



Electroweak Aspects of the Standard Model



Lecture I: Electroweak Theory

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Overview

- 1 **Electroweak phenomenology before the GSW model**
- 2 **The principle of local gauge invariance**
- 3 **The Standard Model of electroweak interaction**



1 Electroweak phenomenology before the GSW model

Some phenomenological facts:

- **discovery of the weak interaction via radioactive β -decay of nuclei:**



- **terminology “weak”:** interaction at low energy has very short range
 \hookrightarrow long life time of weakly decaying particles:

$$\text{strong int.: } \rho \rightarrow 2\pi, \quad \tau \sim 10^{-22}\text{s}$$

$$\text{elmg. int.: } \pi \rightarrow 2\gamma, \quad \tau \sim 10^{-16}\text{s}$$

$$\text{weak int.: } \pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \tau \sim 10^{-8}\text{s}$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu, \quad \tau \sim 10^{-6}\text{s}$$

- **lepton-number conservation:** $\mu^- \not\rightarrow e^- + \gamma$ (BR $\lesssim 10^{-11}$)

$\Rightarrow L_e, L_\mu, L_\tau$ individually conserved:

$$L_e = +1 \text{ for } e^-, \nu_e, \quad L_e = -1 \text{ for } e^+, \bar{\nu}_e, \quad \text{etc.}$$

(For massive ν 's with different masses, only $L_e + L_\mu + L_\tau$ is conserved.)

- **parity violation (Wu et al. 1957):**

e.g.: $K^+ \rightarrow \underbrace{2\pi, 3\pi}_{\text{final states of different parity}}$



\hookrightarrow polarization inversion does not yield inversion of spectra

The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for “**current–current interaction**” of four fermions:

$$\mathcal{L}_{\text{Fermi}}(x) = -2\sqrt{2}G_{\mu}J_{\rho}^{\dagger}(x)J^{\rho}(x), \quad G_{\mu} = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$$

with $J_{\rho}(x) = J_{\rho}^{\text{lep}}(x) + J_{\rho}^{\text{had}}(x) =$ charged weak current

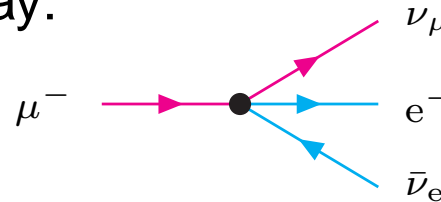
- Leptonic part J_{ρ}^{lep} of J_{ρ} :

$$J_{\rho}^{\text{lep}} = \overline{\psi_{\nu_e}} \gamma_{\rho} \omega_{-} \psi_e + \overline{\psi_{\nu_{\mu}}} \gamma_{\rho} \omega_{-} \psi_{\mu} \quad \omega_{\pm} = \frac{1}{2}(1 \pm \gamma_5) = \text{chirality projectors}$$

- ◇ only left-handed fermions ($\omega_{-}\psi$), right-handed anti-fermions ($\overline{\psi}\omega_{+}$) feel (charged-current) weak interactions \Rightarrow maximal P-violation

- ◇ doublet structure: $\begin{pmatrix} \nu_e \\ e^{-} \end{pmatrix}, \begin{pmatrix} \nu_{\mu} \\ \mu^{-} \end{pmatrix}$, later completed by $\begin{pmatrix} \nu_{\tau} \\ \tau^{-} \end{pmatrix}$

- ◇ $(J^{\text{lep},\rho})^{\dagger} J_{\rho}^{\text{lep}}$ induces muon decay:



- Hadronic part J_ρ^{had} of J_ρ :

Relevant quarks for energies $\lesssim 1 \text{ GeV}$: u, d, s, c
 \hookrightarrow meson ($q\bar{q}$) and baryon (qqq) spectra

Question: doublet structure $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}$?

