Physics beyond the Standard Model

Basics and Phenomenology of Supersymmetry

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- **Problems of the SM: GUTs, SUSY, alternatives**
- **SUSY and the MSSM**
- **<u>The Higgs and SUSY particle spectrum in the MSSM</u>**
- **Higgs and sparticles at colliders**

1. The SUSY spectrum:

The basic Lagrangian, gives the currents and corresponding states:

we need to turn the current eigenstates into the mass eigenstates

- **Charginos:** **mixtures of the charged higgsinos and gauginos**

$$\tilde{W}^{\pm}, \tilde{h}^{\pm}_{2/1} \longrightarrow \chi^{\pm}_{1}, \chi^{\pm}_{2}$$

- **Neutralinos:** **mixtures of the neutral higgsinos and gauginos**

$$\tilde{B}, \tilde{W}^{0}, \tilde{h}^{0}_{2}, \tilde{h}^{0}_{2}, \longrightarrow \tilde{\chi}^{0}_{1}, \tilde{\chi}^{0}_{2}, \tilde{\chi}^{0}_{2}, \tilde{\chi}^{0}_{4}$$

- **Sfermions:** **mixing between LH and RH sfermions of same flavor**

$$\tilde{f}_{L}, \tilde{f}_{R} \longrightarrow \tilde{f}_{1}, \tilde{f}_{2}$$

- **Higgs bosons:** **2 doublets of complex fields $H_1, H_2 \equiv 4$ dof**

$\Rightarrow$ 3 degrees of freedom to generate M_{W^+}, M_{W^-}, M_Z

$\Rightarrow$ 5 degrees of freedom left: $M_h, M_H, M_A, M_{H^+}, M_{H^-}$

To determine the mass and the mixing angle of the physical states

find relevant mass matrices and diagonalize them.... include RC?

1. The SUSY spectrum: charginos

The general chargino mass matrix, in terms of M_2, μ and $\tan\beta$, is

$$\mathcal{M}_{\mathbf{C}} = \begin{bmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{bmatrix} \ , \ \mathbf{s}_\beta \equiv \sin\beta \ \mathbf{etc}$$

diagonalized by: $\mathrm{U}\mathcal{M}_{\mathbf{C}}\mathbf{V}^{-1} \to \mathbf{U} = \mathcal{O}_-, \mathbf{V} = \left\{ \begin{array}{ll} \mathcal{O}_+ & \text{if } \det\mathcal{M}_C > \\ \sigma_3\mathcal{O}_+ & \text{if } \det\mathcal{M}_C < \end{array} \right.$

(Pauli σ_3 to make the $\chi^\pm$ masses positive and $\mathcal{O}_\pm$ rotation matrices)

Simple analytical formulae for the masses $m^\pm_{\chi_{1,2}}$ and mixing angles.

For limiting cases, interpretation much simpler. $\mu \gg \mathbf{M_2}, \mathbf{M_W}$:

$$\mathbf{m}_{\chi_1^\pm} \simeq \mathbf{M_2} - \mathbf{M_W^2}\mu^{-2}\left(\mathbf{M_2} + \mu\mathbf{s_{2\beta}}\right)$$

$$\mathbf{m}_{\chi_2^\pm} \simeq |\mu| + \mathbf{M_W^2}\mu^{-2}\epsilon_\mu\left(\mathbf{M_2 s_{2\beta}} + \mu\right)$$

$$|\mu| \to \infty : \chi_1^\pm \ \mathbf{wino\ with\ m}_{\chi_1^\pm} \simeq \mathbf{M_2} ; \chi_2^\pm \ \mathbf{higgsino\ with\ m}_{\chi_2^\pm} = |\mu$$

In the opposite limit, $M_2 \gg |\mu|, M_Z$, the roles of $\chi_1^\pm, \chi_2^\pm$ are reversed.

1. The SUSY spectrum: neutralinos

For neutralinos, the 4x4 mass matrix depends on $\mu, \mathbf{M_2}, \tan\beta, \mathbf{M_1}$.

In the $(-\mathbf{i}\tilde{\mathbf{B}}, -\mathbf{i}\tilde{\mathbf{W}}_\mathbf{3}, \tilde{\mathbf{H}}^\mathbf{0}_\mathbf{1}, \tilde{\mathbf{H}}^\mathbf{0}_\mathbf{2})$ basis, it is given by

$$\mathcal{M}_\mathbf{N} = \begin{bmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{bmatrix}$$

Diagonalized by a single real matrix Z. Again for $|\mu| \gg \mathbf{M_{1,2}} \gg \mathbf{M_Z}$:

$$\mathbf{m}_{\chi^0_1} \simeq \mathbf{M_1} - \frac{\mathbf{M_Z^2}}{\mu^2}\left(\mathbf{M_1} + \mu \mathbf{s}_{2\beta}\right)\mathbf{s_W^2}$$

$$\mathbf{m}_{\chi^0_2} \simeq \mathbf{M_2} - \frac{\mathbf{M_Z^2}}{\mu^2}\left(\mathbf{M_2} + \mu \mathbf{s}_{2\beta}\right)\mathbf{c_W^2}$$

$$\mathbf{m}_{\chi^0_{3/4}} \simeq |\mu| + \frac{1}{2}\frac{\mathbf{M_Z^2}}{\mu^2}\epsilon_\mu(1 \mp \mathbf{s}_{2\beta})\left(\mu \pm \mathbf{M_2 s_W^2} \mp \mathbf{M_1 c_W^2}\right)$$

For $|\mu| \to \infty$, χ^0_1 is bino (M_1), χ^0_2 wino (M_2) and χ^0_3, χ^0_4 higgsinos (μ).

In the opposite limit, $M_1, M_2 \to \infty$, the roles are again reversed.

1. The SUSY spectrum: gluinos and RGEs

Finally, the gluino mass is identified with $\mathrm{M_3}$ at the tree–level

$$\mathbf{m_{\tilde{g}} = M_3}$$

In constrained models with boundary conditions at the high energy scale $\mathrm{M_U}$, the evolution of the gaugino masses given by RGEs

$$\frac{\mathrm{dM_i}}{\mathrm{d}\log(\mathbf{M_U/Q^2})} = -\frac{\mathbf{g_i^2 M_i}}{\mathbf{16\pi^2}}\mathbf{b_i}\,,\ \mathbf{b_1} = \frac{\mathbf{33}}{\mathbf{5}}\,,\ \mathbf{b_2 = 1}\,,\mathbf{b_3 = -3}$$

where in $\mathrm{b_i}$ all sparticles contribute to the evolution from Q to $\mathrm{M_U}$.
Equations are related to those of the gauge couplings $\alpha_\mathrm{i} = \mathrm{g_i^2}/(4\pi)$.
With inputs at scale $\mathrm{M_Z}$ and common value at $\mathrm{M_U} \sim 2 \times 10^{16}$ GeV, one has for gaugino mass parameters at the weak or SUSY scale $\mathrm{M_S}$:

$$\mathbf{M_3 : M_2 : M_1 \sim \alpha_3 : \alpha_2 : \alpha_1 \sim 6 : 2 : 1}$$

With norm.factor $\frac{5}{3}$ in α_1, we have $\mathrm{M_1} = \frac{5}{3}\tan^2\theta_\mathrm{W}\,\mathrm{M_2} \simeq \frac{1}{2}\mathrm{M_2}$.

