \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) \; = \; \text{ren. self-energy}$$

$$= \Sigma_{\mathrm{finite}}(p^2) - \Sigma_{\mathrm{finite}}(m^2) + (p^2 - m^2)\Sigma'_{\mathrm{finite}}(m^2) \; = \; \text{UV finite}$$

Vertex functions for interactions:

- coupling renormalization: $\lambda_0 \; = \; \lambda + \delta\lambda$

  fix $\delta\lambda$ such that λ assumes a measured value for special kinematics p_i^{exp}

  note: $\Gamma^{\phi\phi\phi\phi} \; = \; Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$

  $\hookrightarrow \; \delta\lambda = -2\delta Z_\phi \lambda - \Lambda(p_1^{\mathrm{exp}}, p_2^{\mathrm{exp}}, p_3^{\mathrm{exp}})$

$\Rightarrow$ Renormalized vertex function is UV finite:

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = \mathrm{i}\lambda + \mathrm{i}\Lambda_{\mathrm{ren}}(p_1, p_2, p_3),$$

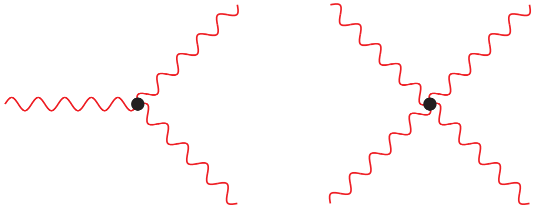
$$\Lambda_{\mathrm{ren}}(p_1, p_2, p_3) = \Lambda_{\mathrm{finite}}(p_1, p_2, p_3) \; - \; \Lambda_{\mathrm{finite}}(p_1^{\mathrm{exp}}, p_2^{\mathrm{exp}}, p_3^{\mathrm{exp}}) \; = \; \text{UV finite}$$

Electroweak Standard Model
–
Radiative Corrections

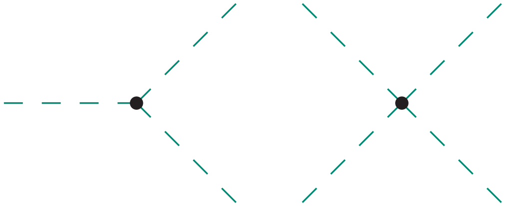
2.1 Loop corrections

Recapitulation of elementary SM couplings (vertices)

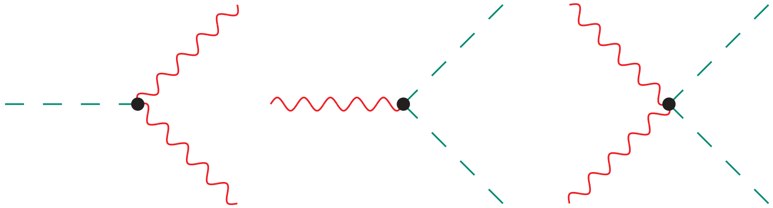
gauge-boson self-couplings:



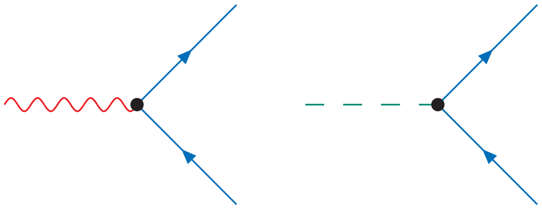
Higgs self-couplings:



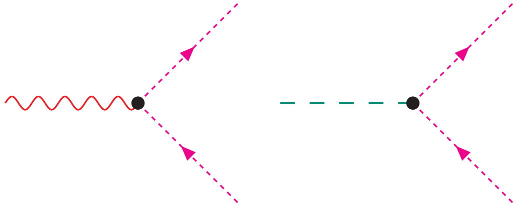
gauge-boson–Higgs couplings:



fermion couplings:



Faddeev–Popov couplings:

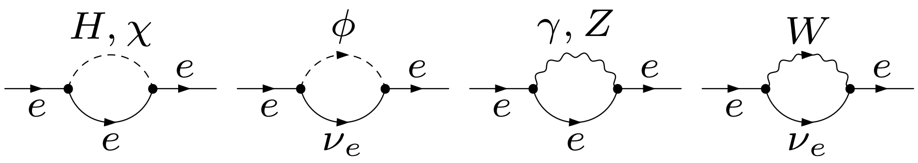


$\Rightarrow$ Large variety of loop diagrams !