Problem: e.g. annihilation of $u\bar{s}$ pair would not be allowed,
 but is observed: $\underbrace{K^+}_{u\bar{s} \text{ pair in quark model}} \rightarrow \mu^+ \nu_\mu$

Solution (Cabibbo 1963):

u-c-mixing and d-s-mixing in weak interaction

\hookrightarrow doublets $\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}$ with $\begin{pmatrix} d' \\ s' \end{pmatrix} = U_C \begin{pmatrix} d \\ s \end{pmatrix},$

orthogonal Cabibbo matrix $U_C = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix},$

empirical result: $\theta_C \approx 13^\circ$

$$J_\rho^{\text{had}} = \overline{\psi}_u \gamma_\rho \omega - \psi_{d'} + \overline{\psi}_c \gamma_\rho \omega - \psi_{s'}$$

Remarks on the Fermi model:

- **universal coupling** G_μ for all transitions
($U_C^\dagger U_C = \mathbf{1}$ is part of universality)
- no (pseudo-)scalar or tensor couplings, such as $(\bar{\psi}\psi)(\bar{\psi}\psi)$, $(\bar{\psi}\psi)(\bar{\psi}\gamma_5\psi)$, etc., necessary to describe low-energy experiments ($E \lesssim 1 \text{ GeV}$)
- **Problems:**
 - ◇ cross sections for $\nu_\mu e \rightarrow \nu_e \mu$, etc., grow for energy $E \rightarrow \infty$ as E^2
↪ **unitarity violation !**
 - ◇ no consistent evaluation of higher perturbative orders possible (no cancellation of UV divergences)
↪ **non-renormalizability !**

“Intermediate-vector-boson (IVB) model”

Idea: “resolution” of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\mathcal{L}_{\text{IVB}} = \mathcal{L}_{0,\text{ferm}} + \mathcal{L}_{0,W} + \mathcal{L}_{\text{int}},$$

$$\mathcal{L}_{0,\text{ferm}} = \overline{\psi}_f (i\not{\partial} - m_f) \psi_f, \quad (\text{summation over } f \text{ assumed})$$

$$\mathcal{L}_{0,W} = -\frac{1}{2} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) (\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_W^2 W_\mu^+ W^{-,\mu},$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad W_\mu^i \text{ real}$$

W^\pm are vector bosons with electric charge $\pm e$ and mass M_W .

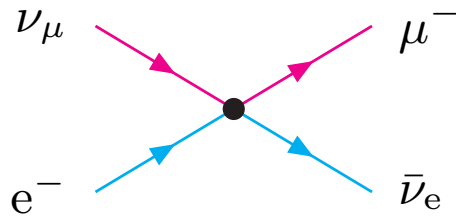
Propagator:
$$G_{\mu\nu}^{WW}(k) = \frac{-i}{k^2 - M_W^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2} \right), \quad k = \text{momentum}$$

Interaction Lagrangian:
$$\mathcal{L}_{\text{int}} = \frac{g_W}{\sqrt{2}} (J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-),$$

 $J^\rho = \text{charged weak current as in Fermi model}$

Four-fermion interaction in process $\nu_\mu e^- \rightarrow \mu^- \nu_e$

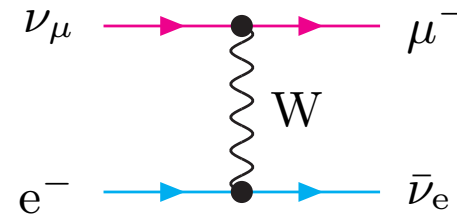
Fermi model:



$$-i2\sqrt{2}G_\mu g_{\rho\sigma}$$

$$\times [\bar{u}_\mu - \gamma^\rho \omega - u_{\nu_\mu}] [\bar{u}_{\nu_e} \gamma^\sigma \omega - u_{e^-}]$$

IVB model:



$$\frac{i}{2} g_W^2 \frac{1}{k^2 - M_W^2} \left(g_{\rho\sigma} - \frac{k_\rho k_\sigma}{M_W^2} \right)$$

$$\times [\bar{u}_\mu - \gamma^\rho \omega - u_{\nu_\mu}] [\bar{u}_{\nu_e} \gamma^\sigma \omega - u_{e^-}]$$

$$\Rightarrow \text{identification for } |k| \ll M_W: \quad 2\sqrt{2}G_\mu = \frac{g_W^2}{2M_W^2}$$