$\mu \gg \mathrm{M_2} \Rightarrow \mathrm{m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim 2m_{\chi_1^0} \sim M_2},\ \mathrm{m_{\chi_3^0} \sim m_{\chi_4^0} \sim m_{\chi_2^\pm}} \sim \mu$.

$\mu \ll \mathrm{M_2} \Rightarrow \mathrm{m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim m_{\chi_1^0}} \sim \mu,\ \mathrm{m_{\chi_4^0} \sim 2m_{\chi_3^0} \sim m_{\chi_2^\pm} \sim M_2}$.

1. The SUSY spectrum: sfermions

Sfermion system described by $\tan\beta, \mu$ and 3 param.for each species: $m_{\tilde{f}_L}, m_{\tilde{f}_R}$ and A_f. For 3d generation, mixing $\propto m_f$ to be included.

$$\mathcal{M}^2_{\tilde{\mathbf{f}}} = \begin{pmatrix} m_f^2 + m_{LL}^2 & m_f X_f \\ m_f X_f & m_f^2 + m_{RR}^2 \end{pmatrix}$$

with the various entries given by

$$m_{LL}^2 = m_{\tilde{f}_L}^2 + (I_f^{3L} - Q_f s_W^2)\, M_Z^2\, c_{2\beta}$$
$$m_{RR}^2 = m_{\tilde{f}_R}^2 + Q_f s_W^2\, M_Z^2\, c_{2\beta}$$
$$X_f = A_f - \mu(\tan\beta)^{-2I_f^{3L}}$$

They are diagonalized by 2×2 rotation matrices of angle θ_f, which turn the current eigenstates $\tilde{f}_L, \tilde{f}_R$ into the mass eigenstates $\tilde{f}_1, \tilde{f}_2$.

$$\mathbf{m^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[m^2_{LL} + m^2_{RR} \mp \sqrt{(m^2_{LL} - m^2_{RR})^2 + 4m_f^2 X_f^2}\,\right]}$$

1. The SUSY spectrum: sfermions

• Note: mixing very strong in the stop sector, $X_t = A_t - \mu \cot\beta$ and generates mass splitting between $\tilde{t}_1, \tilde{t}_2$, leading to a light $\tilde{t}_1$ state

• Mixing in sbottom/stau sectors also for large $X_{b,\tau} = A_{b,\tau} - \mu \tan\beta$. the $\tilde{\tau}_1$ state is in general the lightest sfermion at high $\tan\beta$ values!

In cMSSM with universal m_0 and $m_{1/2}$ at M_{GUT}, the RGEs for scalar masses are simple if Yukawas are small ($c(\tilde{f})$ depend on I, Y, color):

$$m^2_{\tilde{f}_{L,R}} = m_0^2 + \sum_{i=1}^{3} F_i(f) m^2_{1/2} \,, \; F_i = \frac{c_i(f)}{b_i}\left[1 - \left(1 - \frac{\alpha_U}{4\pi} b_i \log\frac{Q^2}{M_U^2}\right)^{-2}\right]$$

$$\tilde{L} : \begin{pmatrix} \frac{3}{10} \\ \frac{3}{2} \\ 0 \end{pmatrix}, \tilde{l}_R : \begin{pmatrix} \frac{6}{5} \\ 0 \\ 0 \end{pmatrix}, \tilde{Q} : \begin{pmatrix} \frac{1}{30} \\ \frac{3}{2} \\ \frac{8}{3} \end{pmatrix}, \tilde{u}_R : \begin{pmatrix} \frac{8}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}, \tilde{d}_R : \begin{pmatrix} \frac{2}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}$$

With inputs at M_Z, $\alpha_U \simeq 0.041$ and $M_U \sim 2 \times 10^{16}$ GeV, one has

$$m^2_{\tilde{q}_i} \sim m_0^2 + 6 m^2_{1/2} \,, \; m^2_{\tilde{\ell}_L} \sim m_0^2 + 0.52 m^2_{1/2} \,, \; m^2_{\tilde{e}_R} \sim m_0^2 + 0.15 m$$

1. The SUSY spectrum: sfermions

For 3d generation squarks, Yukawa couplings to be included!!

Approximate RGE for top squarks [for small $\tan\beta$ values]:

$$\mathbf{m^2_{\tilde{t}_L} = m^2_{\tilde{b}_L} \sim m^2_0 + 6m^2_{1/2} - \frac{1}{3}X_t}$$
$$\mathbf{m^2_{\tilde{t}_R} = m^2_{\tilde{b}_L} \sim m^2_0 + 6m^2_{1/2} - \frac{2}{3}X_t}$$

with the trilinear coupling given by $\mathbf{X_t \sim 1.3m^2_0 + 3m^2_{1/2}}$

$\Rightarrow$ **In contrast to the first two generation sfermions, one has generically a sizable splitting between $\mathbf{m^2_{\tilde{t}_L}}$ and $\mathbf{m^2_{\tilde{t}_R}}$ at the weak scale, due to the running of the large top Yukawa coupling.**

$\Rightarrow$ **Justifies the choice of different soft SUSY–breaking scalar masses and trilinear couplings for the third generation and the first/second generation sfermions [and for sleptons and squarks].**

- **Recal that already from mixing, $\mathbf{m^2_{\tilde{t}_1}}$ lighter than alls squarks**

$\Rightarrow$ **Special status for the top squark....**

2. The Higgs spectrum: scalar potential

In MSSM with two Higgs doublets $H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}$ and $H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$.

