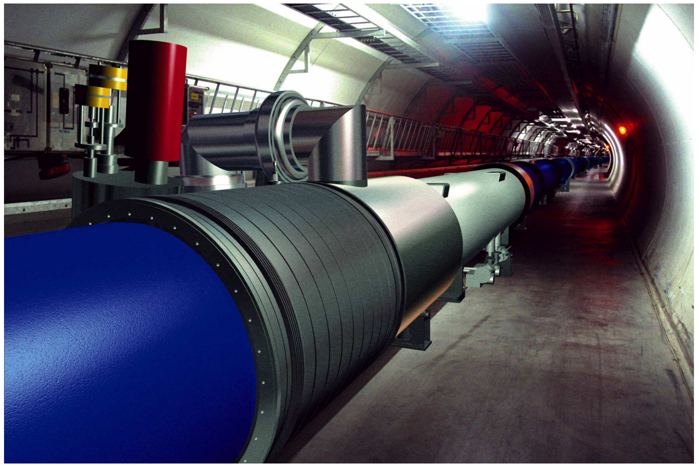
The Standard Model



1. Introduction to the Electroweak Theory
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Introduction to the Electroweak Theory

veni, vidi, vici

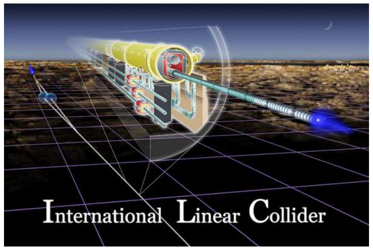
– Gaius Julius Caesar

- Review the SU(2)×U(1) part today
- QCD will be covered tomorrow
- References:
  - ☞ Chris Quigg, *Gauge Theories of the Strong, Weak and Electromagnetic Interactions*
  - ☞ Francis Halzen and Alan Martin, *Quarks and Leptons*

- Tevatron: $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV until 2009

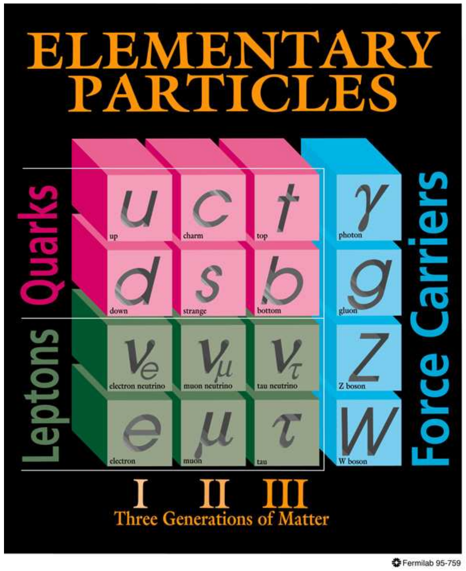
- LHC: pp collisions at $\sqrt{s} = 14$ TeV 2008 – 2014 (?), LHC upgrade (SLHC; 10$\times$ luminosity): 2015 (?) – ?

- ILC: (e^+e^- collisions at $\sqrt{s} = 500$ GeV) 2015 (?) – ?



Fermions

- the pattern of fermions appears to replicate itself 3 times



- masses range from 10^{-3} eV for neutrinos to 171 GeV for the top quark
- we do not understand why there are 3 "generations" of quarks and leptons why not more?
- we do not understand why the fermion masses span such a wide range (14 orders of magnitude)

...and Bosons

- The photon and gluon (QCD SU(3)$_C$ gauge boson) appear to be massless
- but the W and Z bosons are massive:

$$M_W = 80.398 \pm 0.025 \text{ GeV}$$

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

- strong, weak and electromagnetic interactions are described by local gauge theories
- local gauge invariance requires massless gauge bosons
- so, why are the W and Z masses non-zero?