Examples for 2-point functions at one loop: ('t Hooft–Feynman gauge)

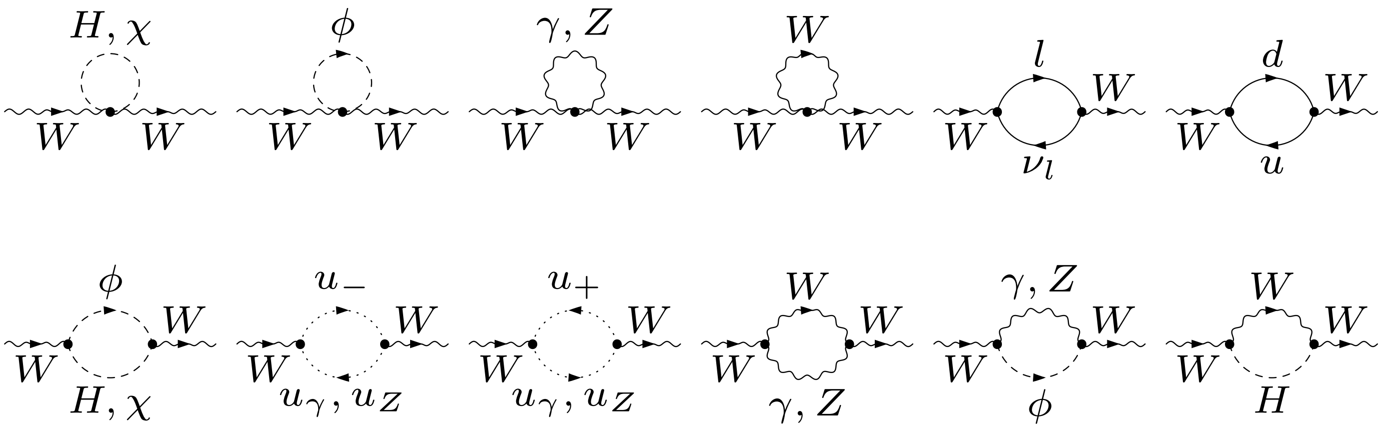
Electron self-energy:

$$\Gamma^{e\bar{e}}(p) \;=\; \mathrm{i}(\not{p} - m_\mathrm{e}) + \mathrm{i}\not{p}\omega_+\Sigma^\mathrm{e}_\mathrm{R}(p^2) + \mathrm{i}\not{p}\omega_-\Sigma^\mathrm{e}_\mathrm{L}(p^2) + \mathrm{i}m_\mathrm{e}\Sigma^\mathrm{e}_\mathrm{S}(p^2)$$



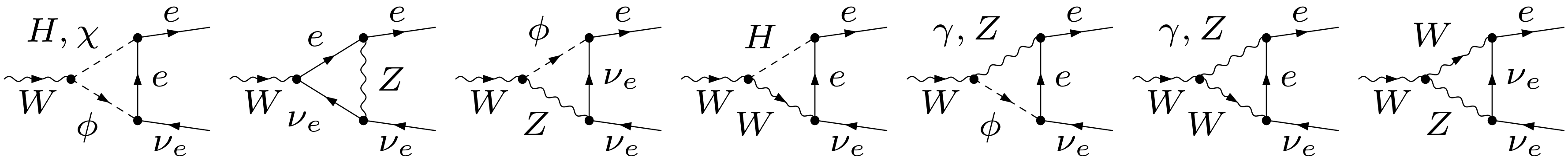
W-boson self-energy:

$$\Gamma^{\mathrm{W}^-\mathrm{W}^+}_{\mu\nu}(k) \;=\; -\mathrm{i}g_{\mu\nu}(k^2 - M_\mathrm{W}^2) - \mathrm{i}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Sigma^\mathrm{W}_\mathrm{T}(k^2) - \mathrm{i}\frac{k_\mu k_\nu}{k^2}\Sigma^\mathrm{W}_\mathrm{L}(k^2)$$