The terms contributing to scalar potential V come from 3 sources:

- D terms, quartic S interactions: $V_D = \frac{1}{2} \sum_a \left(\sum_i g_a S_i^* T^a S_i \right)^2$
- F terms of Superpotential: $V_F = \sum_i \left| \frac{\partial W(z_i)}{\partial z_i} \right|^2 \to \sum_i \left| \frac{\partial W(\phi_j)}{\partial \phi_i} \right|^2$
- Soft terms: $V_{\rm soft} = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + B\mu (H_2.H_1 + {\rm h.c.})$

Adding all $\Rightarrow$ scalar potential involving the Higgs bosons:

$$V_H = \bar{m}_1^2 |H_1|^2 + \bar{m}_2^2 |H_2|^2 - \bar{m}_3^2 \epsilon_{ij} (H_1^i H_2^j + {\rm h.c.})$$
$$+ \frac{g_2^2 + g_1^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_2^2 |H_1^* H_2|^2$$

with $\overline{m}_1^2 = |\mu|^2 + m_1^2,\ \overline{m}_2^2 = |\mu|^2 + m_2^2,\ \overline{m}_3^2 = B\mu$

2. The Higgs spectrum: scalar potential

- Development in terms of components $H_1 = (H_1^0, H_1^-)$, $H_2 = (H_2^+, H_2^0$

$$V_H = \overline{m}_1^2(|H_0^1|^2 + |H_1^+|^2) + \overline{m}_2^2(|H_2^0|^2 + |H_2^-|^2) - \overline{m}_3^2(H_1^+ H_2^- - H_1^0 H_2^0 +$$

$$+\frac{g_2^2+g_1^2}{8}(|H_1^0|^2 + |H_1^+|^2 - |H_2^0|^2 - |H_2^-|^2)^2 + \frac{g_2^2}{2}|H_1^{+*} H_1^0 + H_2^{0*} H_2^-|^2$$

- Now require that the minimum of V_H breaks $G_{\rm SM} \to U(1)_{\rm QED}$.

$$\langle 0 | \mathrm{Re}(H_1^0) | 0 \rangle = v_1 \;\; , \;\; \langle 0 | \mathrm{Re}(H_2^0) | 0 \rangle = v_2 \;\; , \;\; \tan\beta = v_2/v_1$$

So at $V_H^{\rm min}$ we have $\langle H_1^+ \rangle$ =0 and at $\frac{\partial V}{\partial H_1^+} = 0$ we have $\langle H_2^- \rangle$=0.

This is good for QED, and we can ignore the fields H_1^+, H_2^- to simplify.

The relevant part of the scalar potential is then simply given by:

$$V_H = \overline{m}_1^2 |H_1^0|^2 + \overline{m}_2^2 |H_2^0|^2 + \overline{m}_3^2 (H_1^0 H_2^0 + \mathrm{hc}) + \frac{g_2^2 + g_1^2}{8}(|H_1^0|^2 - |H_2^0|^2)^2$$

2. The Higgs spectrum: scalar potential

Some remarks on this scalar potential:

$$V_H = \overline{m}_1^2|H_1^0|^2 + \overline{m}_2^2|H_2^0|^2 + \overline{m}_3^2(H_1^0 H_2^0 + \mathrm{hc}) + \frac{M_Z^2}{4v^2}(|H_1^0|^2 - |H_2^0|^2)^2$$

- **Quartic couplings fixed in terms of the gauge couplings, only 3 free parameters: $\overline{m}_1^2, \overline{m}_2^2, \overline{m}_3^2$ (6 para and a phase in a general 2HDM).**
- **$m_{1,2}^2 + |\mu|^2$ real, only $B\mu$ can be complex. But any phase in $B\mu$ can be absorbed in phases of $H_1, H_2 \Rightarrow V_H$ (MSSM) conserves CP.**
- **If $B\mu$ is zero, all other terms are positive and thus $V_H = 0$ only if $\langle H_1^0 \rangle = \langle H_2^0 \rangle = 0$. To have SSB (without CCB), we need $\overline{m}_{1,2,3} \neq 0$**

$\Rightarrow$ **Connection of gauge symmetry breaking and SUSY breaking!!**

More precisely: in SM, SSB takes place with ad hoc choice $\mu^2 < 0$.
In MSSM, $m_{H_i}^2 > 0$ at M_{GUT} but $t/\tilde{t}$ in RGE make $m_{H_i}^2 < 0$ at M_Z: radiative breaking of the electroweak symmetry (i.e. through RC).

$\Rightarrow$ **Symmetry breaking more natural and elegant than in SM.**

2. The Higgs spectrum: Higgs masses

To obtain the physical Higgs fields and their masses from potential $\mathrm{V_H}$, develop $\mathrm{H_1 = (H_1^0, H_1^-)}$ and $\mathrm{H_2 = (H_2^+, H_2^0)}$ into real (CP-even+charged Higgses) and imaginary (CP-odd Higgs+Goldstone bosons) parts and diagonalize the 2×2 mass matrices:

$$\mathcal{M}_{\mathbf{ij}}^{\mathbf{2}} = \frac{1}{2}\partial^2 \mathbf{V_H}/\partial \mathbf{H_i} \partial \mathbf{H_j}|_{\langle \mathrm{Re}(\mathbf{H^0_{1,2}})\rangle = \mathbf{v_{1,2}}, \langle \mathrm{Im}(\mathbf{H^0_{1,2}})\rangle = 0, \langle \mathbf{H^\pm_{1,2}}\rangle = 0}$$

To obtain masses $\mathrm{M_1, M_2}$ and mixing angle θ, two useful relations:

$$\mathrm{Tr}(\mathcal{M}^2) = \mathbf{M_1^2 + M_2^2} \ , \ \ \mathrm{Det}(\mathcal{M}^2) = \mathbf{M_1^2 M_2^2}$$

$$\sin 2\theta = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}} \ , \ \cos 2\theta = \frac{\mathcal{M}_{11}-\mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}}$$

First note if you perform the first derivative of the scalar potential V_H:

$$\mathbf{V_H = \overline{m}_1^2 |H_1^0|^2 + \overline{m}_2^2 |H_2^0|^2 + \overline{m}_3^2 (H_1^0 H_2^0 + \mathrm{hc}) + \frac{M_Z^2}{4v^2} (|H_1^0|^2 - |H_2^0|^2)^2}$$

you have, at the minimum, $\partial \mathrm{V_H}/\partial \mathrm{H_{1,2}} = 0$, leading to the relations:

$$\mathbf{\bar{m}_1^2} = -\mathbf{\bar{m}_3^2} \tan\beta - \frac{1}{2}\mathbf{M_Z^2} \cos(2\beta) \ , \ \mathbf{\bar{m}_2^2} = -\mathbf{\bar{m}_3^2} \cot\beta + \frac{1}{2}\mathbf{M_Z^2} \cos(2\beta)$$

2. The Higgs spectrum: Higgs masses

The second derivative of V_H give you the relevant mass matrices:

$$\mathcal{M}_{\mathbf{R}}^{\mathbf{2}} = \left[\begin{array}{cc} -\bar{m}_3^2 \tan\beta + M_Z^2 \cos^2\beta & \bar{m}_3^2 - M_Z^2 \sin\beta\cos\beta \\ \bar{m}_3^2 M_Z^2 \sin\beta\cos\beta & -\bar{m}_3^2 \mathrm{cot}\beta + M_Z^2 \sin^2\beta \end{array} \right]$$

$$\mathcal{M}_{\mathbf{I}}^{\mathbf{2}} = \left[\begin{array}{cc} -\bar{m}_3^2 \tan\beta & \bar{m}_3^2 \\ \bar{m}_3^2 & -\bar{m}_3^2 \mathrm{cot}\beta \end{array} \right]$$

