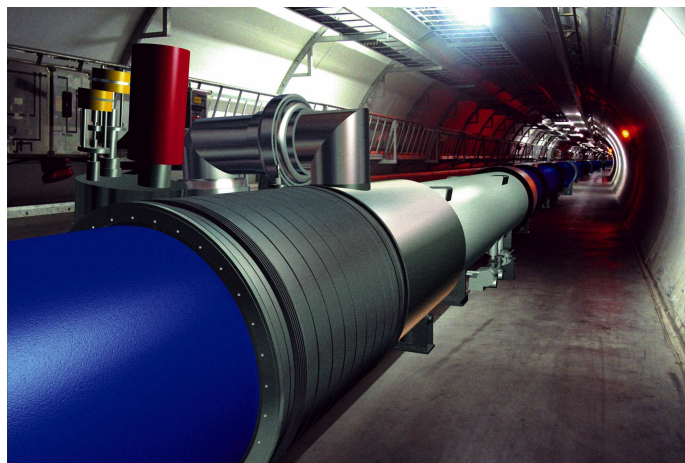


# The Standard Model



1. Introduction to the Electroweak Theory
2. Introduction to Perturbative QCD and LHC theory
3. Probing the Standard Model at the LHC I
4. Probing the Standard Model at the LHC II

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## Introduction to Perturbative QCD and LHC Theory

- Review perturbative QCD and LHC theory today
- References:
  - ☞ Chris Quigg, *Gauge Theories of the Strong, Weak and Electromagnetic Interactions*
  - ☞ Francis Halzen and Alan Martin, *Quarks and Leptons*
  - ☞ Vernon Barger and Roger Phillips, *Collider Physics*

## a little bit of history ...

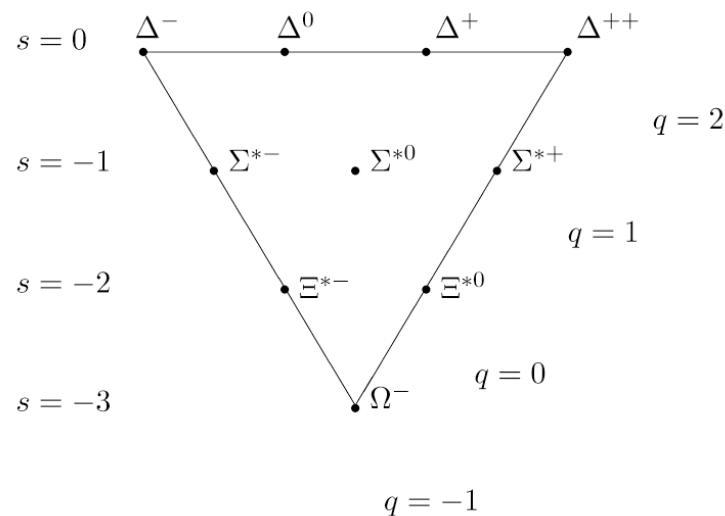
- after the discovery of the proton it became clear that there had to be a new interaction, much stronger than the electromagnetic interaction, which binds protons inside the nucleus
- strong force needed to be short range (otherwise we would see it like the e.m. force)
- after the discovery of the pion, it was thought that the pion is the carrier of the strong force
- however, it was very difficult to construct a working theory of strong interactions:
  - ☞ non-renormalizable because carrier is massive
  - ☞ proton – pion coupling too large for perturbation theory to make sense
  - ☞ ... and many other reasons

- meanwhile, experimentalists were busy discovering new resonances  $\Delta, \Sigma, \eta, \Xi, \dots$

- Gell-Mann proposed to classify baryons and mesons in representations of the group  $SU(3)$ , using isospin and strangeness as a basis

☞ baryons form octets (**8**) and decuplets (**10**) under  $SU(3)$

decuplet:



☞ mesons form octets and singlets

- neither of them is the fundamental representation of SU(3) (which is a triplet, **3**)

- can interpret mesons as bound states of new fundamental fermions (**quarks**) which form a triplet under SU(3) and their antiparticles:

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$$

- baryons can be interpreted as bound states of three quarks:

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$$

- problems:

- ☞ need a new interaction to bind quarks in baryons and mesons

- ☞ the spin statistics theorem is violated for quarks:

- take the  $\Delta^{++} = uuu$  state

- the space part of the wavefunction is totally symmetric (ground state)

- the spin part of the wavefunction is totally symmetric:  $\text{spin}(\Delta)=3/2$

- the SU(3) part of the wavefunction is totally symmetric: three  $u$ -quarks

- the spin statistics theorem requires the wave function of fermions to be totally antisymmetric

- solution to both problems: introduce a new degree of freedom: **color**
- each quark comes in three colors; quarks form a triplet under  $SU(3)_c$
- postulate that only color singlet states bound states are observed (**confinement hypothesis**):

$$|\text{meson}\rangle \sim \delta_{ij} |q_i \bar{q}'_j\rangle$$

$$|\text{baryon}\rangle \sim \epsilon_{ijk} |q_i q'_j q''_k\rangle$$

the color singlet wavefunction for three quarks now is totally anti-symmetric

- gauging  $SU(3)_c$  provides the new interaction which binds quarks in hadrons; force carrier: **gluons**
- the strong interaction which binds protons (and neutrons) inside nuclei is nothing but a remnant (similar to the van der Waals force) of the gluon interaction between quarks