QED

- QED: U(1) (so-called Abelian) local gauge theory with single spin 1 gauge field A_μ
- Lagrangian: ($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$)

$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu(\partial_\mu - ieA_\mu) - m\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

ψ: fermion field; e: electric charge m: fermion mass

- $\mathcal{L}$ is invariant under local gauge transformations (α is a function of x):

$$\psi \to \psi' = e^{i\alpha(x)}\psi; \qquad A_\mu \to A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha$$

- A mass term for A_μ

$$\mathcal{L}_{mass} = \frac{1}{2} M_A^2 A^\mu A_\mu$$

would destroy gauge invariance (ie. invariance of the Lagrangian under gauge transformations)

- for general gauge groups (SU(n) etc.): gauge bosons must be massless
- gauge invariance is the guiding principle
- to generate a mass for gauge bosons, the local gauge symmetry has to be broken such that renormalizability (ie. the theoretical consistency of the theory) is preserved
- this leads to spontaneous symmetry breaking

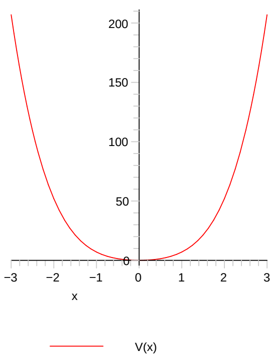
Spontaneous Symmetry Breaking:
The Abelian Higgs Model

- Start with a toy model: a single complex scalar field ϕ coupled to a U(1) gauge field
- Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 - V(\phi)$$

where $D_\mu = \partial_\mu - ieA_\mu$, and

$$V(\phi) = \mu^2|\phi|^2 + \lambda|\phi|^4$$

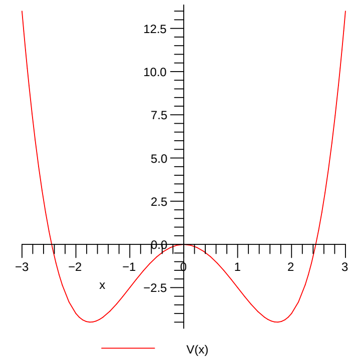


$\lambda > 0$

- if $\mu^2 > 0$:
  - ☞ $V(\phi)$ has a unique minimum at $\phi = 0$
  - ☞ $m_\phi = \mu$ and $M_A = 0$

- if $\mu^2 < 0$:
  - ☞ minimum of $V(\phi)$ occurs at

$$\langle\phi\rangle = \frac{v}{\sqrt{2}} = \sqrt{-\frac{\mu^2}{2\lambda}}$$



  - ☞ vacuum breaks U(1) symmetry
  - ☞ rewrite

$$\phi = \frac{1}{\sqrt{2}}\,\exp(i\frac{\chi}{v})(v+h)$$

χ and h are two real scalar fields and correspond to the two degrees of freedom of the complex ϕ field

  - ☞ the Lagrangian now becomes:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A^\mu A_\mu + \frac{1}{2}(\partial_\mu h\partial^\mu h + \partial_\mu\chi\partial^\mu\chi) + \mu^2h^2$$
$$-evA_\mu\partial^\mu\chi + \ h,\, \chi \text{ interaction terms}$$

- $M_A = ev$
- $m_h = \sqrt{-2\mu^2}$ (recall: $\mu^2 < 0$)
- $m_\chi = 0$ (χ is a so-called Goldstone Boson)
- how to interpret the $-evA_\mu\partial^\mu\chi$ term?
- remove by gauge transformation

$$A'_\mu = A_\mu - \frac{1}{ev}\partial_\mu\chi$$

- new Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}e^2v^2A'^\mu A'_\mu + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h)$$

In this (so-called unitary) gauge, $\mathcal{L}$ contains only physical particles

- χ is gone! It has been "eaten" by the gauge boson field

Spontaneous Symmetry Breaking: Pocket Guide

Spontaneous breaking of a U(1) gauge symmetry by a non-zero vacuum expectation value of a complex scalar field results in massive gauge boson and one real, massive scalar field. The second scalar field (the Goldstone boson) is eaten by the gauge boson field and is transformed into its longitudinal component

The Non-Abelian Higgs Mechanism

- now we generalize from a U(1) to a SU(n) gauge group
- scalar field is in n-dimensional fundamental representation of SU(n)
- gauge fields A^a_μ are in $n^2 - 1$ dimensional adjoint representation
- Lagrangian:

$$\begin{aligned} \mathcal{L} &= (D_\mu \Phi^\dagger)(D^\mu \Phi) - V(\Phi) \\ V(\Phi) &= \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \end{aligned}$$

- $\mathcal{L}$ is invariant under ($i, j = 1, \ldots, n$; $a = 1, \ldots, n^2 - 1$; ϵ^a are small parameters, g is the coupling constant, τ^a are the group generators [$\tau^a = \sigma^a/2$ for SU(2)])

$$\begin{aligned} \Phi_i &\rightarrow (1 - i\epsilon^a \tau^a)_{ij} \Phi_j \\ D_\mu \Phi &= (\partial_\mu - i g \tau^a A^a_\mu) \Phi \end{aligned}$$