For the CP–odd case, since $\mathrm{Det}\mathcal{M}_{\mathbf{I}}^{\mathbf{2}} = \mathbf{0}$, one eigenvalue is zero (the Goldstone) and the other corresponds to the CP-odd Higgs (A) mass:

$$\mathbf{M_A^2} = -\mathbf{\bar{m}_3^2}(\tan\beta + \mathrm{cot}\beta) = -\mathbf{2\bar{m}_3^2}/\sin\mathbf{2}\beta$$

The mixing angle θ is, in fact, just the angle β:

$$\left(\begin{array}{c} G^0 \\ A \end{array} \right) = \left(\begin{array}{cc} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{array} \right) \left(\begin{array}{c} \mathrm{Im}(H_1^0) \\ \mathrm{Im}(H_2^0) \end{array} \right)$$

2. The Higgs spectrum: Higgs masses

In the case of the CP–even Higgs bosons, determinant and trace give:

$$\mathrm{Det}\mathcal{M}_{\mathbf{R}}^{\mathbf{2}} = \mathbf{M_A^2 M_Z^2 c_{2\beta}^2} \equiv \mathbf{M_h^2 M_H^2}$$

$$\mathrm{Tr}\mathcal{M}_{\mathbf{R}}^{\mathbf{2}} = \mathbf{M_A^2} + \mathbf{M_Z^2} \equiv \mathbf{M_h^2} + \mathbf{M_H^2}$$

To obtain the CP–even Higgs masses, solve the 2d order equation:

$$\mathbf{M_h^2(M_A^2+M_Z^2-M_h^2) = M_A^2 M_Z^2 c_{2\beta}^2} \Rightarrow \mathbf{M_h^4 - M_h^2(M_A^2+M_Z^2) + M_A^2 M_Z^2 c_{2\beta}^2}$$

The two solutions are then (h is the lightest CP-even Higgs boson):

$$\mathbf{M_{h,H}^2} = \frac{\mathbf{1}}{\mathbf{2}}\left[\mathbf{M_A^2} + \mathbf{M_Z^2} \mp \sqrt{(\mathbf{M_A^2}+\mathbf{M_Z^2})^{\mathbf{2}} - \mathbf{4M_A^2 M_Z^2}\cos^{\mathbf{2}}\mathbf{2}\beta}\right]$$

The mixing angle α which rotates the fields is ($-\frac{\pi}{2} \leq \alpha \leq 0$)

$$\tan\mathbf{2}\alpha = \frac{\mathbf{2}\mathcal{M}_{\mathbf{12}}}{\mathcal{M}_{\mathbf{11}}-\mathcal{M}_{\mathbf{22}}} = \frac{-(\mathbf{M_A^2}+\mathbf{M_Z^2})\sin\mathbf{2}\beta}{(\mathbf{M_Z^2}-\mathbf{M_A^2})\cos\mathbf{2}\beta} = \tan\mathbf{2}\beta\,\frac{\mathbf{M_A^2}+\mathbf{M_Z^2}}{\mathbf{M_A^2}-\mathbf{M_Z^2}}$$

In the case of the charged Higgs bosons, one obtains similarly to A:

$$\mathbf{M_{H^\pm}^2} = \mathbf{M_A^2} + \mathbf{M_W^2}$$

and the mixing angle θ is, also as for A, just the angle β.

2. The Higgs spectrum: Higgs masses

We have an important constraint on the lightest MSSM h boson mass:

$$\mathbf{M_h \leq \min(M_A, M_Z) \cdot |\cos 2\beta| \leq M_Z}$$

besides some other (also important) relations for H,A and $\mathrm{H^\pm}$:

$$\mathbf{M_H > \max(M_A, M_Z) \ \text{and} \ M_{H^\pm} > M_W}$$

If we send M_A to infinity, we will have for Higgs masses and α:

$$\mathbf{M_h \sim M_Z |\cos 2\beta| \ , \ M_H \sim M_{H^\pm} \sim M_A \ , \qquad \alpha \sim \frac{\pi}{2} - \beta}$$

This is the decoupling regime: all Higgses are heavy except for h.

The h boson is lighter than $\mathrm{M_Z}$ and should have been seen at LEP2

(we have $\sqrt{\mathrm{s}}_{\mathrm{LEP2}} \sim \mathbf{200\ GeV} \ > \mathbf{M_h + M_Z} \sim \mathbf{180}$ GeV).

So what happened in this case? Maybe the MSSM is already ruled out?

No! This relation holds only at first order (tree–level) and there are

strong couplings involved, in particular the htt and $h\tilde{t}\tilde{t}$ couplings.

$\Rightarrow$ Calculation of radiative corrections to $\mathrm{M_h}$ necessary.

2. The Higgs spectrum: Higgs masses

Radiative corrections very important in the MSSM Higgs sector!

A large activity for the RC calculation in the last 15 years (ask SH!).

- **Dominant corrections are due to top (s)quark at one-loop level**

$$\mathbf{\Delta M_h^2 = \frac{3g^2}{2\pi^2}\frac{m_t^4}{M_W^2}\log\frac{m_{\tilde{t}}^2}{m_t^2}}$$

It depends on $\mathbf{m_t^4}$ and $\log(\mathbf{m_{\tilde{t}}^2/m_t^2})$, and is large: $\underline{\mathbf{M_h^{max}\to M_Z{+}40}}$ GeV!

This explains why the h boson has not been observed at LEP2.

- **The full one–loop corrections have been calculated:**

– the parameters μ, A_t and A_b appear at the subleading level.

– the h boson mass is maximal (minimal) for $A_t \sim 2M_{\tilde{Q}}(0)$.

- **Approximate calculation for the dominant two–loop radiative corrections (in the effective potential approach; see SH again):**

– dominant QCD RC large but absorbed by $m_t|^{\rm pole} \to m_t|^{\overline{\rm MS}}$.