## The QCD Lagrangian

- starting from the  $SU(2)$  part of the electroweak Lagrangian, it is straightforward to construct the QCD Lagrangian:
  - ➡ replace Pauli matrices by Gell-Mann matrices  $\lambda_a$  (generators of  $SU(3)$ ),  $a = 1, \dots, 8$
  - ➡ replace  $\epsilon_{ijk}$  by  $SU(3)$  structure constants  $f_{abc}$  (totally antisymmetric),  $a, b, c = 1, \dots, 8$
  - ➡ for explicit form of  $\lambda_a$  and  $f_{abc}$  see eg. the textbook by Quigg
  - ➡ couple lefthanded and righthanded quarks to gluons with equal strength
  - ➡ add a possible mass term for quarks (not forbidden by gauge invariance)



- Lagrangian:

$$\mathcal{L} = \bar{q} (i\gamma^\mu \partial_\mu - m) q - g_s \left( \bar{q} \gamma^\mu \frac{\lambda_a}{2} q \right) G_\mu^a - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

$q$ : quark spinor (triplet in  $SU(3)_c$  space);  $m$ : quark mass;  $g_s$ : QCD coupling constant,  $G_\mu^a$ ,  $a = 1, \dots, 8$ ,  $SU(3)_c$  gauge fields (gluons),

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c$$

- Remarks:

☞  $SU(3)_c$  remains unbroken → gluons are massless

☞ because  $SU(3)$  is a non-abelian group ( $f_{abc} \neq 0$ ), gluons interact with themselves: three- and four-gluon vertices, similar to self-interactions of  $W$ 's and  $Z$ 's

☞ the self-interaction of gluons causes **asymptotic freedom**, and (most likely) confinement

☞ **asymptotic freedom**:  $g_s \rightarrow 0$  at short distances or high energies  
→ strong interactions can be treated perturbatively at high energies

## The QCD Running Coupling Constant

- Once higher order contributions to the gauge boson propagator are taken into account the coupling constant in gauge theories becomes momentum dependent (**running coupling constant**)
- proceed step by step:
  - ☞ consider higher order corrections to photon propagator in QED
  - ☞ derive expression for running QED coupling
  - ☞ finally consider QCD case
- ready for a little math? Ok, here we go...

- one-loop corrections to photon propagator (electron loop):  
use QED Feynman rules:

$$\text{Diagram} = \frac{(-i)}{q^2} I_{\mu\nu} \frac{(-i)}{q^2}$$

$q$ : photon momentum, and

$$I_{\mu\nu}(q^2) = - \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[ (ie\gamma_\mu) \frac{i(\not{p} + m)}{p^2 - m^2} (ie\gamma_\nu) \frac{i(\not{p} - q + m)}{(p - q)^2 - m^2} \right]$$

$p$ : loop momentum;  $m$ : mass of electron in the loop  
after a little algebra...

$$I_{\mu\nu}(q^2) = -ig_{\mu\nu} q^2 I(q^2) + \dots$$

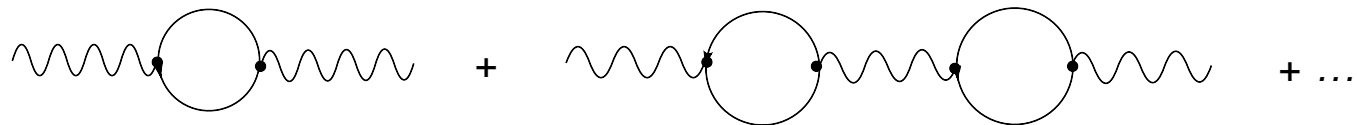
$$I(q^2) = \frac{\alpha}{3\pi} \int_{m^2}^{\infty} \frac{dp^2}{p^2} - \frac{2\alpha}{\pi} \int_0^1 dz z(1-z) \log \left( 1 - \frac{q^2 z(1-z)}{m^2} \right)$$

$$\alpha = e^2/4\pi$$

- there is a little problem with  $I(q^2)$ :  
it diverges! Now what?
- cutoff integration at  $\Lambda^2$ . Then, for large  $(-q^2)$ ,

$$I(q^2) = \frac{\alpha}{3\pi} \log \left( \frac{\Lambda^2}{-q^2} \right)$$

- now we can sum the whole series of bubble diagrams:



$$= \frac{1}{1 + I(q^2)} \frac{g_{\mu\nu}}{q^2}$$

- since the photon propagator always appears between two fermion vertices, one can absorb the factor  $1/(1 + I(q^2))$  in  $e^2$  or  $\alpha$ :

$$\alpha(Q^2) = \frac{\alpha_0}{1 - \frac{\alpha_0}{3\pi} \log\left(\frac{Q^2}{\Lambda^2}\right)}$$

$$Q^2 = -q^2$$

- now suppose we know  $\alpha(\mu^2)$  for a certain momentum  $\mu$  (renormalization point). Then

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log\left(\frac{Q^2}{\mu^2}\right)}$$

The dependence on the artificial cutoff parameter  $\Lambda$  is gone!