Consequences for the high-energy behaviour:

- k^ρ terms: $\bar{u}_{\nu_e} \not{k} \omega - u_{e^-} = \bar{u}_{\nu_e} (\not{p}_e - \not{p}_{\nu_e}) \omega - u_{e^-} = m_e \bar{u}_{\nu_e} \omega - u_{e^-}$
 \hookrightarrow no extra factors of scattering energy E
- propagator $1/(k^2 - M_W^2) \sim 1/E^2$ for $|k| \sim E \gg M_W$
 \hookrightarrow damping of amplitude in high-energy limit by factor $1/E^2$

$$\Rightarrow \text{cross section } \underset{E \rightarrow \infty}{\sim} \text{const}/E^2, \quad \Rightarrow \text{No unitarity violation !}$$

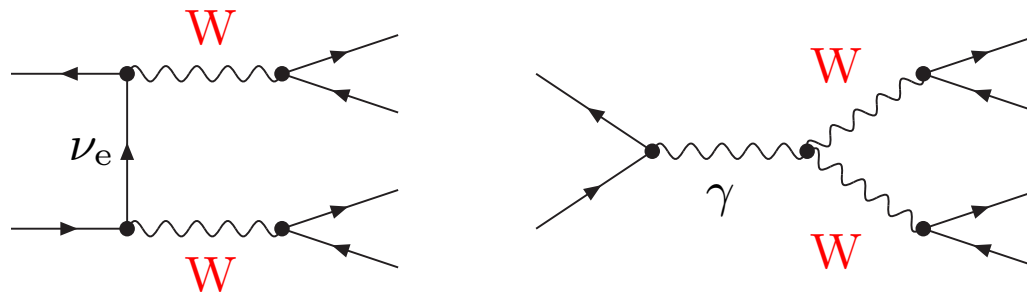
Comments on the IVB model:

- Formal similarity with QED interaction: $J^\rho W_\rho^+ + \text{h.c.} \longleftrightarrow j_{\text{elmg.}}^\rho A_\rho$
- Intermediate vector bosons can be produced, e.g.

$$\underbrace{u\bar{d}}_{\text{in pp collision}} \longrightarrow \underbrace{W^+ \rightarrow f\bar{f}'}_{W^\pm \text{ unstable}} \quad (\text{discovery 1983 at CERN})$$

• Problems:

- ◇ **unitarity violations** in cross sections with longitudinal W bosons, e.g.



- ◇ **non-renormalizability**
(no consistent treatment of higher perturbative orders)

↪ **Solution by spontaneously broken gauge theories !**

2 The principle of local gauge invariance

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\text{ferm}} = \overline{\psi}_f(i\cancel{\partial} - m_f)\psi_f$ has **global phase symmetry**:

$$\psi_f \rightarrow \psi'_f = \exp\{-iQ_f e\theta\}\psi_f, \quad \overline{\psi}_f \rightarrow \overline{\psi}'_f = \overline{\psi}_f \exp\{+iQ_f e\theta\}$$

with **space-time-independent group parameter θ**

“Gauging the symmetry”: **demand local symmetry, $\theta \rightarrow \theta(x)$**

To maintain local symmetry, extend theory by **“minimal substitution”**:

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + iQ_f e A^\mu(x) = \text{“covariant derivative”},$$
$$A^\mu(x) = \text{spin-1 gauge field (photon)}.$$

Transformation property of photon $A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu\theta(x)$ ensures

- $D_\mu\psi_f \rightarrow (D_\mu\psi_f)' = D'_\mu\psi'_f = \exp\{-iQ_f e\theta\}(D_\mu\psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\text{QED}} = \overline{\psi}_f(i\cancel{\partial} - Q_f e \cancel{A} - m_f)\psi_f - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting fields with “internal symmetry”:

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G :

$$\Phi \rightarrow \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-igT^a\theta^a\} = \text{unitary},$$

$$T^a = \text{group generators}, \quad [T^a, T^b] = iC^{abc}T^c, \quad \text{Tr} \{T^a T^b\} = \frac{1}{2}\delta^{ab}$$