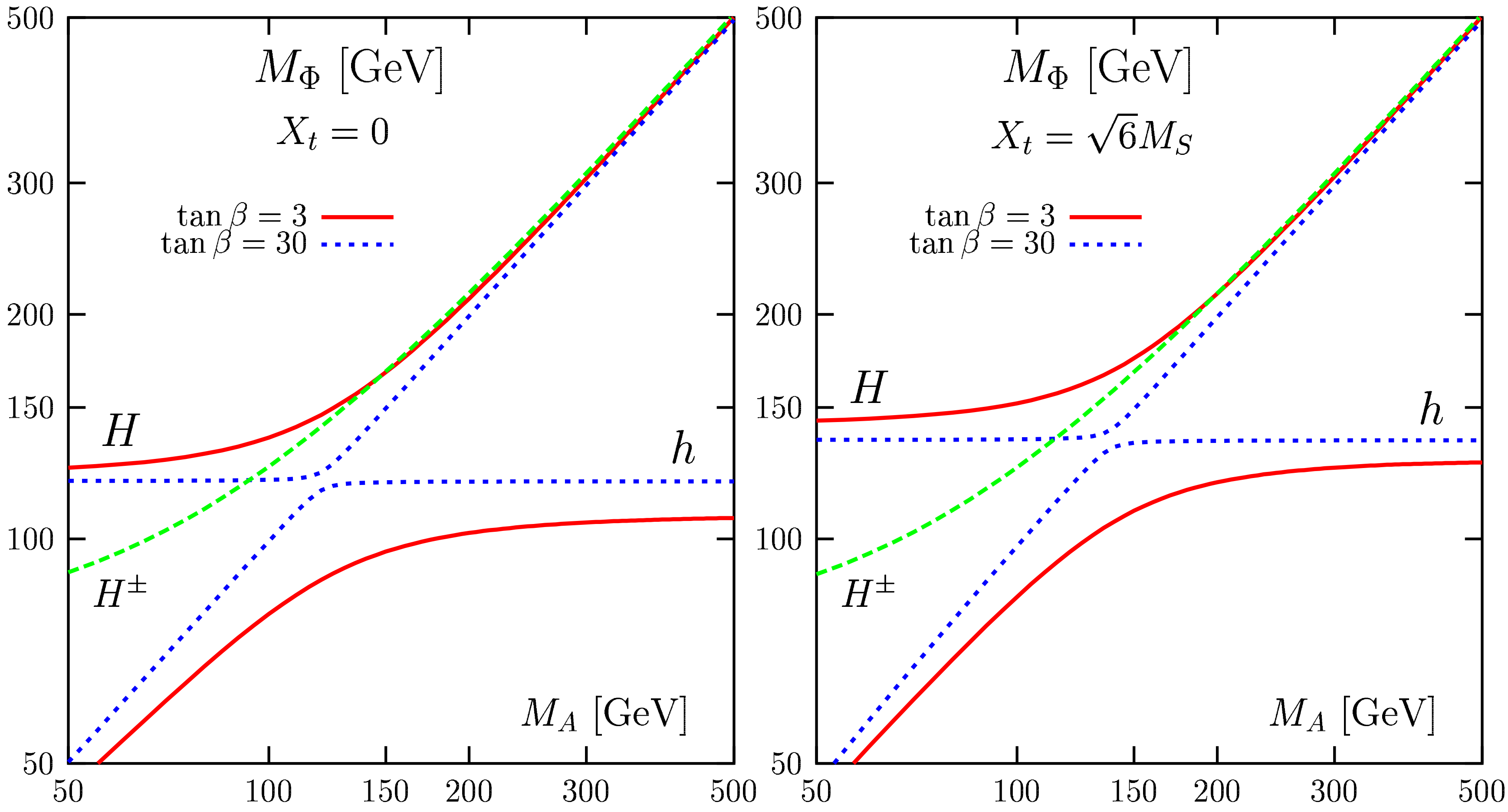
– Yukawa corrections rather small in the limit $M_h = 0$.

2. The Higgs spectrum: Higgs masses

- **Using full 1–loop and the 2–loop RC in effective potential approach:**

– $\mathcal{O}(\alpha_t \alpha_S)$: **including squark mixing and gluino loops.**

– $\mathcal{O}(\alpha_t^2)$: **including mixing and** $\mathcal{O}(\alpha_b \alpha_S), \mathcal{O}(\alpha_\tau \alpha_S)$.



3. Spectrum and constraints

Determination of spectrum:

– RGEs (two loops, numerics)

– EWSB and $V_{\rm soft}$ (iterations)

– Masses, couplings, RC

Sophisticated RGE programs:

– example of SuSpect

(Kneur, Moultaka, AD)

– other programs also exist:

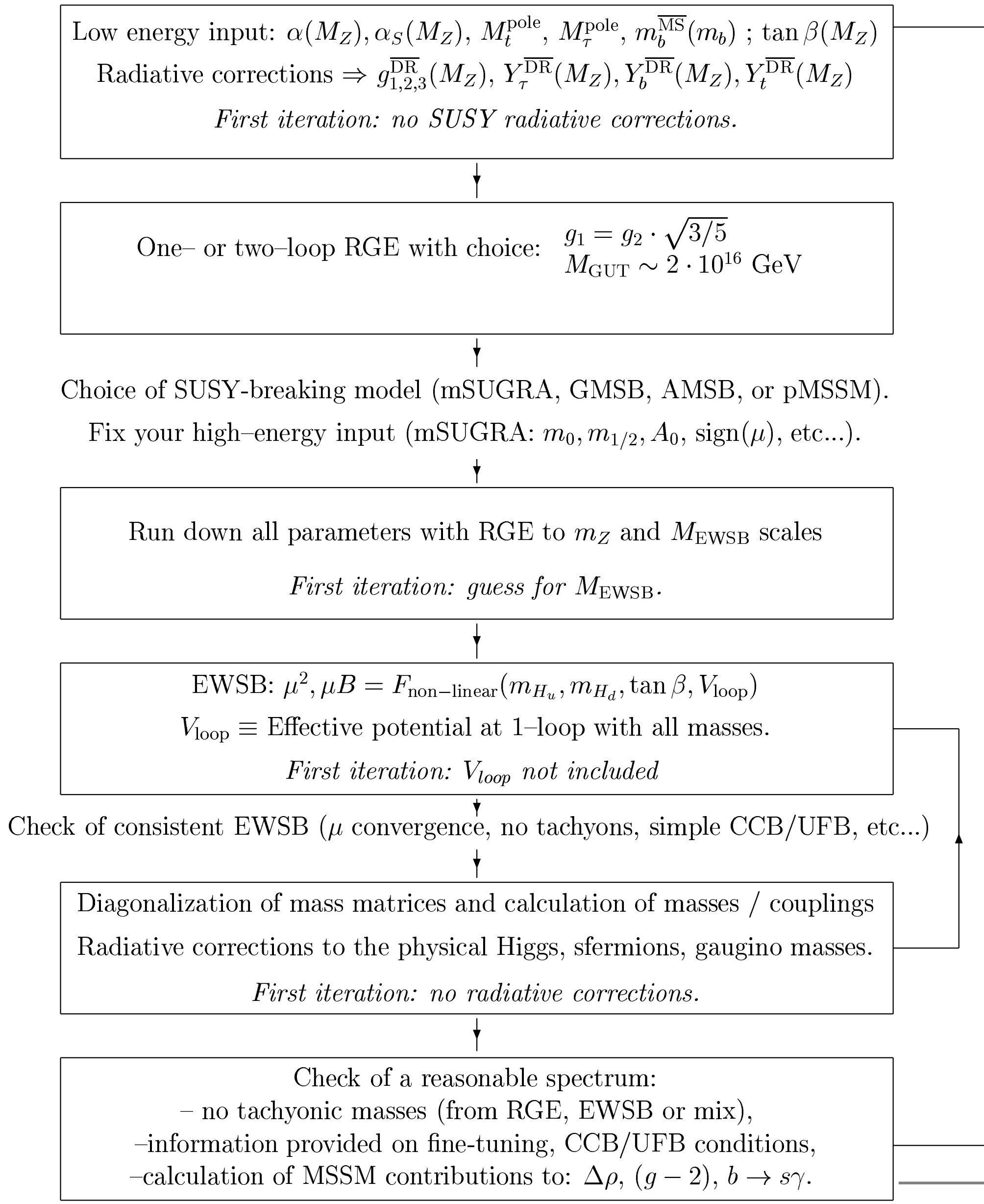
(Isajet, SoftSUSY, Spheno, ...)