- Note the QED coupling constant **increases** with  $Q^2$ :

$$\alpha(m_e^2) = 1/137, \alpha(M_Z^2) \approx 1/128.89$$

- now we take a look at the QCD coupling constant
- because of the gluon self-couplings, gluon loops contribute in addition to quark loops to the gluon propagator at higher order:



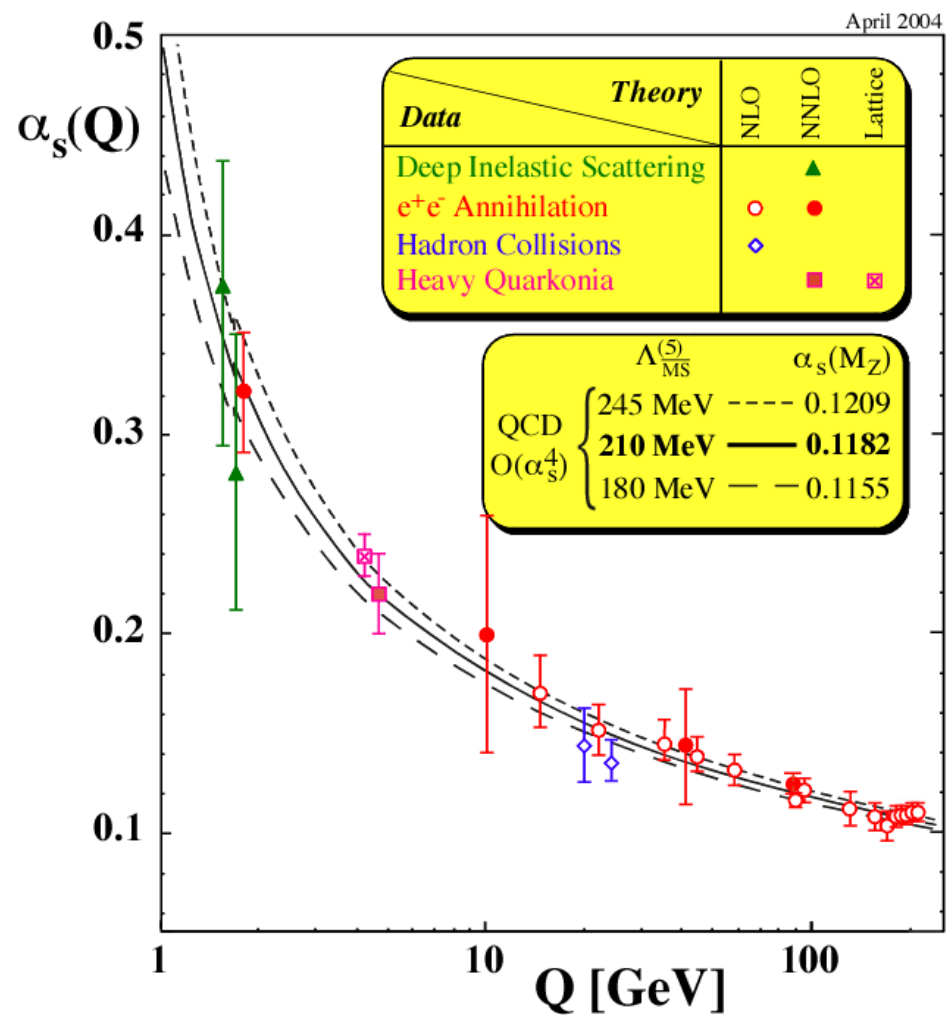
- the contribution of the gluon loop has the **opposite** sign of that of the quark loop:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi} (33 - 2n_f) \log\left(\frac{Q^2}{\mu^2}\right)}$$

$n_f$ : number of quark flavors

- if  $n_f < 16$ ,  $\alpha_s \rightarrow 0$  for  $Q^2 \rightarrow \infty$

- comparison with data (Bethke)



## How to calculate cross sections at the LHC

- now we know that we can calculate QCD processes using perturbation theory at the LHC
- from yesterday's lecture we also know that the coupling constant for electroweak interactions is sufficiently small to use perturbation theory
- using Feynman rules we now can ( at least in principle) calculate whatever process we want and at whatever order in perturbation theory we want
- but these calculations are done for incoming and outgoing quarks and gluons  
we have to connect the incoming quarks and gluons with the colliding protons, and the outgoing particles with the observed hadronic jets



- to lowest order in perturbation theory, each outgoing quark or gluon is identified with a hadronic jet, provided they are well separated in **pseudo-rapidity – azimuth space**

$$\Delta R = [\Delta\eta^2 + \Delta\Phi^2]^{-1/2} > R_{min}$$

$\Phi$  angle in plane perpendicular to beam axis (azimuthal angle);  $\eta$ : pseudo-rapidity

$$\eta = -\log(\tan(\theta/2)) = \frac{1}{2} \log \left( \frac{E + p_z}{E - p_z} \right)$$