- \mathcal{L}_Φ is invariant under G : $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu \Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad (m = \text{common boson mass}, \lambda = \text{coupling strength})$$

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) \rightarrow \mathcal{L}_\Phi(\Phi, D_\mu \Phi) \quad \text{with } D_\mu = \partial_\mu + igT^a A_\mu^a(x),$$

g = gauge coupling, T^a = generator of G in Φ representation, $A_\mu^a(x)$ = gauge fields

Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu\Phi)$ local invariant if $D_\mu\Phi \rightarrow (D_\mu\Phi)' = D'_\mu\Phi' = U(\theta)(D_\mu\Phi)$
 $\Rightarrow T^a A'_\mu{}^a = UT^a A_\mu^a U^\dagger - \frac{i}{g}U(\partial_\mu U^\dagger), \quad A_\mu^a A^{a,\mu} = \text{not gauge invariant}$
infinitesimal form: $\delta A_\mu^a = gC^{abc}\delta\theta^b A_\mu^c + \partial_\mu\delta\theta^a$
- covariant definition of field strength: $[D_\mu, D_\nu] = igT^a F_{\mu\nu}^a$
 $\Rightarrow T^a F_{\mu\nu}^a \rightarrow T^a F'_{\mu\nu}{}^a = UT^a F_{\mu\nu}^a U^\dagger, \quad F_{\mu\nu}^a F^{a,\mu\nu} = \text{gauge invariant}$
explicit form: $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gC^{abc}A_\mu^b A_\nu^c$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}F_{\mu\nu}^a F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu\Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2, A^4$ in F^2 part
 \hookrightarrow cubic and quartic gauge-boson self-interactions
- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction \rightarrow unification of interactions
- mass term $M^2(A_\mu^a A^{a,\mu})$ for gauge bosons forbidden by gauge invariance
 \hookrightarrow gauge bosons of unbroken Yang–Mills theory are massless

Quantum chromodynamics — gauge theory of strong interactions

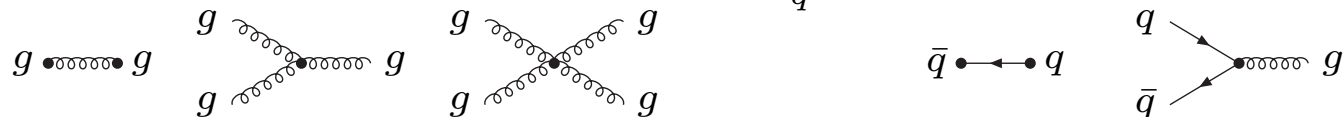
- **Gauge group:** $SU(3)_c$, $\dim. = 8$
 structure constants f^{abc} , gauge coupling g_s , $\alpha_s = \frac{g_s^2}{4\pi}$
- **Gauge bosons:** 8 massless gluons g with fields $A_\mu^a(x)$, $a = 1, \dots, 8$
- **Matter fermions:** quarks q (spin- $\frac{1}{2}$) with flavours $q = d, u, s, c, b, t$ in fundamental representation:

$$\psi_q(x) \equiv q(x) = \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix} = \text{colour triplet}$$

$$T^a = \frac{\lambda^a}{2}, \quad \text{Gell-Mann matrices } \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

- **Lagrangian:**

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + \sum_q \bar{\psi}_q (i\not{D} - m_q) \psi_q \\ &= -\frac{1}{4} \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f^{abc} A_\mu^b A_\nu^c \right)^2 + \sum_q \bar{\psi}_q \left(i\not{\partial} - g_s \frac{\lambda^a}{2} A^a - m_q \right) \psi_q \end{aligned}$$



3 The Standard Model of electroweak interaction (Glashow–Salam–Weinberg model)

3.1 The gauge group for electroweak interaction

Why unification of weak and elmg. interaction ?

- similarity: spin-1 fields couple to matter currents formed by spin- $\frac{1}{2}$ fields
- elmg. coupling of charged W^\pm bosons

γ, W^+, W^- as gauge bosons of group $SU(2)$? – No!