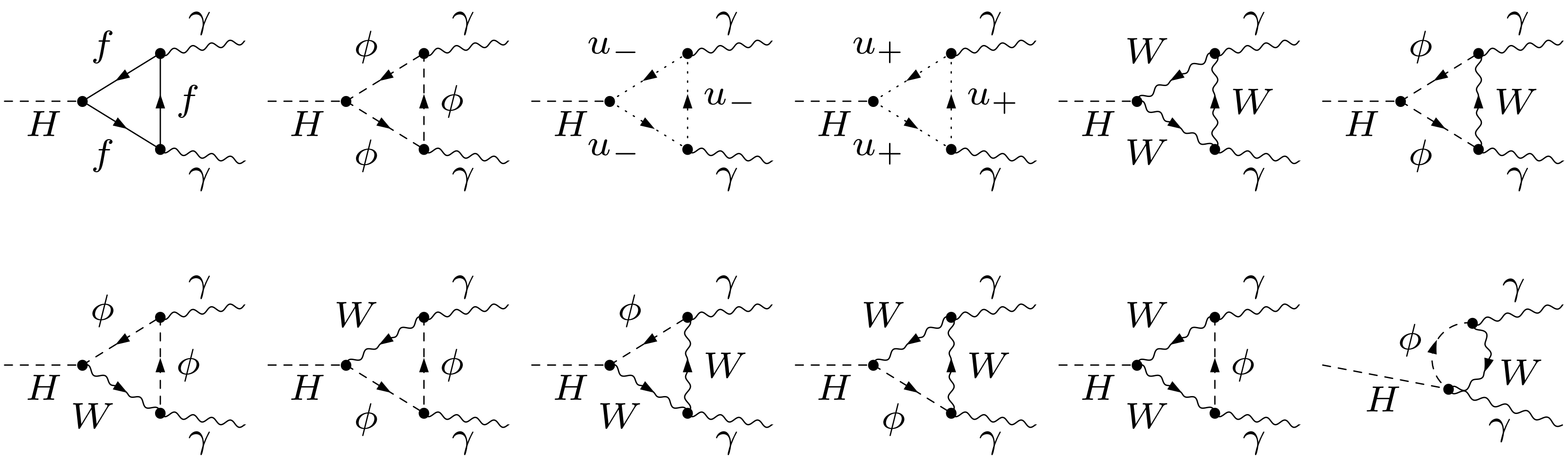


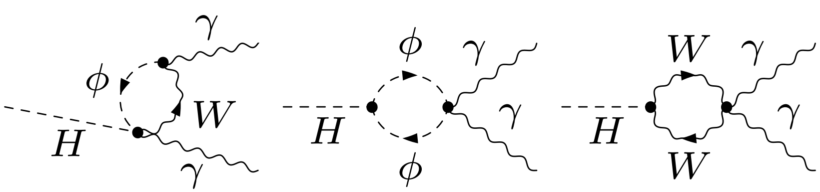
Examples for 3-point functions at one loop:

$\mathrm{We}\nu_\mathrm{e}$ vertex correction:



$\mathrm{H}\gamma\gamma$ vertex (loop induced):





$+$ diagrams (if different) with photons interchanged

2.2 Renormalization

Bare input parameters: $e_0, M_{\mathrm{W},0}, M_{\mathrm{Z},0}, M_{\mathrm{H},0}, m_{f,0}, V_{ij,0}$

Renormalization transformation:

- Parameter renormalization:

$$e_0 = (1+\delta Z_e)e,$$

$$M_{\mathrm{W},0}^2 = M_\mathrm{W}^2 + \delta M_\mathrm{W}^2, \quad M_{\mathrm{Z},0}^2 = M_\mathrm{Z}^2 + \delta M_\mathrm{Z}^2, \qquad M_{\mathrm{H},0}^2 = M_\mathrm{H}^2 + \delta M_\mathrm{H}^2,$$

$$m_{f,0} = m_f + \delta m_f, \qquad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad \text{(both } V_{ij,0}, V_{ij} \text{ unitary)}$$