$\theta$ : scattering angle;  $p_z$ : momentum in beam ( $z$ ) direction;  $E$ : energy

- to connect protons with quarks and gluons we need to know the probability that a quark or gluon is carrying a certain fraction of the proton momentum

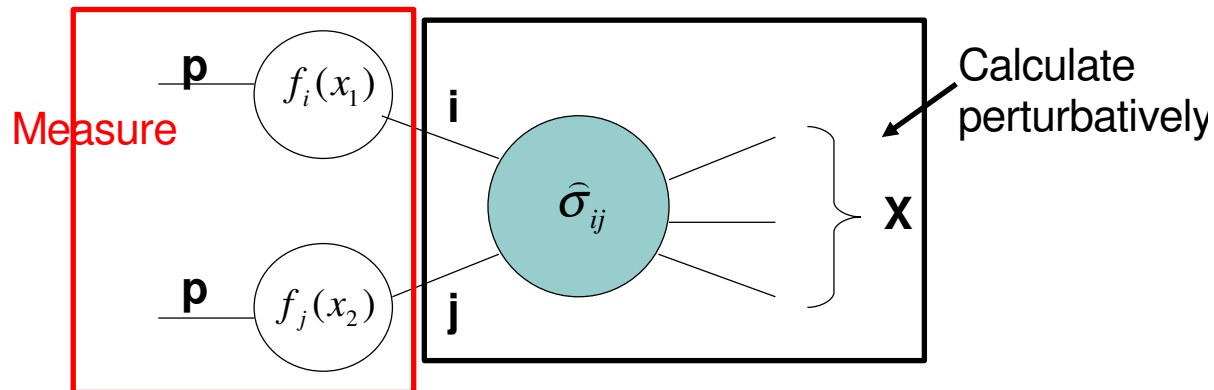
**provided by parton distribution functions (PDFs),  $f_i(x, \mu_f)$**

# Master Formula

- the mother of all LHC calculations:

$$\sigma(pp \rightarrow X) = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_f) f_j(x_2, \mu_f) \hat{\sigma}(ij \rightarrow X)$$

$x_{1,2}$ : momentum fractions carried by incoming quarks/gluons;  $\mu_f$ : factorization scale;  $\hat{\sigma}$ : partonic cross section (that's what we calculate from Feynman diagrams using Feynman rules)



## Sidebar: PDF's

- PDF's cannot be calculated perturbatively  
→ determined by experiment (at a given  $\mu_f$ )
- QCD predicts **evolution of PDF's via the Altarelli-Parisi equations**, *ie* once we know the PDF's for a certain scale, QCD predicts them for all other scales
- PDF's are **universal**: PDF's determined *eg* at HERA can be directly used for LHC calculations
- PDF's for valence and sea quarks are different!
- PDF's come in form of canned FORTRAN routines by the MRST and CTEQ Collaborations

## Generic QCD Cross Section

- focus on QCD cross section for a moment
- calculate  $\hat{\sigma}$  as power series in  $\alpha_s$

$$\hat{\sigma}(ij \rightarrow X) = \alpha_s^n(\mu_r) \sum_{k=0}^{\infty} \alpha_s^k(\mu_r) \hat{\sigma}_k(ij \rightarrow X)$$

$\mu_r$ : renormalization scale;  $n$ : depends on final state  $X$

- if one could do an all-order calculation the cross section  $\sigma$  would be independent of  $\mu_r$  and  $\mu_f$
- $k = 0$ : leading order (LO)  
 $k = 1$ : next-to-leading order (NLO), etc.

## QCD Cross Sections at the LHC

- Understanding physics discoveries at the LHC (and the Tevatron), requires accurate SM predictions, in particular for QCD cross sections *this can be difficult*
- the LHC will copiously produce final states with many particles (for example in supersymmetry)
- need precise QCD calculations, together with Monte Carlo techniques to connect to real world, *ie.* jets
- LO cross sections suffer from large uncertainties caused by the dependence on  $\mu_r$  and  $\mu_f$ , especially for final states with many particles
- including higher order (NLO and NNLO) QCD corrections reduces the scale dependence of cross sections

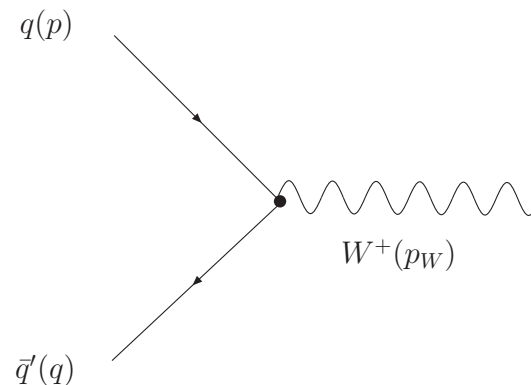
## Example: $W$ production at LO

- calculate cross section in center of mass (=laboratory) frame. Proton momenta:

$$P_1 = \frac{\sqrt{s}}{2}(1, 0, 0, 1); \quad P_2 = \frac{\sqrt{s}}{2}(1, 0, 0, -1)$$