Reason: charge operator Q cannot be $SU(2)$ generator, since $\text{Tr} \{Q\} \neq 0$

for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$, etc.

Possible way out: additional heavy fermions like E^+ as partner to e^- ?

↪ no experimental confirmation !

Minimal solution: $SU(2)_I \times U(1)_Y$

- $SU(2)_I$ → weak isospin group with gauge bosons W^+ , W^- , W^0
- $U(1)_Y$ → weak hypercharge with gauge boson B

W^0 and B carry identical quantum numbers

↪ two neutral gauge bosons γ , Z as mixed states

Experiment: 1973 discovery of neutral weak currents at CERN

↪ indirect confirmation of Z exchange

1983 discovery of W^\pm and Z bosons at CERN

3.2 Fermion sector and minimal substitution

Multiplet structure:

Distinguish between left-/right-handed parts of fermions: $\psi^L = \omega_- \psi$, $\psi^R = \omega_+ \psi$

- ψ^L couple to W^\pm → group ψ^L into $SU(2)_I$ doublets, weak isospin $T_I^a = \frac{\sigma^a}{2}$
- ψ^R do not couple to W^\pm → ψ^R are $SU(2)_I$ singlets, weak isospin $T_I^a = 0$
- $\psi^{L/R}$ couple to γ in the same way
 ↪ adjust coupling to $U(1)_Y$ (i.e. fix weak hypercharges $Y^{L/R}$ for $\psi^{L/R}$)
 such that elmg. coupling results: $\mathcal{L}_{\text{int,QED}} = -Q_f e \bar{\psi}_f \not{A} \psi_f$

Fermion content of the SM:

(ignoring possible right-handed neutrinos)

			T_I^3	Q		
leptons:	$\Psi_L^L =$	$\begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix},$	$\begin{pmatrix} \nu_\mu^L \\ \mu^L \end{pmatrix},$	$\begin{pmatrix} \nu_\tau^L \\ \tau^L \end{pmatrix},$	$+\frac{1}{2}$	0
	$\psi_i^R =$	$e^R,$	$\mu^R,$	$\tau^R,$	0	-1
quarks: (Each quark exists in 3 colours!)	$\Psi_Q^L =$	$\begin{pmatrix} u^L \\ d^L \end{pmatrix},$	$\begin{pmatrix} c^L \\ s^L \end{pmatrix},$	$\begin{pmatrix} t^L \\ b^L \end{pmatrix},$	$+\frac{1}{2}$	$+\frac{2}{3}$
	$\psi_u^R =$	$u^R,$	$c^R,$	$t^R,$	0	$+\frac{2}{3}$
	$\psi_d^R =$	$d^R,$	$s^R,$	$b^R,$	0	$-\frac{1}{3}$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\text{ferm}} = i\bar{\psi}_f \not{\partial} \psi_f = i\bar{\Psi}_L^L \not{\partial} \Psi_L^L + i\bar{\Psi}_Q^L \not{\partial} \Psi_Q^L + i\bar{\psi}_l^R \not{\partial} \psi_l^R + i\bar{\psi}_u^R \not{\partial} \psi_u^R + i\bar{\psi}_d^R \not{\partial} \psi_d^R$$

Minimal substitution:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig_2 T_1^a W_\mu^a + ig_1 \frac{1}{2} Y B_\mu = D_\mu^L \omega_- + D_\mu^R \omega_+,$$

$$D_\mu^L = \partial_\mu - \frac{ig_2}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} g_2 W_\mu^3 - g_1 Y^L B_\mu & 0 \\ 0 & -g_2 W_\mu^3 - g_1 Y^L B_\mu \end{pmatrix},$$

$$D_\mu^R = \partial_\mu + ig_1 \frac{1}{2} Y^R B_\mu$$

Photon identification:

“Weinberg rotation”: $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},$ $c_W = \cos \theta_W, s_W = \sin \theta_W,$
 $\theta_W = \text{weak mixing angle}$

$$D_\mu^L \Big|_{A_\mu} = -\frac{i}{2} A_\mu \begin{pmatrix} -g_2 s_W - g_1 c_W Y^L & 0 \\ 0 & g_2 s_W - g_1 c_W Y^L \end{pmatrix} \stackrel{!}{=} ie A_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1 \rightarrow g_2 = \frac{e}{s_W}$