Note: renormalization of $c_\mathrm{W}, s_\mathrm{W}$ fixed by on-shell condition $c_\mathrm{W} = \frac{M_\mathrm{W}}{M_\mathrm{Z}}$
(s_W is *not* a free parameter if M_W, M_Z are used as input parameters)

- Field renormalization

$$W_0^\pm = \sqrt{Z_W}\, W^\pm, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad H_0 = \sqrt{Z_H}\, H,$$

$$\psi_{f,0}^\mathrm{L} = \sqrt{Z_{ff'}^\mathrm{L}}\, \psi_{f'}^\mathrm{L}, \qquad \psi_{f,0}^\mathrm{R} = \sqrt{Z_{ff'}^\mathrm{R}}\, \psi_{f'}^\mathrm{R}$$

Note: matrix renormalization necessary to account for loop-induced mixing

Renormalization conditions:

- **Mass renormalization:**

  on-shell definition: mass^2 is location of pole in propagator

  $\hookrightarrow\ \delta M_\mathrm{W}^2 = \mathrm{Re}\{\Sigma_\mathrm{T}^W(M_\mathrm{W}^2)\}$, similar expressions for $\delta M_\mathrm{Z}^2, \delta M_\mathrm{H}^2, \delta m_f$

  Note: ◇ location of pole is complex for unstable particles

  $\hookrightarrow$ subtlety in all-orders definition, but not relevant at one loop
  (gauge-invariant definition: mass^2 as real part of pole location)

  ◇ other definitions of quark masses often more appropriate
  (running masses, masses in effective field theories)

- **Field renormalization:** (bosons and leptons)

  ◇ residues of propagators (diagonal, transverse parts) normalized to 1

  $\hookrightarrow\ \delta Z_W = -\mathrm{Re}\{\Sigma_\mathrm{T}^{W\,\prime}(M_\mathrm{W}^2)\}$,
  similar expressions for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{\mathrm{L/R}}$

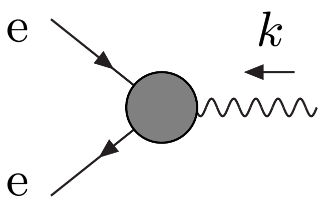
  ◇ suppression of mixing propagators on particle poles

  $\hookrightarrow$ fixes non-diagonal constants $\delta Z_{AZ}, \delta Z_{ZA}, \delta Z_{ff'}^{\mathrm{L/R}}\ (f \neq f')$

  Note: problems for unstables particles beyond one loop
  (field-renormalization constants become complex)

Renormalization conditions: (continued)

- Charge renormalization: define e in Thomson limit



$A_\mu \xrightarrow{k\to 0} \mathrm{i}e\gamma_\mu$ for on-shell electrons

$\Rightarrow$ $e =$ elementary charge of classical electrodynamics

fine-structure constant $\alpha(0) = \dfrac{e^2}{4\pi} = 1/137.03599976$

Gauge invariance relates δZ_e to photon wave-function renormalization:

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_\mathrm{W}}{2c_\mathrm{W}}\delta Z_{ZA}$$

- Quark-field and CKM-matrix renormalization $\rightarrow$ fixes $\delta Z^{\mathrm{L/R}}_{qq'}, \delta V_{ij}$

  rotation to mass eigenstates;

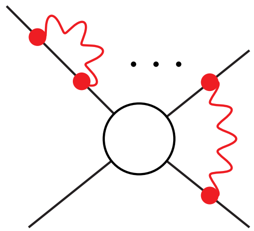
  CKM part requires a careful (non-trivial) investigation
  of mixing self-energies, mass eigenstates, LSZ reduction, etc.

General result: all renormalization constants can be obtained from self-energies.