$\sqrt{s} = 14$  TeV LHC center of mass energy

- Feynman diagram for LO  $W^+$  production



$$p = x_1 P_1; \quad q = x_2 P_2; \quad \hat{s} = x_1 x_2 s$$

- Matrix element (ignore Cabibbo mixing)

$$\mathcal{M} = -\frac{g}{2\sqrt{2}} \bar{v}(q) \gamma^\mu (1 - \gamma_5) u(p) \epsilon_\mu^W(p_W)$$

$u, v$ :  $q, \bar{q}'$  spinors;  $\epsilon_\mu^W$ :  $W$  polarization vector;  $g$ :  $SU(2)_L$  weak coupling constant

- after a little algebra (really, its not that bad!), summing over the  $W$  polarizations, and averaging over the spin and colors of the incoming (anti)quarks

$$|\overline{\mathcal{M}}|^2 = \frac{g^2}{12} \hat{s}$$

- now do phase space integration:

$$\hat{\sigma}_{LO} = \frac{1}{2\hat{s}} (2\pi) |\overline{\mathcal{M}}|^2 \frac{d^3 p_W}{2E_W} \delta^4(p + q - p_W) = \frac{\pi g^2}{12} \delta(\hat{s} - M_W^2)$$

- At LO, the  $W$  has no transverse momentum

## Automated LO Calculations

- For more complicated processes, such as  $pp \rightarrow W^+W^+jjjj$ , there may be **thousands** of Feynman diagrams which have to be calculated
- doing this by hand is, hmmm, not very efficient
- for complicated final states and to implement detector acceptance cuts and detector resolution effects use (adaptive) Monte Carlo integration
- LO cross sections are well suited for automatic evaluation:
  - ☞ figure out which Feynman diagrams contribute
  - ☞ translate diagrams into amplitudes
  - ☞ integrate, and write events to disk for further processing (*eg.* interface with shower Monte Carlo)
- my personal favorites: MadEvent (**Stelzer, Maltoni et al.**) and ALPGEN (**Mangano et al.**)



- ALPGEN (<http://mlm.home.cern.ch/mlm/alpgen/>):

- ☞ **advantage**: directly evaluates matrix element from path integral → program is very fast

- ☞ **dis-advantage**: processes fixed by authors; the user cannot add his/her own processes

- MadEvent (<http://madgraph.hep.uiuc.edu/>):

- ☞ very flexible: user can choose process

- ☞ BSM physics scenarios are built in

- ☞ program parallelizes + web interface

- both programs interface with shower MC's

# Recipe for NLO Calculations

- calculation of the NLO amplitude squared involves
  - ☞ virtual (loop diagrams) and real (tree level) contributions
  - ☞ removal of ultraviolet and infrared divergencies

- master formula:

$$\hat{\sigma}^{NLO} = \hat{\sigma}^{LO} + \frac{\alpha_s(\mu_r)}{4\pi} \delta\hat{\sigma}^{NLO}$$

- contributions to  $\delta\hat{\sigma}^{NLO}$ :
  - ☞ 1-loop virtual corrections
  - ☞ real gluon and quark emission  
new parton subprocesses may contribute
  - ☞ need PDF's and  $\alpha_s$  which include NLO corrections

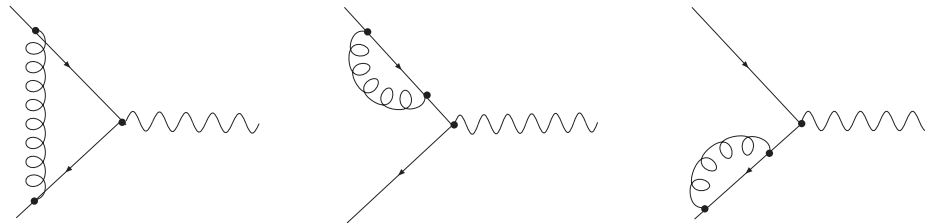
## $W$ Production at NLO

- The  $\mathcal{O}(\alpha_s)$  virtual corrections originate from the interference between the LO matrix element (ME),  $\mathcal{M}_{LO}$ , and the 1-loop virtual ME,  $\mathcal{M}_{NLO}^{virt}$ :

$$\hat{\sigma}^{virt} = \int (dPS) 2\text{Re}(\mathcal{M}_{LO}^* \mathcal{M}_{NLO}^{virt})$$

$dPS$ : phase space measure

- 1-loop diagrams contributing to NLO  $W$  production



- these diagrams lead to infinities

## Virtual Corrections

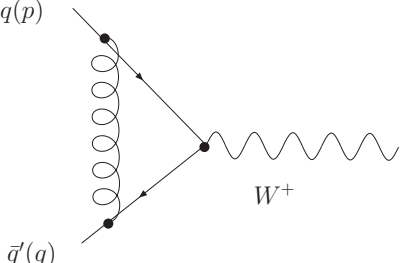
- have to evaluate traces of Dirac matrices
- reduce tensor integrals to scalar integrals (Passarino-Veltman)
- calculate scalar integrals using **dimensional regularization** to handle divergencies; integrate in  $d = 4 - 2\epsilon$  dimensions
- UV divergencies are poles in  $1/\epsilon_{UV}$  and are regularized in a given renormalization scheme
- infrared divergencies are poles in powers of  $1/\epsilon_{IR}$   
**IR poles cancel when virtual and real corrections, and NLO PDF's are combined**

confused?



lets do an example ...