- normalize $Y^{L/R}$ such that $g_1 = \frac{e}{c_W}$

$\hookrightarrow Y$ fixed by “Gell-Mann–Nishijima relation”: $Q = T_1^3 + \frac{Y}{2}$

Fermion–gauge-boson interaction:

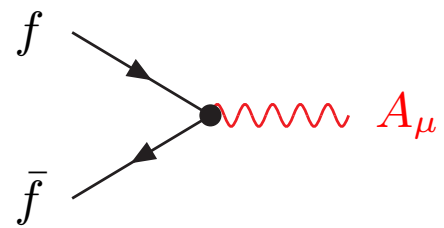
$$\mathcal{L}_{\text{ferm, YM}} = \frac{e}{\sqrt{2}s_W} \overline{\Psi}_F^L \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} \Psi_F^L + \frac{e}{2c_W s_W} \overline{\Psi}_F^L \sigma^3 Z \Psi_F^L$$

$$- e \frac{s_W}{c_W} Q_f \overline{\psi}_f Z \psi_f - e Q_f \overline{\psi}_f A \psi_f \quad (f=\text{all fermions}, F=\text{all doublets})$$

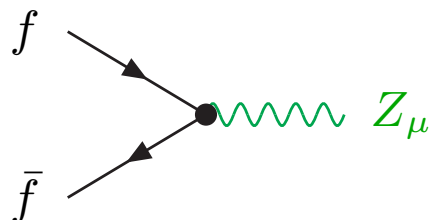
Feynman rules:



$$\frac{ie}{\sqrt{2}s_W} \gamma_\mu \omega_-$$



$$-iQ_f e \gamma_\mu$$



$$ie\gamma_\mu (g_f^+ \omega_+ + g_f^- \omega_-) = ie\gamma_\mu (v_f - a_f \gamma_5)$$

$$\text{with } g_f^+ = -\frac{s_W}{c_W} Q_f, \quad g_f^- = -\frac{s_W}{c_W} Q_f + \frac{T_{I,f}^3}{c_W s_W},$$

$$v_f = -\frac{s_W}{c_W} Q_f + \frac{T_{I,f}^3}{2c_W s_W}, \quad a_f = \frac{T_{I,f}^3}{2c_W s_W}$$

3.3 Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu}$$

Field-strength tensors:

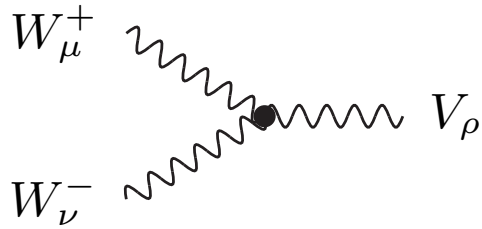
$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

$$\begin{aligned} \mathcal{L}_{\text{YM}} = & -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\ & - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ & + \text{(trilinear interaction terms involving } AW^+W^-, ZW^+W^-) \\ & + \text{(quadrilinear interaction terms involving} \\ & \quad AAW^+W^-, AZW^+W^-, ZZW^+W^-, W^+W^-W^+W^-) \end{aligned}$$

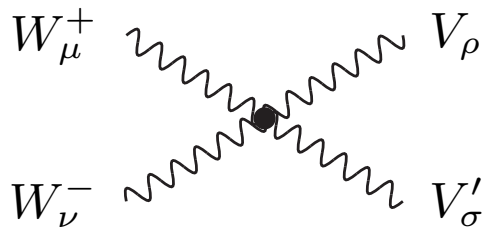
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$ieC_{WWV} \left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu \right]$$

$$\text{with } C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_W}{s_W}$$



$$ie^2 C_{WWVV'} \left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho} \right]$$

$$\text{with } C_{WW\gamma\gamma} = -1, \quad C_{WW\gamma Z} = \frac{c_W}{s_W},$$

$$C_{WWZZ} = -\frac{c_W^2}{s_W^2}, \quad C_{WWWW} = \frac{1}{s_W^2}$$

3.4 Higgs sector and spontaneous symmetry breaking

Idea: spontaneous breakdown of $SU(2)_I \times U(1)_Y$ symmetry $\rightarrow U(1)_{\text{emg}}$ symmetry
 \hookrightarrow masses for W^\pm and Z bosons, but γ remains massless