Viable parameter space:

– choose inputs, param. scan

– impose known constraints

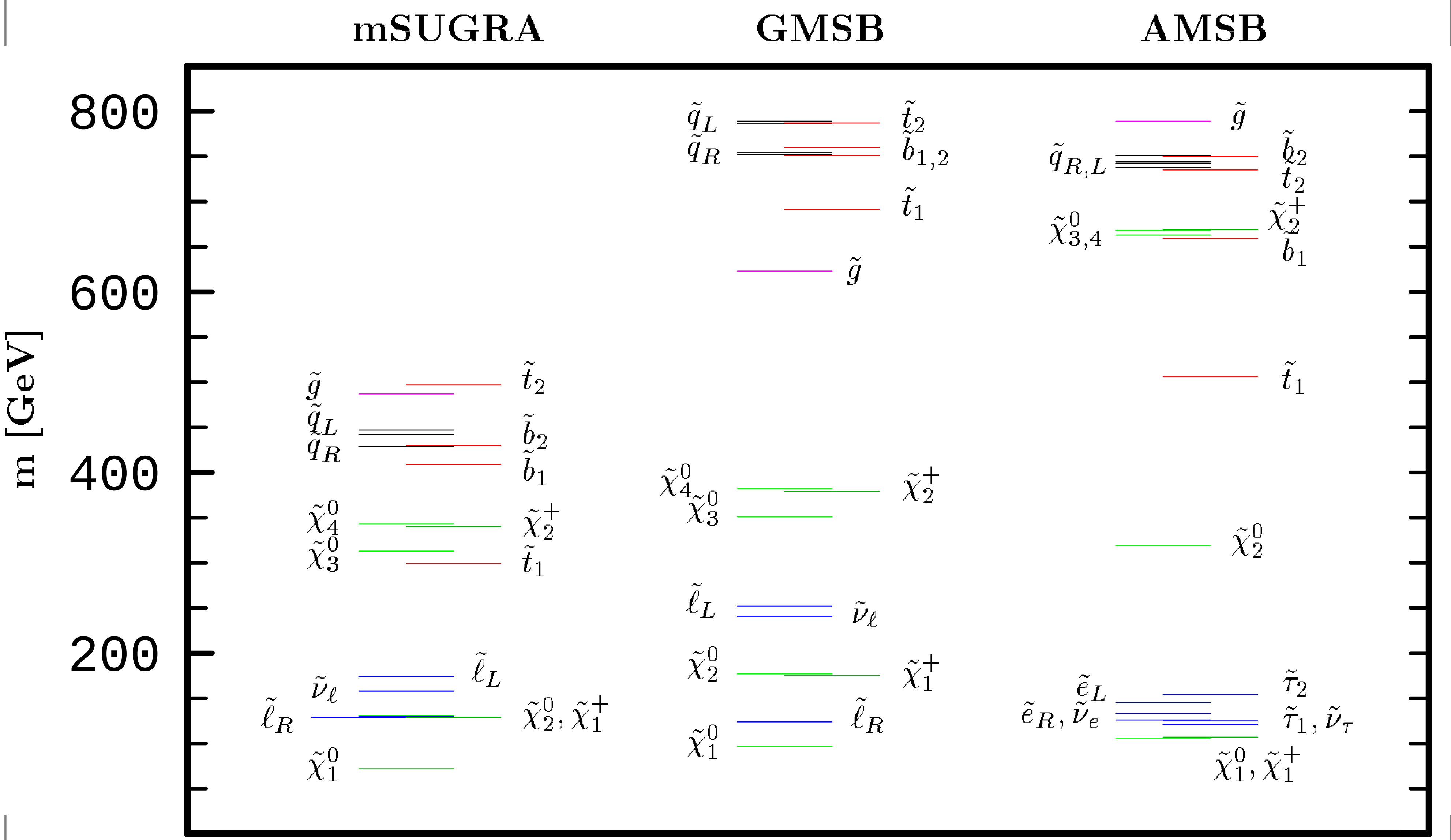
(Th, Experimental, DM, ...)



3. Spectrum and constraints: Theoretical constraints

- **No RGE problems:**
  - **Perturbative couplings/No Landau poles**
  - **Non tachyonic sfermions (in particular for 3d generation)**
  - **Consistent unification of gauge couplings**
- **Proper implementation of EWSB:**
  - **Non tachyonic A boson or μ parameter**
  - **Convergent/stable value of μ after several iterations**
  - **Vacuum non CCB nor UFB**
- **Reasonnable SUSY spectrum:**
  - **Non tachyonic sfermions from mixing**
  - **Higgs masses not NaN**
  - **The LSP is the lightest neutralino χ_1^0**

3. Spectrum and constraints: examples of spectra in cMSSM



. Spectrum and constraints: direct experimental constraints

Bounds from $\tilde{P}$ searches:

- **Bounds from LEPI/LEPII:**
  $m_{\tilde{\chi}_1^\pm} \gtrsim 104$ GeV
  $m_{\tilde{f}} \gtrsim 100$ GeV
  with $\tilde{f} = \tilde{t}_1, \tilde{b}_1, \tilde{l}^\pm, \tilde{\nu}$

- **Bounds from the Tevatron:**
  $m_{\tilde{g}} \gtrsim 300$ GeV
  $m_{\tilde{q}_{1,2}} \gtrsim 260$ GeV
  with $\tilde{q} = \tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}, \tilde{b}$