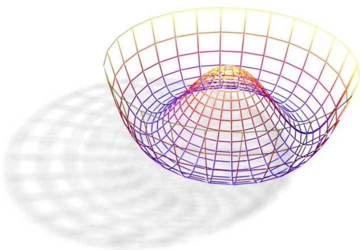
The SM Higgs Mechanism

- Now consider the SM SU(2)×U(1) gauge group
- introduce one complex Higgs SU(2) doublet

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- if $\mu^2 < 0$ in $V(\Phi)$, then spontaneous symmetry breaking occurs
- minimum of $V(\Phi)$ at

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$



- choice of minimum breaks SU(2)×U(1)
- why is $\mu^2 < 0$?

The SM W and Z Boson Masses

- now couple the Higgs field to the SU(2) and U(1) gauge bosons:

$$D_\mu = \partial_\mu - i\frac{g}{2}\sigma^a W_\mu^a - i\frac{g'}{2}B_\mu$$

W^a, $a = 1, 2, 3$: SU(2) gauge bosons;
B: U(1) gauge boson
g: SU(2) coupling constant,
g': U(1) coupling constant

- gauge boson mass terms (... after some algebra)

$$(D_\mu \Phi^\dagger)(D^\mu \Phi) = \cdots + \frac{v^2}{8}\left(g^2 (W_\mu^1)^2 + g^2 (W_\mu^2)^2 + (-gW_\mu^3 + g' B_\mu)^2\right) + \ldots$$

- mass eigenstates (A is the photon):

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 + W_\mu^2) \qquad M_W = \frac{gv}{2}$$

$$Z_\mu^0 = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \qquad M_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2}$$

$$A_\mu = \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}} \qquad M_A = 0$$

- Define weak mixing angle:

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

- W and Z masses are related then by

$$M_W = M_Z \cos\theta_W$$

- summary and remarks:
  - ☞ start with one complex scalar SU(2) doublet (4 degrees of freedom): this is the minimal case
  - ☞ Higgs vacuum expectation value breaks SU(2)$\times$U(1)$\rightarrow$U(1)$_{em}$
  - ☞ The $W^{\pm}$ and Z^0 bosons acquire mass
  - ☞ Three Goldstone bosons are absorbed into the W and Z
  - ☞ One massive scalar (Higgs) boson remains

Leptons in the SM

- W exchange is responsible for muon decay: $\mu \to \nu_\mu \bar{\nu}_e e$
- only left-handed quarks and leptons couple to the W boson
  Parity is maximally violated
- left-handed leptons form SU(2) doublets

$$\psi_L = \begin{pmatrix} \nu_L = \frac{1-\gamma_5}{2}\nu \\ e_L = \frac{1-\gamma_5}{2}e \end{pmatrix}$$

  same for other lepton generations
- right-handed charged leptons are SU(2) singlets
- there are no right-handed neutrinos in the SM
  - ☞ neutrinos are massless in SM
  - ☞ discovery of non-zero ν mass is evidence for physics beyond the SM

- Couple leptons to the SU(2)×U(1) gauge fields

$$\mathcal{L}_{lep} = \bar{\Psi}_L i\gamma^\mu \left(\partial_\mu - i\frac{g'}{2} Y B_\mu - i\frac{g}{2}\sigma^a W^a_\mu\right)\Psi_L + \bar{\ell}_R i\gamma^\mu \left(\partial_\mu - i\frac{g'}{2} Y B_\mu\right)\ell_R$$

ℓ: charged lepton; $Y = -1$: its weak hypercharge

- write in terms of the mass eigenstates $W^\pm$, Z and A:

$$\mathcal{L}_{lep} = kin.\, terms - \left[\frac{g}{\sqrt{2}}\bar{\ell}_L \gamma_\mu \nu_{\ell L} W^+_\mu + h.c.\right] - e\bar{\ell}\gamma^\mu \ell A_\mu - \frac{g}{2\cos\theta_W}\left(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\left[-1 + 2\sin^2\theta_W\right]\ell_L + 2\sin^2\theta_W \ell_R \gamma^\mu \ell_R\right) Z_\mu$$