2.3 IR divergences and photon bremsstrahlung

Consider processes with charged external particles, e.g., $\mathrm{e^+e^-} \to \mu^+\mu^-$

- Virtual corrections: loop diagrams

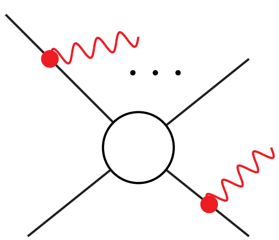


IR divergences from soft virtual photons ($q \to 0$)

$$\int \frac{\mathrm{d}^4 q \ldots}{(q^2 - m_\gamma^2)(2qp_1)(2qp_2)} \;\to\; C \ln(m_\gamma)$$

- "Real" corrections: photon bremsstrahlung

$$\int \frac{\mathrm{d}^3\mathbf{q}}{2q_0} \left| \quad \right|^2$$



IR divergences from soft real photons ($\mathbf{q} \to 0$)

$$\int \frac{\mathrm{d}^3\mathbf{q} \ldots}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \;\to\; -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

$\hookrightarrow$ virtual and soft-photonic corrections cannot be discussed separately

$\leftrightarrow$ related to limited experimental resolution of soft photons

$\Rightarrow$ Cross-section predictions necessarily depend on treatment of photon emission (energy and angular cuts)

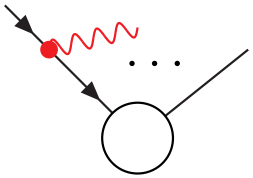
Separation of soft and hard photons:

Why? cancellation of $\ln(m_\gamma)$ terms delicate in practice, but terms are universal

- soft photons, $m_\gamma < E_\gamma < \Delta E \ll Q =$ typical scale of the process
  $\hookrightarrow$ correction is universal factor δ_{soft} to Born cross section
  relatively simple analytical expression with explicit $C\ln(\Delta E/m_\gamma)$ terms
- hard photons, $E_\gamma > \Delta E$
  $\hookrightarrow$ Monte Carlo integration of full radiative process, but with $m_\gamma = 0$
  $-C\ln(\Delta E)$ terms emerge numerically

$\ln(\Delta E)$ contributions cancel numerically in sum for small ΔE up to $\mathcal{O}(\Delta E/E)$

Calculation of soft-photon factor:



$$= \quad A(p-q)\frac{\mathrm{i}(\not{p}-\not{q}+m_f)}{(p-q)^2-m_f^2}(\mathrm{i}Q_f e)\not{\varepsilon}_\gamma^* u_f(p)$$

$$\underset{q\to 0}{\sim} \quad -Q_f e\,\frac{\varepsilon_\gamma^* p}{qp}A(p)u_f(p) \;=\; -Q_f e\,\frac{\varepsilon_\gamma^* p}{qp}\mathcal{M}_{\mathrm{Born}}$$

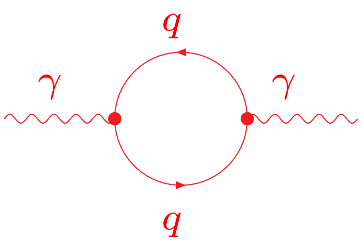
"Eikonal factorization" holds for all charged particles (spin $0, \frac{1}{2}, 1$)

$$\Rightarrow \quad \delta_{\mathrm{soft}} = -\frac{\alpha}{2\pi^2}\int_{m_\gamma<q_0<\Delta E}\frac{\mathrm{d}^3\mathbf{q}}{2q_0}\sum_{i,j}\frac{(\pm Q_i)(\pm Q_j)(p_i p_j)}{(qp_i)(qp_j)}$$

(i = particle with charge Q_i incoming($+$) or outgoing ($-$))

2.4 The universal radiative corrections $\Delta\alpha$ and $\Delta\rho$

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q

for $|s|$ in GeV range and below (non-perturbative regime)

$\hookrightarrow$ charge-renormalization constant δZ_e sensitive to m_q

Solution: fit hadronic part of $\Delta\alpha(s) \;=\; -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\}$ and thus of δZ_e

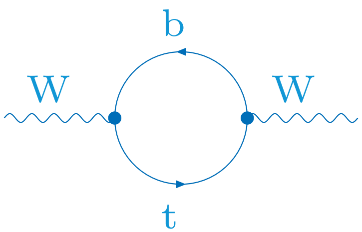
via dispersion relations to $R(s) \;=\; \dfrac{\sigma(\mathrm{e^+e^-} \to \text{hadrons})}{\sigma(\mathrm{e^+e^-} \to \mu^+\mu^-)}$ Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\alpha(s) \;=\; \dfrac{\alpha(0)}{1-\Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