- **Possible refinements:**
  – (almost) stable χ_1^+ at LEPII
  – degenerate $\tilde{t}_1, \tilde{\tau}_1$ with LSP
  – $\tilde{t}_1$ with large Δm at Tevatron

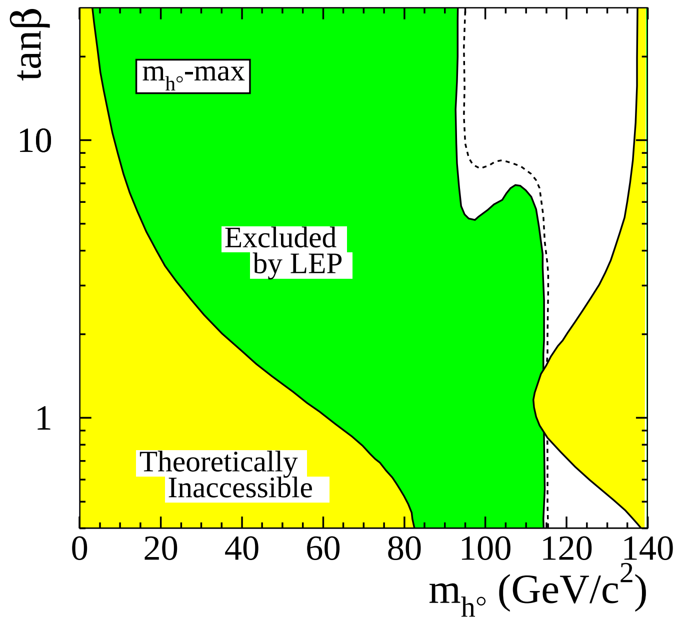
Bounds from Higgs searches at LEPII:

$M_A \gg M_Z \Rightarrow M_h > 114$ **GeV**

$M_A \sim M_Z \Rightarrow M_h, M_A \gtrsim 92$ **GeV**

– **Slightly depend on m_t, H mixing, ...**

– **Include a $\Delta^{\rm th} M_h \sim 3$ GeV error.**



(Excluded boundary to be fitted)

Note: include 1.7σ Higgs signal??

Spectrum and constraints: indirect experimental constrain

- **High precision electroweak measurements:** agree with SM

  Large $(\tilde{t}, \tilde{b})$ mass splitting might generate large contributions:

  $\Delta^{\rm SUSY}\rho = \Pi_{ZZ}(0)/M_Z^2 - \Pi_{WW}(0)/M_W^2 \lesssim 2.2 \cdot 10^{-3}$

  (loose constraints from direct SUSY contributions to $Zb\bar{b}$ vertex)

- **The $(g-2)_\mu$ constraint:** 2.5σ away from SM (only e^+e^- data)

  Might be accounted for by $\tilde{\mu}$–χ^0 and $\tilde{\nu}_\mu$–$\chi^\pm$ loop contributions

  $1.06 \cdot 10^{-9} \leq \frac{1}{2} g_\mu^{\rm SUSY} \leq 4.36 \cdot 10^{-9}$

  (OK with SM if+ τ data: $-5.7{\cdot}10^{-10} \leq \frac{1}{2} g_\mu^{\rm SUSY} \leq 4.7{\cdot}10^{-9}$)

- **The $b \to s\gamma$ constraint:** experimental value agrees with SM

  Strong constraints on the t–$H^\pm$ and $\tilde{t}$–$\chi^\pm$ loop contributions

  $2.65 \cdot 10^{-4} \leq B(b \to s\gamma) \leq 4.45 \cdot 10^{-4}$

  (might be aleviated with a small amount of flavor violation)

- **The $b \to s\ell^+\ell^-$ constraint:** not very stringent in mSUGRA yet

3. Spectrum and constraints: the dark matter constraint

- **WMAP measurement of temperature anisotropies in CMB, ...**

  $\Omega_{\rm DM}\, h^2 \simeq 0.113 \pm 0.009 \Rightarrow 0.09 \leq \Omega_{\rm DM}\, h^2 \leq 0.14$ **at 99% CL**

- **In the MSSM, LSP neutralino χ_1^0 is best candidate for CDM**

  – electrically neutral and (often maybe too) weakly interacting

  – stable if R–parity is conserved

  – massive: $m_{\chi_1^0} \gtrsim 50$ GeV in constrained models (mSUGRA)

- **Calculation of $\Omega_{\chi_1^0}\, h^2 \propto \langle v\sigma(\chi\chi \to {\rm SM\, part.})\rangle^{-1}$ complicated:**
  - **Many final states ($\Phi = h, H, A, H^\pm$; $f = \ell, q$; $V = W, Z, \gamma$)**

    $$\chi_1^0\chi_1^0 \to f\bar{f}, VV, \Phi_i\Phi_j, \Phi_i V \ \ {\rm etc....}$$

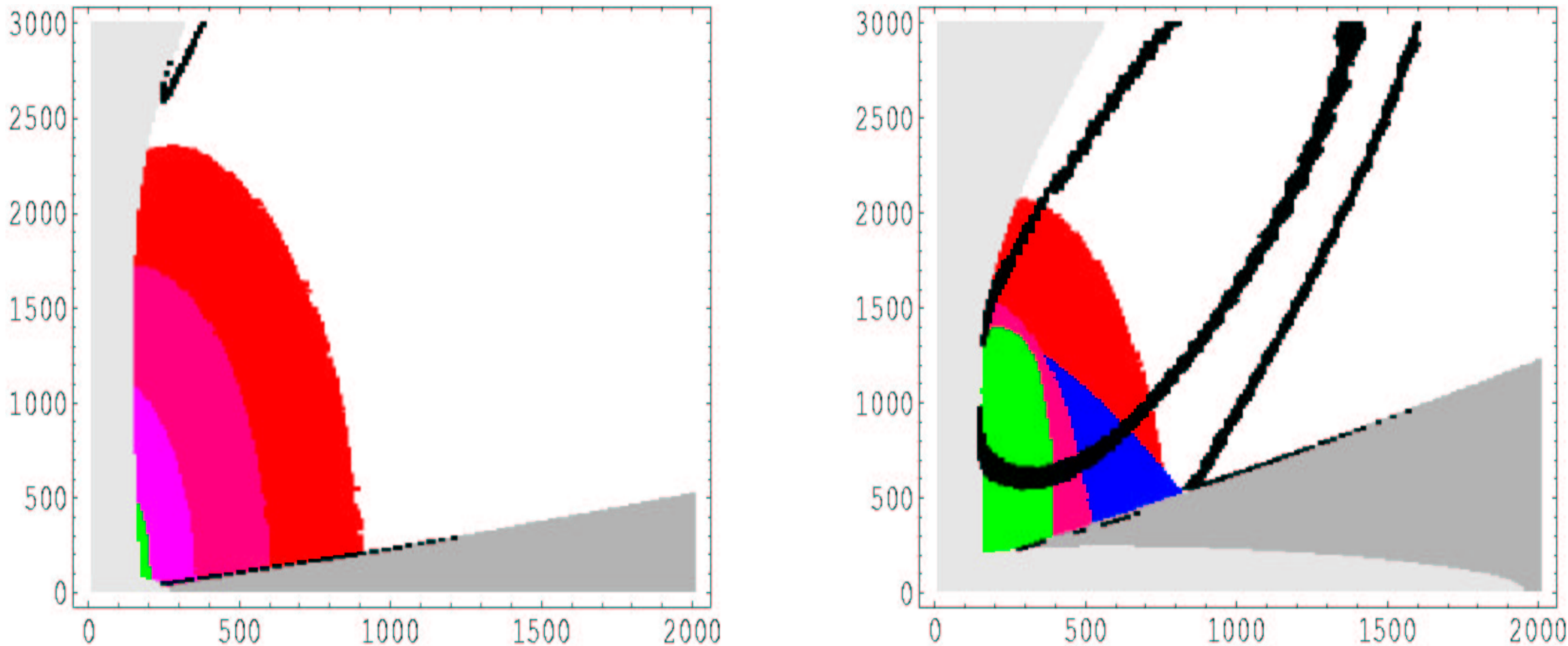
  - **Several channels are present; for example in $\chi_1^0\chi_1^0 \to f\bar{f}$:**
    t–channel $\tilde{f}$, s–channel Z and s–channel A, h, H exchanges
  - **Co-annihilation processes with NLSP taken into account:**