…now add quarks

- left-handed quarks form SU(2) doublets

$$Q_L = \begin{pmatrix} u_{iL} \\ d_{iL} \end{pmatrix}$$

  $i = 1, 2, 3$: color index

- right-handed quarks are SU(2) singlets
- weak hypercharges:

$$Y = 2Q_{em} - I_3$$

  Q_{em}: electric charge

  I_3: 3rd component of weak isospin

- numerically: $Y_{Q_L} = 1/3$, $Y_{u_R} = 4/3$, $Y_{d_R} = 2/3$

quark couplings to W and Z

- couplings to W analogous to lepton case
- couplings to Z boson

$$\mathcal{L}_{Z\bar{q}q} = -\frac{g}{4\cos\theta_W}\bar{q}\gamma^\mu\left[L_q(1-\gamma_5)+R_q(1+\gamma_5)\right]qZ_\mu$$

with

$$\begin{aligned} L_q &= I_3 + 2Q_{em}\sin^2\theta_W \\ R_q &= 2Q_{em}\sin^2\theta_W \end{aligned}$$

- we are not done yet, quarks and leptons at this point are still massless!

Generating Fermion Masses

- generic fermion mass term in Lagrangian:

$$\mathcal{L}_{mass} = m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

forbidden by SU(2)×U(1) gauge invariance

- but we can add a coupling to the scalar field Φ which is SU(2)×U(1) invariant

$$\mathcal{L}_d = -\lambda_d \bar{Q}_L \Phi d_R + h.c.$$

d: down quark field λ_d: Yukawa coupling

- in terms of the physical Higgs field:

$$\mathcal{L}_d = -\lambda_d \frac{1}{\sqrt{2}} \left(\bar{u}_L \bar{d}_L\right) \begin{pmatrix} 0 \\ v+h \end{pmatrix} d_R + h.c.$$

- mass of down quark:

$$m_d = -\frac{\lambda_d v}{\sqrt{2}}$$

- Yukawa coupling for up quark:

$$\mathcal{L}_u = -\lambda_u \bar{Q}_L \Phi_c u_R + h.c.$$

where $\Phi_c = i\sigma_2 \Phi^*$

- for 3 generations:

$$\mathcal{L}_{yuk} = -\frac{v+h}{\sqrt{2}} \sum_{\alpha,\beta=1}^{3} \left(\lambda_u^{\alpha\beta} \bar{u}_L^\alpha u_R^\beta + \lambda_d^{\alpha\beta} \bar{d}_L^\alpha d_R^\beta \right) + h.c.$$

α, β run over the three fermion generations

Diagonalizing the Mass Matrix

- Unitary matrices U and V diagonalize mass matrix

$$u_L^\alpha = U_u^{\alpha\beta} u_L^{m\beta} \qquad d_L^\alpha = U_d^{\alpha\beta} d_L^{m\beta}$$
$$u_R^\alpha = V_u^{\alpha\beta} u_R^{m\beta} \qquad d_R^\alpha = V_d^{\alpha\beta} d_R^{m\beta}$$

  u^m, d^m: mass eigenstates

- weak and mass eigenstates are different!
- charged weak current (coupling to W):

$$J^{+\mu} = \frac{1}{\sqrt{2}} \bar{u}_L^\alpha \gamma^\mu d_L^\alpha = \frac{1}{\sqrt{2}} \bar{u}_L^{m\alpha} \gamma^\mu (U_u^\dagger V_d)_{\alpha\beta} d_L^{m\beta}$$

  $U_u^\dagger V_d$ is the quark mixing (or CKM) matrix

- the neutral weak current (coupling to Z) is still flavor diagonal

SM Parameters and Deficiencies

- three gauge couplings
- Higgs mass and coupling λ
- fermion masses and mixing angles
- The SM does not predict
  - ☞ why we have 3 generations of fermions
  - ☞ why the fermion masses span so many orders of magnitude
- Radiative corrections tend to drive the Higgs mass up to the Planck scale (so-called hierarchy problem)

The Standard Model is incomplete

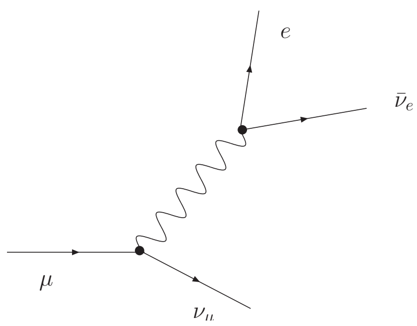
Muon Decay

- now we can calculate the muon decay rate. After a little algebra we find:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

where the Fermi constant, G_F, is given by:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$



- from the muon lifetime $\tau_\mu = 1/\Gamma_\mu \approx 2.2 \times 10^{-6}$ s:

$$G_F = 1.16637(1) \times 10^{-5}\ \text{GeV}^{-2}$$

- if we know the coupling constant g, we can predict the W mass...