- evaluate vertex correction to  $W$  production using dimensional regularization



$$\mathcal{M}^{virt} = \int \frac{d^d k}{(2\pi)^d} \frac{\bar{v}(q)\gamma_\mu(k-q)\gamma^\nu(k+q)\gamma^\mu u(p)}{k^2(k+p)^2(k-q)^2}$$

- for  $k^2 \rightarrow 0$ :

$$\frac{1}{k^2(k+p)^2(k-q)^2} \approx \frac{1}{k^2(k \cdot p)(k \cdot q)}$$

$k \rightarrow 0$ : infrared divergence

$(k \cdot p) \rightarrow 0, (k \cdot q) \rightarrow 0$ : collinear divergencies

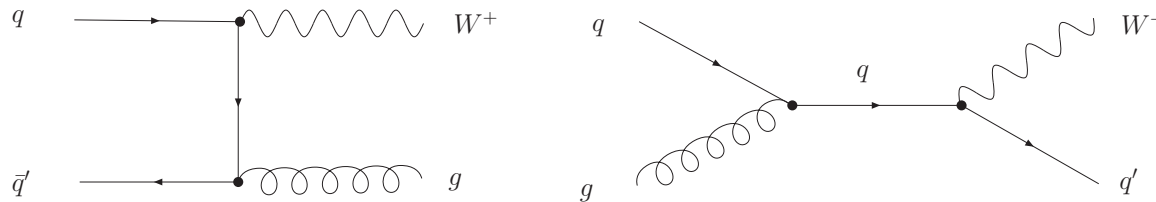
- after some algebra... ( $d = 4 - 2\epsilon$ )

$$\mathcal{M}^{virt} = \mathcal{M}^{LO} \frac{\alpha_s(\mu_r)}{4\pi} \frac{4}{3} \left( \frac{4\pi}{M_W^2} \right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{2}{3}\pi^2 \right)$$

the virtual contribution is proportional to the lowest order ME

## Real Corrections

- have to calculate  $q\bar{q}' \rightarrow W^+ g$  and  $qg \rightarrow W^+ q'$



- illustrate important points with

$$|\overline{\mathcal{M}(q\bar{q}' \rightarrow W^+ g)}|^2 = 2g^2 g_s^2 \frac{4}{3} (1-\epsilon) \left[ (1-\epsilon) \left( \frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} \right) - \frac{2M_W^2 \hat{s}}{\hat{t}\hat{u}} - 2\epsilon \right]$$

$\hat{t} = (p - p')^2 \sim p_T^2$ ,  $\hat{u} = (q - p')^2$ ,  $p'$  momentum of outgoing quark/gluon;  $p_T$  transverse momentum of  $W$

- cross section diverges for small  $\hat{t}$ , ie. small  $W$   $p_T$
- real corrections generate transverse momentum for  $W$

- integrating over phase space, the  $1/\hat{t}$  term manifests as  $1/\epsilon$  terms:

$$\hat{\sigma}(q\bar{q}' \rightarrow W^+g) = \frac{\pi\alpha\alpha_s}{6\sin^2\theta_W\hat{s}} \frac{4}{3} \left(\frac{4\pi}{M_W^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ \frac{2}{\epsilon^2}\delta(1-z) - \frac{2}{\epsilon} \frac{1+z^2}{(1-z)_+} + \text{finite terms} \right]$$

$$z = M_W^2/\hat{s}$$

- now compute  $\hat{\sigma}^{virt}$  in terms of  $z$ :

$$\hat{\sigma}^{virt} = \frac{\pi\alpha\alpha_s}{6\sin^2\theta_W\hat{s}} \frac{4}{3} \left(\frac{4\pi}{M_W^2}\right)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left[ -\frac{2}{\epsilon^2}\delta(1-z) - \frac{3}{\epsilon}\delta(1-z) + \text{finite terms} \right]$$

- the  $1/\epsilon^2$  terms cancel (general feature), leaving  $1/\epsilon$  terms from the collinear divergence
- these will cancel with collinear divergence from NLO corrections to PDF's  $\rightarrow$  cross section is finite



$$qg \rightarrow W^+ q'$$

- squared ME for  $qg \rightarrow W^+ q'$ :

$$|\overline{\mathcal{M}(qg \rightarrow W^+ q')}|^2 = -8g^2 g_s^2 \left( \frac{\hat{s}}{\hat{t}} + \frac{\hat{t}}{\hat{s}} + \frac{2M_W^2 \hat{u}}{\hat{s}\hat{t}} \right)$$

collinear singularity for  $\hat{t} \rightarrow 0$

- integrating over phase space, and translating singularities into  $1/\epsilon$  poles:

$$\hat{\sigma}(qg \rightarrow W^+ q') = \frac{g^2 g_s^2}{192\pi} [z^2 + (1-z)^2] \left\{ -\frac{1}{\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} + \log \left( \frac{M_W^2 (1-z)^2}{4\pi z} \right) \right\} + \text{finite terms}$$