Note: choice of scalar extension of massless model involves freedom

GSW model:

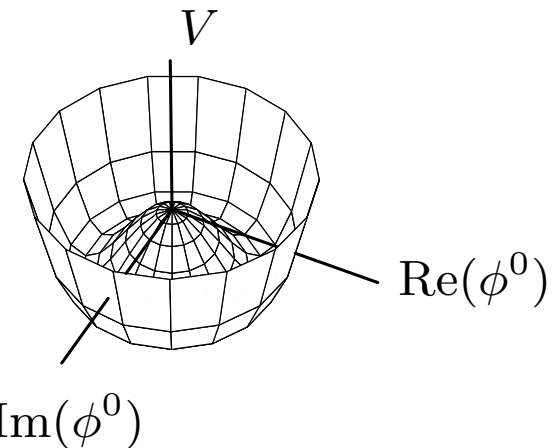
Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$= SU(2)_I \times U(1)_Y$ symmetric

$$V(\Phi) = \text{minimal for } |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant \Rightarrow spontaneous symmetry breaking

emg. gauge invariance unbroken, since $Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

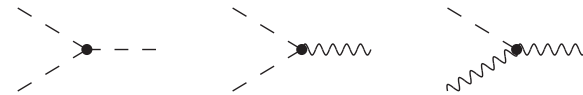
Field excitations in Φ :

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}} \left(v + H(x) + i\chi(x) \right) \end{pmatrix}$$

Gauge-invariant Lagrangian of Higgs sector: $(\phi^- = (\phi^+)^\dagger)$

$$\begin{aligned} \mathcal{L}_H &= (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad \text{with } D_\mu = \partial_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a + i \frac{g_1}{2} B_\mu \\ &= (\partial_\mu \phi^+) (\partial^\mu \phi^-) - \frac{iev}{2s_W} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4s_W^2} W_\mu^+ W^{-,\mu} \\ &\quad + \frac{1}{2} (\partial\chi)^2 + \frac{ev}{2c_W s_W} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4c_W^2 s_W^2} Z^2 + \frac{1}{2} (\partial H)^2 - \mu^2 H^2 \end{aligned}$$

+ (trilinear SSS , SSV , SVV interactions)



+ (quadrilinear $SSSS$, $SSVV$ interactions)



Implications:

- gauge-boson masses: $M_W = \frac{ev}{2s_W}$, $M_Z = \frac{ev}{2c_W s_W} = \frac{M_W}{c_W}$, $M_\gamma = 0$
- physical Higgs boson H : $M_H = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons ϕ^\pm, χ : unphysical degrees of freedom

3.5 Fermion masses and Yukawa couplings

Ordinary Dirac mass terms $m_f \bar{\psi}_f \psi_f = m_f (\bar{\psi}_f^L \psi_f^R + \bar{\psi}_f^R \psi_f^L)$ not gauge invariant

↪ introduce fermion masses by (gauge-invariant) Yukawa interaction

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\text{Yuk}} = -\bar{\Psi}_L^L G_l \psi_l^R \Phi - \bar{\Psi}_Q^L G_u \psi_u^R \tilde{\Phi} - \bar{\Psi}_Q^L G_d \psi_d^R \Phi + \text{h.c.}$$

- $G_l, G_u, G_d = 3 \times 3$ matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ = charge conjugate Higgs doublet, $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms = bilinear terms in \mathcal{L}_{Yuk} , obtained by setting $\Phi \rightarrow \Phi_0$:

$$\mathcal{L}_{m_f} = -\frac{v}{\sqrt{2}} \bar{\psi}_l^L G_l \psi_l^R - \frac{v}{\sqrt{2}} \bar{\psi}_u^L G_u \psi_u^R - \frac{v}{\sqrt{2}} \bar{\psi}_d^L G_d \psi_d^R + \text{h.c.}$$