Tree Level Calculations are not Sufficient

- we also know:

$$e = g \sin\theta_W; \qquad e = \sqrt{4\pi\alpha}; \qquad \sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$$M_Z = 91.1875 \pm 0.0021 \text{ GeV}$$

from LEP, and

$$\alpha = 1/137.0359895(61)$$

from $(g-2)_e$ and the Quantum Hall Effect

- thus:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{\pi\alpha}{2\left(1 - \frac{M_W^2}{M_Z^2}\right) M_W^2}$$

- solve for M_W:

$$M_W^2 = \sqrt{2}\pi \frac{\alpha}{G_F} \left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2} G_F M_Z^2}} \right)^{-1}$$

- use your pocket calculator to predict M_W:

$$M_W = 80.939 \text{ GeV}$$

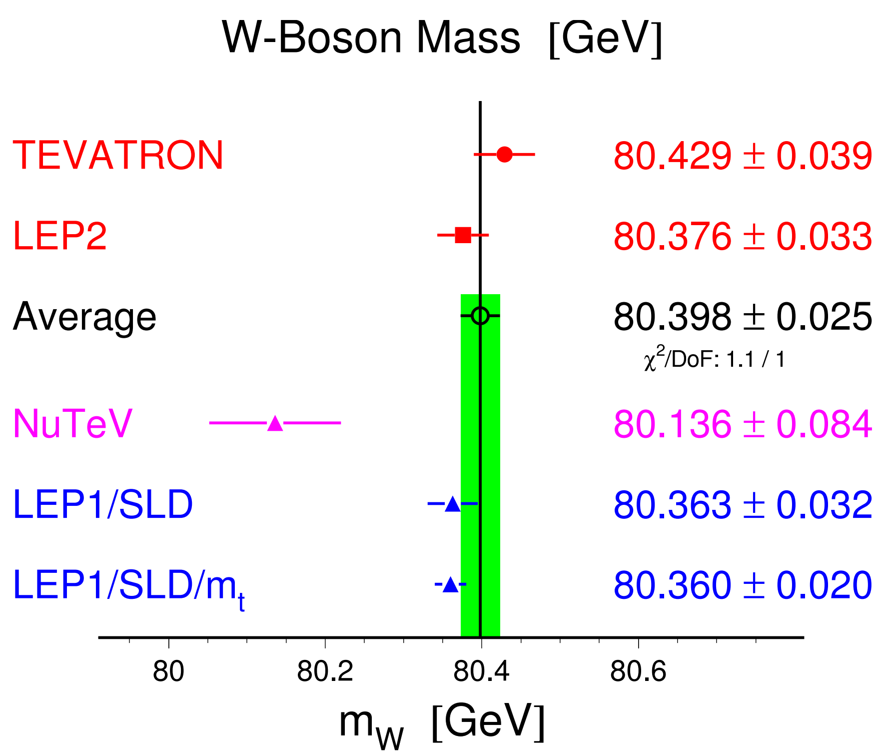
- experimental value:

$$M_W = 80.398 \pm 0.025 \text{ GeV}$$

- need to go beyond tree level calculations

World Average for W Mass

- Direct measurements from Tevatron and LEP2
- Indirect measurements from LEP/SLC and NuTeV
- direct and indirect measurements are in good agreement



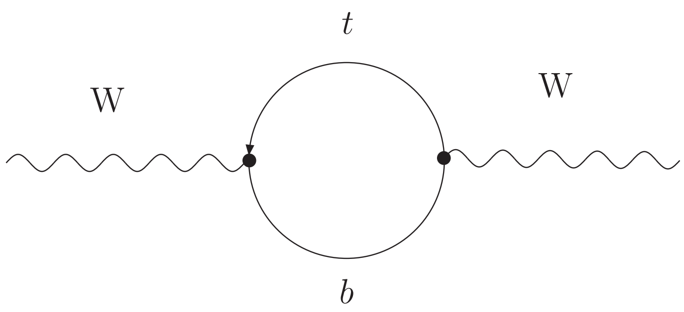
The World Beyond Tree Level

- At one-loop level, corrections can be represented by Δr:

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2\theta_W} \frac{1}{1-\Delta r}$$

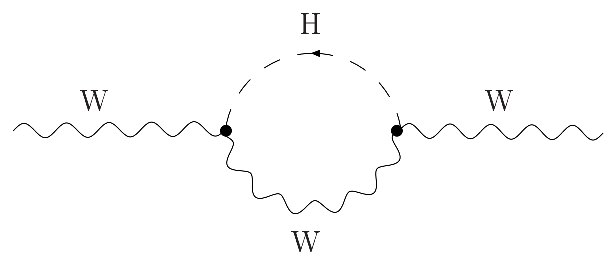
- Δr receives contributions from top quark loops which are quadratic in m_t

$$\Delta r_t = -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \cot^2\theta_W$$



- ...and logarithmic in the Higgs mass, M_H

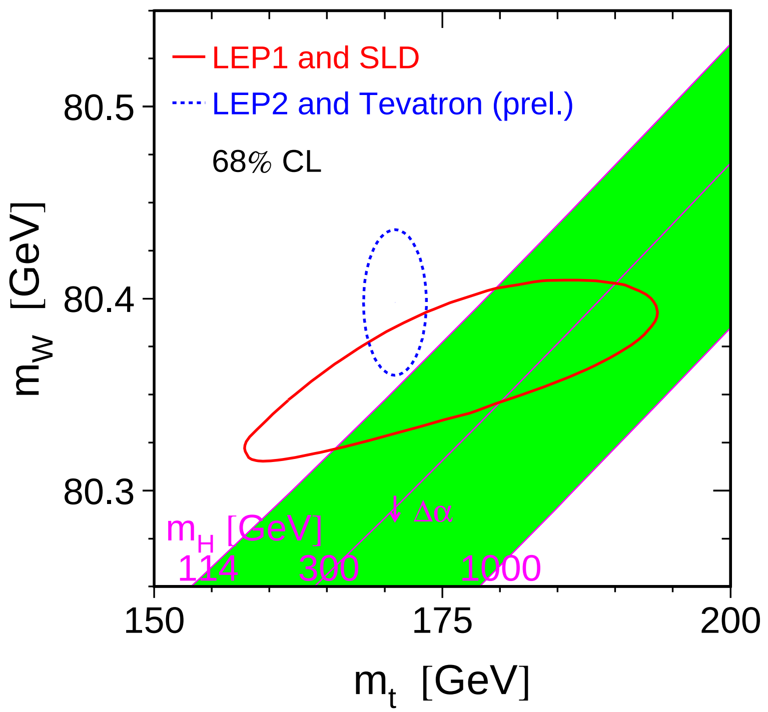
$$\Delta r_H = \frac{11 G_F M_W^2}{24\sqrt{2}\pi^2}\left(\log\left(\frac{M_H^2}{M_W^2}\right) - \frac{5}{6}\right)$$