- divergencies cancel with those from QCD corrections to PDF's

## Putting it all together

- In principle, one can now put all the pieces together, do all integrations analytically, and get a finite result for the total cross section that is not very interesting
- What do we want?
  - ☞ distributions ( $p_T$ , rapidity, inv. mass, etc.) at NLO
  - ☞ implement acceptance cuts and detector resolution effects
  - ☞ interface with shower MC if possible (watch out for double counting!)
- Solution: Monte Carlo integration, numerical cancellation of singularities

## NLO Monte Carlo Techniques for $2 \rightarrow n$ Processes

- Phase space slicing: map  $1/\epsilon$  terms into phase space cutoffs
  - ☞ partition phase space into soft, collinear and finite regions by introducing theoretical cutoffs  $\delta_s$  and  $\delta_c$
  - ☞ evaluate  $2 \rightarrow n + 1$  diagrams in soft gluon approximation; cancel IR  $1/\epsilon$  terms analytically; the rest is part of the  $2 \rightarrow n$  contribution
  - ☞ evaluate  $2 \rightarrow n + 1$  diagrams for small  $|\hat{t}|$  and  $|\hat{u}|$  in leading pole approximation, factorize collinear singularities into PDF's: the remainder is part of the  $2 \rightarrow n$  contribution
  - ☞ now evaluate  $2 \rightarrow n$  and  $2 \rightarrow n + 1$  numerically via MC: the total NLO cross section is independent of the cut-off parameters
- Subtraction method:
  - ☞ subtract a function with the same soft/collinear singularities as NLO cross section
  - ☞ do integral of the difference numerically

# Phase Space Slicing under the Magnifying Glass

- Write the  $2 \rightarrow n + 1$  part as

$$\hat{\sigma}(2 \rightarrow n + 1) = \hat{\sigma}^{soft} + \hat{\sigma}^{hard\ col.} + \hat{\sigma}^{hard\ non-col.}$$

- Soft singularities:

$$\hat{\sigma}^{soft} : E_g < \delta_s \frac{\sqrt{\hat{s}}}{2}$$

- Collinear singularities:

$$\hat{\sigma}^{hard\ col.} : E_g > \delta_s \frac{\sqrt{\hat{s}}}{2} \quad \text{and} \quad 1 - \cos \theta_g < \delta_c$$

- $\hat{\sigma}^{soft}$  and  $\hat{\sigma}^{hard\ col.}$  are calculated analytically

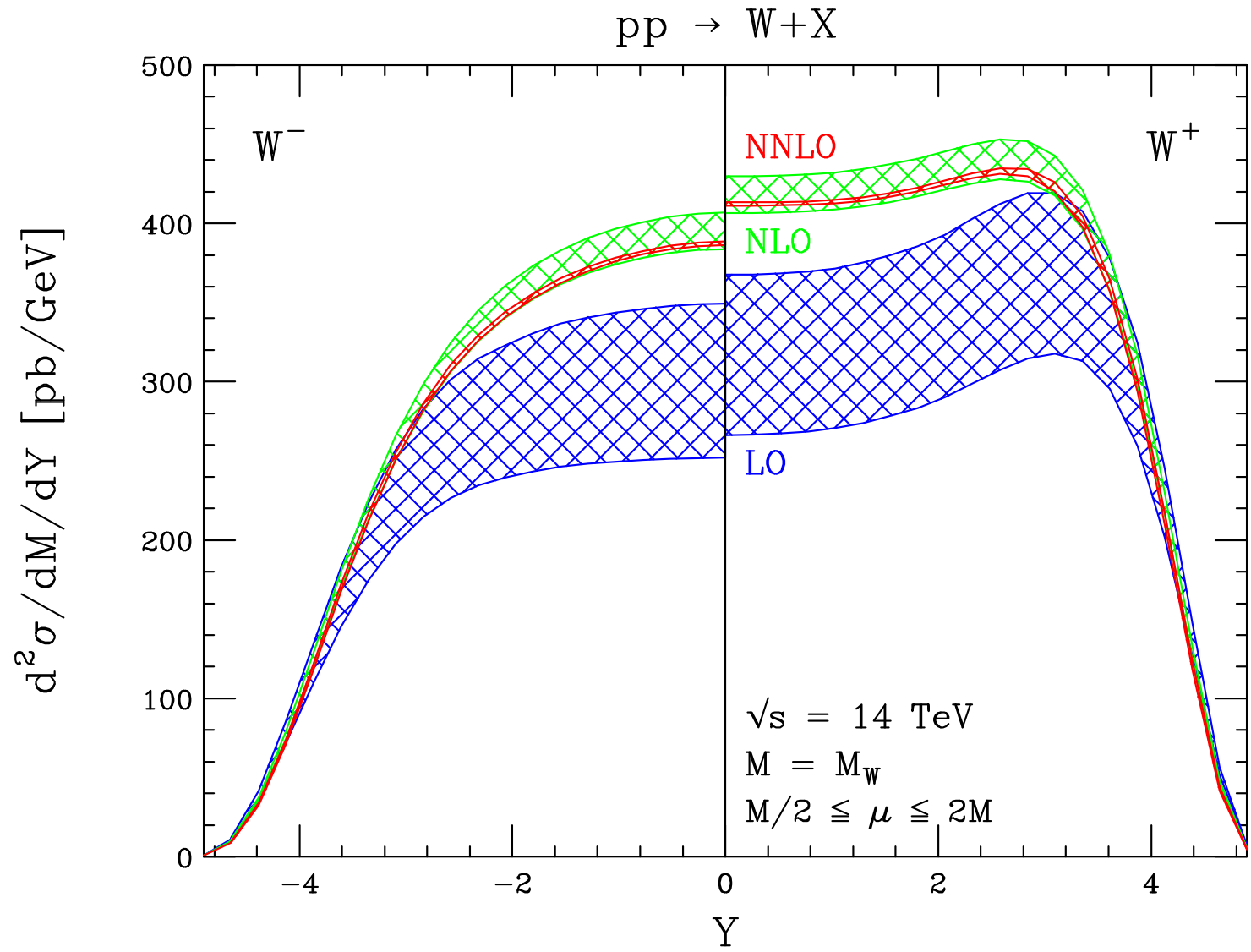
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## to summarize...