↪ diagonalization by unitary field transformations ($f = l, u, d$)

$$\hat{\psi}_f^{L/R} \equiv U_f^{L/R} \psi_f^{L/R} \quad \text{such that} \quad \frac{v}{\sqrt{2}} U_f^L G_f (U_f^R)^\dagger = \text{diag}(m_f)$$

$$\Rightarrow \text{standard form:} \quad \mathcal{L}_{m_f} = -m_f \bar{\hat{\psi}}_f^L \hat{\psi}_f^R + \text{h.c.} = -m_f \bar{\hat{\psi}}_f \hat{\psi}_f$$

Quark mixing:

- ψ_f correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$ correspond to mass eigenstates,
for **massless neutrinos** define $\hat{\psi}_\nu^L \equiv U_l^L \psi_\nu^L \rightarrow$ **no lepton-flavour changing**

Yukawa and gauge interactions in terms of mass eigenstates:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & -\frac{\sqrt{2}m_l}{v} \left(\phi^+ \overline{\hat{\psi}_{\nu_l}^L} \hat{\psi}_l^R + \phi^- \overline{\hat{\psi}_l^R} \hat{\psi}_{\nu_l}^L \right) + \frac{\sqrt{2}m_u}{v} \left(\phi^+ \overline{\hat{\psi}_u^R} V \hat{\psi}_d^L + \phi^- \overline{\hat{\psi}_d^L} V^\dagger \hat{\psi}_u^R \right) \\ & - \frac{\sqrt{2}m_d}{v} \left(\phi^+ \overline{\hat{\psi}_u^L} V \hat{\psi}_d^R + \phi^- \overline{\hat{\psi}_d^R} V^\dagger \hat{\psi}_u^L \right) - \frac{m_f}{v} i \text{sgn}(T_{I,f}^3) \chi \overline{\hat{\psi}_f} \gamma_5 \hat{\psi}_f \\ & - \frac{m_f}{v} (v + H) \overline{\hat{\psi}_f} \hat{\psi}_f, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{ferm, YM}} = & \frac{e}{\sqrt{2}s_W} \overline{\hat{\Psi}_L^L} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} \hat{\psi}_L^L + \frac{e}{\sqrt{2}s_W} \overline{\hat{\Psi}_Q^L} \begin{pmatrix} 0 & V W^+ \\ V^\dagger W^- & 0 \end{pmatrix} \hat{\psi}_Q^L \\ & + \frac{e}{2c_W s_W} \overline{\hat{\Psi}_F^L} \sigma^3 \not{Z} \hat{\Psi}_F^L - e \frac{s_W}{c_W} Q_f \overline{\hat{\psi}_f} \not{Z} \hat{\psi}_f - e Q_f \overline{\hat{\psi}_f} \not{A} \hat{\psi}_f \end{aligned}$$

- only charged-current coupling of quarks modified by $V = U_u^L (U_d^L)^\dagger =$ **unitary**
(Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- **Higgs–fermion coupling strength** $= \frac{m_f}{v}$

Features of the CKM mixing:

- $V = 3$ -dim. generalization of Cabibbo matrix U_C
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase
↪ complex phase is the only source of CP violation in SM

counting:

$$\begin{aligned} & \left(\begin{array}{c} \text{\#real d.o.f.} \\ \text{in } V \end{array} \right) - \left(\begin{array}{c} \text{\#unitarity} \\ \text{relations} \end{array} \right) - \left(\begin{array}{c} \text{\#phase diffs. of} \\ u\text{-type quarks} \end{array} \right) - \left(\begin{array}{c} \text{\#phase diffs. of} \\ d\text{-type quarks} \end{array} \right) - \left(\begin{array}{c} \text{\#phase diff. between} \\ u\text{- and } d\text{-type quarks} \end{array} \right) \\ & = 18 - 9 - 2 - 2 - 1 = 4 \end{aligned}$$

- no flavour-changing neutral currents in lowest order,
flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
("Glashow–Iliopoulos–Maiani mechanism")