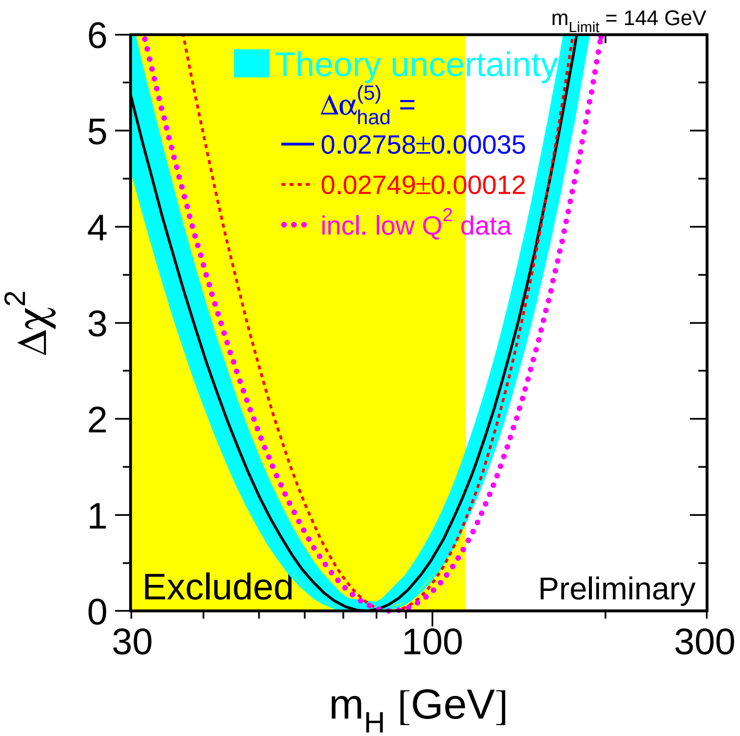


- precision measurements are much more sensitive to m_t than to M_H
- if M_W and m_t are measured, one can constrain M_H (indirect constraint)
- direct constraint from LEP2 ($e^+e^- \to ZH$):

$$M_H > 114.4 \text{ GeV}$$

Data Prefer a Light Higgs Boson





$M_H < 189$ GeV at 95% confidence level

Measuring the Weak Mixing Angle

- the "blueband" plot also takes into account other indirect constraints on M_H (in addition to the $M_W - m_t$ measurement), in particular the measurement of $\sin^2\theta_W$
- when going beyond tree level it is advantageous to define the weak mixing angle as

$$\sin^2\theta^f_{eff} = \frac{1}{4}\left(1 + Re\frac{v_f}{a_f}\right)$$

where v_f and a_f are the vector and axial vector couplings of the fermion f (including the so-called factorizable one-loop vertex corrections). These couplings are defined via the Zff vertex function

$$\Gamma(Zff) = i\bar{f}\gamma^\mu(v_f + a_f\gamma_5)fZ_\mu$$

- one can also write:

$$\sin^2\theta^f_{eff} = \left(1 - \frac{M_W^2}{M_Z^2}\right)(1 + \Delta\kappa(M_H))$$

where $\Delta\kappa(M_H)$ parametrizes the one-loop corrections

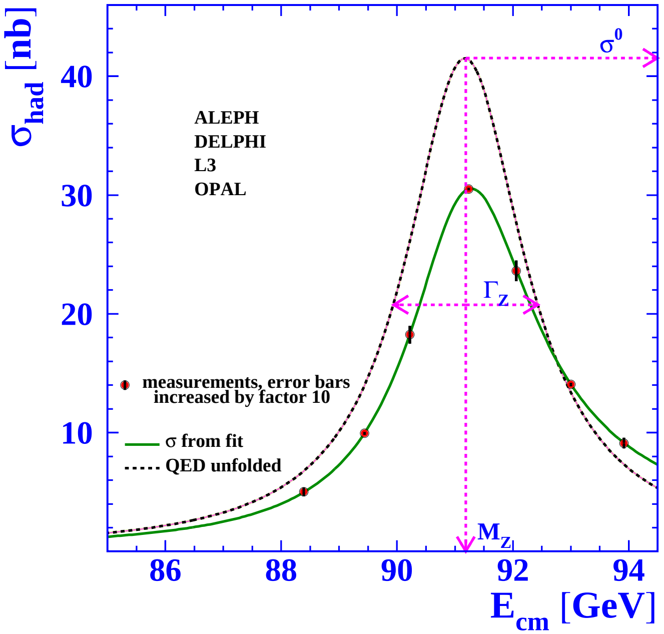
- $\Delta\kappa$ depends logarithmically on M_H
- experimentally for leptons ($f = \ell$)

$$\sin^2\theta^\ell_{eff} = 0.23140 \pm 0.00014$$

dominated by LEP asymmetry measurements at the Z pole

The SM under the magnifying glass

- The SM has been tested at the quantum level (one-loop and beyond) at LEP and SLC in the 1990's
- 2×10^7 Z boson events were analyzed at LEP and 5×10^5 at SLD with a polarized electron beam
- observables measured:
  - ☞ Z line-shape parameters: σ, M_Z, Γ_Z
  - ☞ Z branching ratios
  - ☞ asymmetries



Veni, Vidi, Vici

- Electroweak theory is precision theory
- The SM works at the 1% level. Strong constraints on any beyond the SM theory.

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ^0_{had} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21586
R_c	0.1721 ± 0.0030	0.1722
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.398 ± 0.025	80.374
Γ_W [GeV]	2.140 ± 0.060	2.091
m_t [GeV]	170.9 ± 1.8	171.3