- calculate real and virtual contributions
- each is divergent, which is difficult for numerical evaluation
- make each separately finite by adding adding and/or subtracting pieces
- make use of collinear/pole and soft approximation

# Scale Dependence

- The dependence of the NLO cross section on the renormalization and factorization scales is in general smaller than that for LO
- One can show that, at a given order in perturbation theory, scale dependence is a higher order effect
- Scale dependence thus can be used as an indication of how big higher order effects are
- as an example, return to  $W$  production:
  - ➡  $W$  production known at NNLO in QCD (Anastasiou et al.)
  - ➡ need NNLO PDF's for that
  - ➡ scale uncertainty at LO:  $\approx 50\%$
  - ➡ scale uncertainty at NLO:  $\approx 10\%$
  - ➡ scale uncertainty at NNLO:  $\approx 1\%$



## Higher Order Calculations ...

- ...are hard: for multi-particle final states, matrix elements become very complicated
- multi-particle final states are relevant for LHC background processes
- much has been accomplished in recent years, but much remains to be done
- many new developments in the last few years, for example: [sector decomposition](#), [antenna subtraction](#), [\(semi\)numerical evaluation of loop integrals](#), etc.
- frontier now is 1-loop  $2 \rightarrow 4$  and 2-loop  $2 \rightarrow 2$  processes



- time ordered LHC shopping list

- ☞ need for  $10 - 30 \text{ fb}^{-1}$  (2008-2010):

- full NLO QCD corrections to  $pp \rightarrow t\bar{t} \rightarrow b\bar{b} + 4f$

- NLO QCD corrections to  $t\bar{t}\gamma, W/Z + \geq 3 \text{ jets}$  production

- NNLO QCD corrections to PDF's, 2-jet production

- ☞ need for  $300 \text{ fb}^{-1}$  (2012-2013):

- NLO QCD corrections to  $gg \rightarrow HH, t\bar{t}W, t\bar{t}Z$  production

- NLO EW corrections are needed for all hard scattering processes

- ☞ need for  $3000 \text{ fb}^{-1}$  ( $> 2015$ ):

- NLO QCD corrections to  $WWWjj, jj\gamma\gamma, Q\bar{Q}\gamma j$  production

- probably many more processes as time and physics knowledge base evolves

## Tools

- naturally, we would like to automatize higher order calculations
- Grace-1 loop is a tool which fully automatizes electroweak 1-loop corrections
- for QCD, such a tool does not exist (yet), but there are a host of semi-automatic programs: FeynArts, FeynCalc, FormCalc, LoopTools, etc
- big stumbling block: avoid numerical instabilities arising from vanishing Gram determinants
- but we want more...

## Nature is more complicated than NLO

- NLO ME calculations are not sufficient:
  - ☞ there is only one additional parton with respect to LO **need more**
  - ☞ in many processes, large logarithms from the soft, collinear and/or threshold regions appear **have to be resummed**
  - ☞ fixed order calculations do not include hadronization
- want NLO calculations merged into shower MC's such as PYTHIA, HERWIG and SHERPA:  
MC@NLO does that (**Frixione, Webber**)
- complications:
  - ☞ have to avoid double counting
  - ☞ NLO ME calculations usually have negative event weights (experimentalists hate that): POWHEG (**Nason et al.**) and VINCIA (**Giele, Kosower, Skands**) attempt to solve that problem