$\hookrightarrow$ large effects from bottom–top loops in W self-energy Veltman '77



$$\Delta\rho_{\mathrm{top}} \sim \frac{\Sigma^{ZZ}_{\mathrm{T}}(0)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{WW}_{\mathrm{T}}(0)}{M_{\mathrm{W}}^2} \sim \frac{3G_\mu m_{\mathrm{t}}^2}{8\sqrt{2}\pi^2}$$

Radiative corrections to muon decay

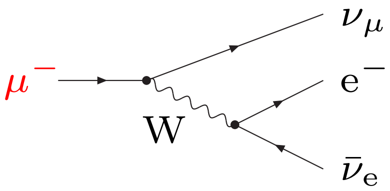
Precision calculation of M_W via μ decay

$\hookrightarrow$ M_W as function of $\alpha(0)$, G_μ, M_Z and the quantity Δr

$$M_\mathrm{W}^2\left(1-\frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha(0)}{\sqrt{2}G_\mu}(1+\Delta r)$$

Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

Lowest order:

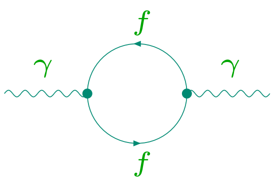


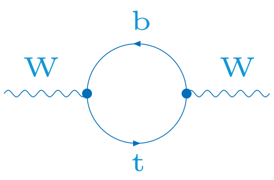
$\mathcal{O}(\alpha)$ corrections:

Sirlin '80, Marciano, Sirlin '80

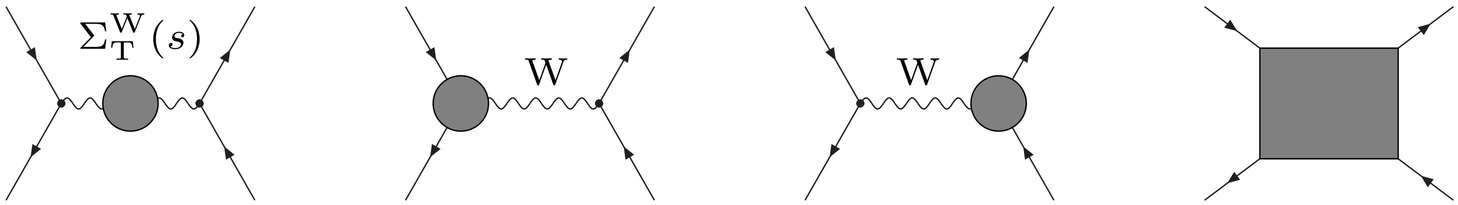
$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha(M_\mathrm{Z}^2) - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top} + \Delta r_\mathrm{rem}(M_\mathrm{H})$$

$\Delta\alpha(M_\mathrm{Z}^2)$	$\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top}$	$\Delta r_\mathrm{rem}(M_\mathrm{H})$
$\sim 6\%$	$\sim 3\%$	$\sim 1\%$
$\alpha\ln(m_f/M_\mathrm{Z})$	$G_\mu m_\mathrm{t}^2$	$\alpha\ln(M_\mathrm{H}/M_\mathrm{Z})$



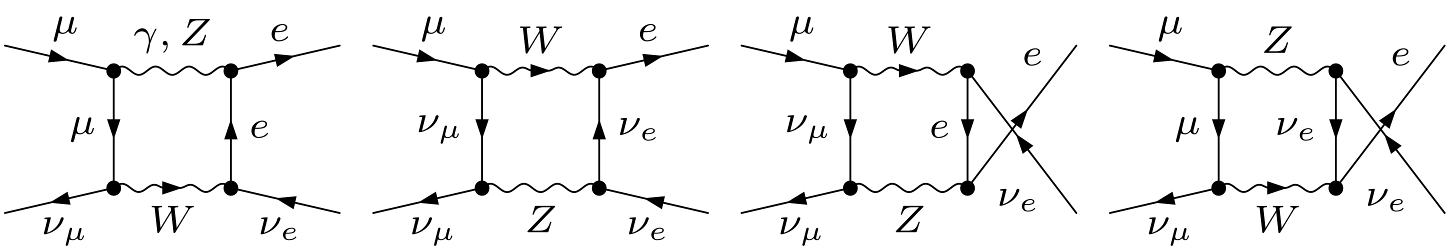


Virtual correction – 1-loop diagrams:

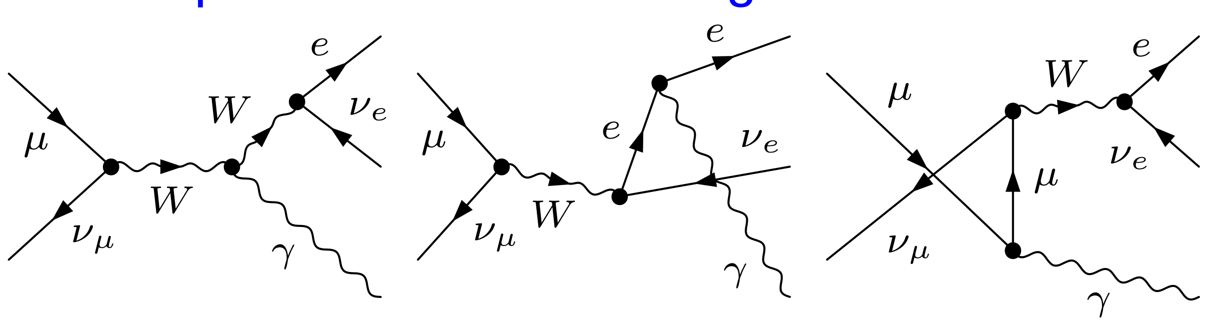


W self-energy | $\mathrm{W}l\nu_l$ vertex correction | box diagrams

e.g.:

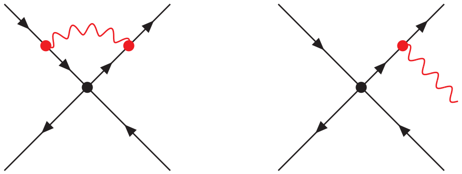


Real correction – 1-photon bremsstrahlung:



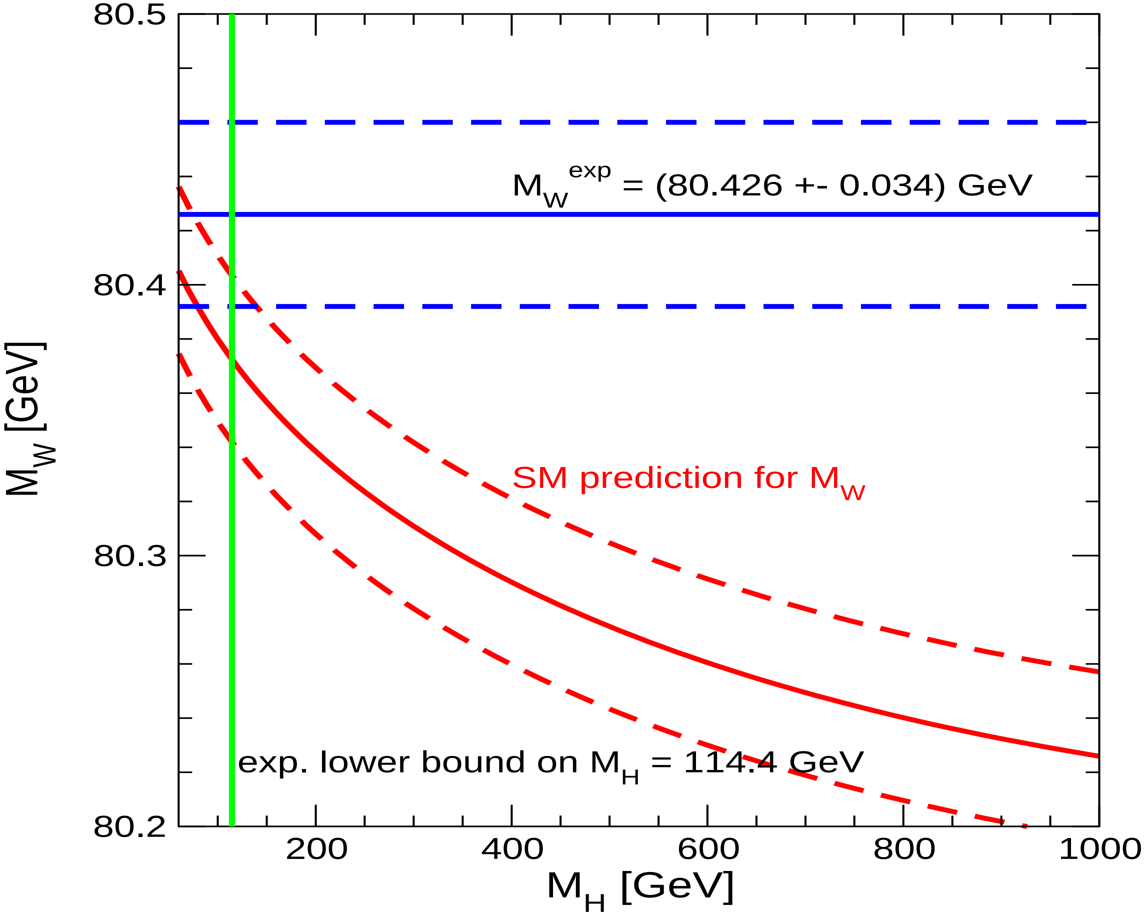
Consistent use of G_μ:

Photonic QED corrections are treated in the Fermi model and subtracted from Δr



etc.

State-of-the-art prediction of M_W from muon decay:



Hollik et al. ’03

Theoretical uncertainty:

status ’00: $\Delta M_\mathrm{W} \sim 6\,\mathrm{MeV}$

status ’06: $\Delta M_\mathrm{W} \sim 4\,\mathrm{MeV}$

Experimental error:

status ’06: $\Delta M_\mathrm{W} \sim 29\,\mathrm{MeV}$

ILC(?): $\Delta M_\mathrm{W} \sim 7\,\mathrm{MeV}$

Prediction includes:

- full electroweak corrections of $\mathcal{O}(\alpha)$ (1-loop level)
- full electroweak corrections of $\mathcal{O}(\alpha^2)$ (2-loop level)
  (v.Ritbergen,Stuart ’98; Seidensticker,Steinhauser ’99;
  Freitas,Hollik,Walter,Weiglein ’00-’02; Awramik,Czakon ’02/’03; Onishchenko,Veretin ’02)
- various improvements by universal corrections to ρ-parameter

Literature

- Textbooks:
  - ◇ Böhm/Denner/Joos: “Gauge Theories of the Strong and Electroweak Interaction”
  - ◇ Collins: “Renormalization”
  - ◇ Itzykson/Zuber: “Quantum Field Theory”
  - ◇ Peskin/Schroeder: “An Introduction to Quantum Field Theory”
  - ◇ Weinberg: “The Quantum Theory of Fields, Vol. 1: Foundations”;
    “The Quantum Theory of Fields, Vol. 2: Modern Applications”
- (Incomplete) list of articles on techniques for radiative corrections:
  - ◇ one-loop integrals:
    G. ’t Hooft and M. Veltman, Nucl. Phys. B 153 (1979) 365;
    G. Passarino and M. Veltman, Nucl. Phys. B 160 (1979) 151;
    W. Beenakker and A. Denner, Nucl. Phys. B 338 (1990) 349;
    A. Denner, U. Nierste and R. Scharf, Nucl. Phys. B 367 (1991) 637;
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    A. Denner and S. Dittmaier, arXiv:1005.2076 [hep-ph].
  - ◇ renormalization of the electroweak SM:
    K. I. Aoki, Z. Hioki, M. Konuma, R. Kawabe and T. Muta, Prog. Theor. Phys. Suppl. 73 (1982) 1;
    M. Böhm, W. Hollik and H. Spiesberger, Fortsch. Phys. 34 (1986) 687;
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    A. Denner, Fortsch. Phys. 41 (1993) 307;
    A. Denner, S. Dittmaier and G. Weiglein, Nucl. Phys. B 440 (1995) 95.
  - ◇ IR structure of photon radiation:
    D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. 13 (1961) 379.