$$\chi_1^0 + \tilde{P} \to X + Y \ \ {\rm and} \ \ \tilde{P} + \tilde{P}^{(*)} \to X + Y \ \ {\rm if} \ m_{\tilde{P}} \sim m_\chi$$

3. Spectrum and constraints: an example of a scan

An $(m_{1/2}, m_0)$ scan with $A=0, \mu>0, m_t=172.5$ GeV:

$\mathbf{m_0}$ $\tan\beta = 10$ $\tan\beta = 50$

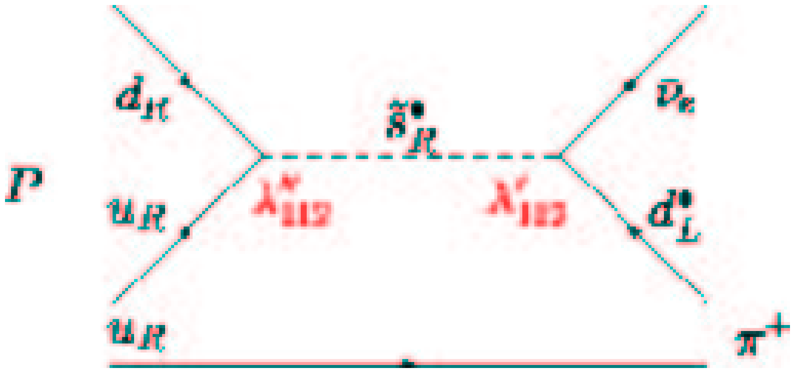


Generically, four (known) regions with the required ammount of DM: $\mathbf{m_{1/}}$

bulk region (excluded), focus point, co-annihilation, A/h pole regions

4. Extensions of MSSM: Rp violation

To avoid fast P decay, we do not need both L and B conservation



In most general W, include ΔL=1 or ΔB=1 interactions:

$$W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk}L_iL_j\bar{e}_k + \lambda'_{ijk}L_iQ_j\bar{d}_k + \mu'_iL_iH_u$$
$$W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k$$

P decay modes and experimental limits on $\not{B}$ and $\not{L}$ imply $\lambda^{,','}_{ijk} \ll 1$.

- **However, at least 45 new parameters in the general case.**
- **no stable LSP and thus no SUSY DM candidate...**
- **But, rich phenomenology (e.g. s channel sfermion production)**
- **enters in neutrino phenomenology and adresses small ν masses**

4. Extensions of the MSSM: CP violation

One can allow for some CP–violating parameters, in particular:

- Complex $\mathbf{M_1}, \mathbf{M_2}, \mathbf{M_3}$ (some phases rotated away) and μ
- Complex trilinear $\mathbf{A_f}$ couplings, in particular $\mathbf{A_f}$.

The MSSM Higgs sector stays CP–conserving at the tree–level but complex parameters enter at the one–loop level through μ and $\mathbf{A_t}$.

- CP violation is needed for (direct) baryogenesis in MSSM
- However, many new parameters will enter in the general case
- Complicates the determination of spectrum but less fine-tunning!
- Strongly constrained by data ($n_{\rm edm}..$) and needs cancelations
- No sign yet of any additionnaly from $\not{C}$P in B–factories etc...

One can also allow for flavor non–diagonal interactions, howver:

- Parameters strongly constrained from FCNC, K, B physics...
- Only adds complications/parameters (no theory motivation)...

4. Extensions of the MSSM: NMSSM

The μ problem: μ enters EWSB and the determination of $\mathrm{M_Z}$.

It must be of order SUSY–breaking parameters such as $\mathrm{M_{H_1}, M_{H_2}}$.

But μ is a SUSY preserving parameter, comes from $\mathbf{W} \propto \mu \mathbf{\hat{H}_1 \hat{H}_2}$,

and, a priori, no reason for having $\mu \propto \mathrm{M_Z, M_{SUSY}} \ll \mathrm{M_{GUT}}$....

Solution: μ is related to a vev of an additional field S with $\langle S \rangle = s$

NMSSM: introduce a gauge singlet superfield $\hat{S}$ into superpotential

$$\mathbf{W} = \mathbf{W}_{\mathrm{MSSM}} + \lambda \mathbf{\hat{H}_1 \hat{H}_2 \hat{S}} + \frac{\mathbf{1}}{\mathbf{3}} \kappa \mathbf{\hat{S}}$$

Extended spectrum in NMSSM compared to MSSM:

- one additional neutralino state: $\Rightarrow \chi^0_{1,...,5}$
- two additional Higgs particles $\Rightarrow \mathbf{H_1, H_2, H_3,\ A_1, A_2,\ H^+, H^-}$

$\Rightarrow$ less constrained and fine tuned model, richer phenomenology...

Ex: upper bound on h mass is $\mathrm{M_h^{NMSSM}} = \mathrm{M_h^{MSSM}} + \mathbf{20}$–40 GeV.

LEP searches bounds are not valid and h lighter than 100 GeV.