Physics beyond the Standard Model

Basics and Phenomenology of Supersymmetry

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- **Problems of the SM: GUTs, SUSY, alternatives**
- **SUSY and the MSSM**
- **The Higgs and SUSY particle spectrum in the MSSM**
- **Higgs and sparticles at colliders**

1. SUSY–GUTs: general features

Low–energy SUSY comes to the rescue!!

SUSY-GUTs are the most attractive extensions of the SM

For each SM particle, there is a SUSY partner with spin–$\frac{1}{2}$ diff.

(there are also two Higgs doublets, see the discussions later).

Some theoretical advantages of SUSY theory, are:

- It is the largest symmetry that an S–matrix can have

(so, we go further in our quest for/use of symmetries in Nature..).

- If SUSY gauged, we get a spin–$\frac{3}{2}$ gravitino and spin–2 graviton

(and therefore can include gravitational interaction: conceptual SM pb!).

- It is a natural part of Superstrings (theory of everything!?).

If SUSY is realized at low energies (which means $\mathrm{M}_{\mathrm{SUSY}} \sim 1$ TeV),

it solves the problems discussed in the case of non–SUSY GUTs!!!

Let us take the example of the MSSM, for instance (details later):

1. SUSY–GUTs: gauge coupling unification

The new SUSY particles will contribute to the running of $g_{1,2,3}$

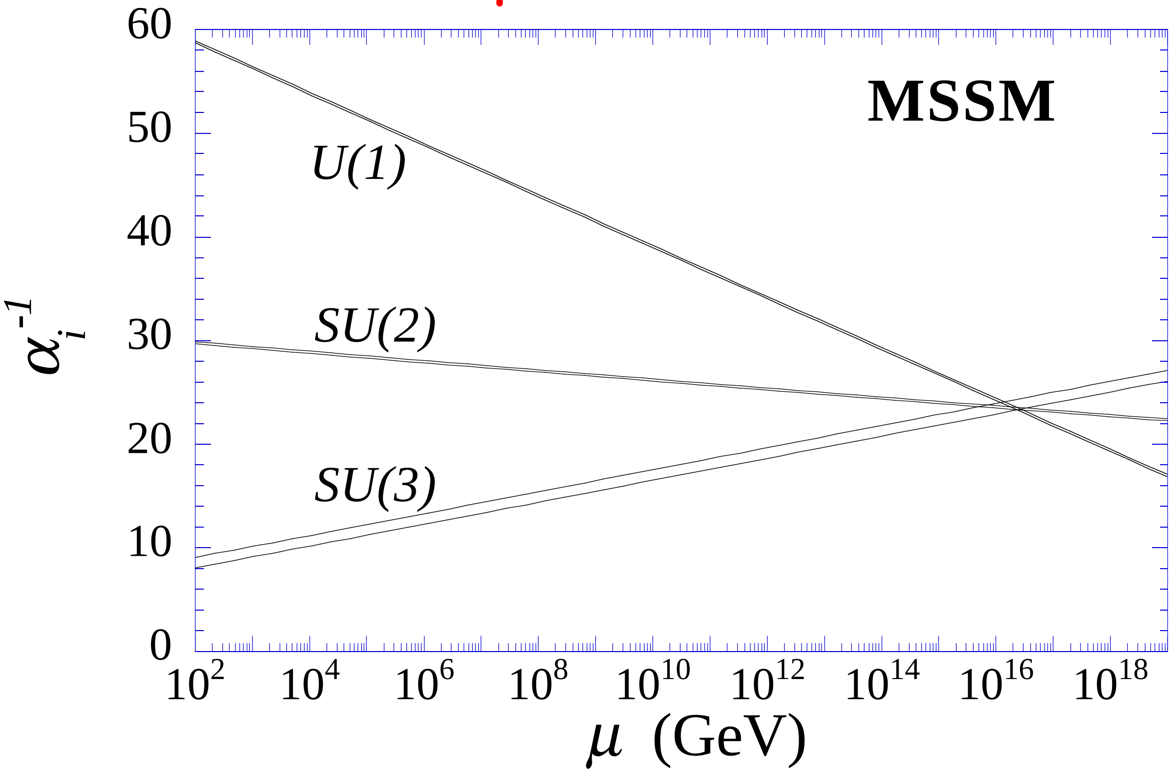
(one has to add a contribution of $b_N = -2N/3$ for gauginos)

$$b_3^{\rm MSSM} = (11 - 2\tfrac{6}{3} - 2 - \tfrac{12}{6} = 7 - 5 = 3$$

$$b_2^{\rm MSSM} = (\tfrac{22}{3} - 4 - \tfrac{1}{6} + (-\tfrac{4}{3} - \tfrac{12}{6} - \tfrac{2}{3} - \tfrac{1}{6}) = \tfrac{19}{6} - \tfrac{25}{6} = -1$$

$$b_1^{\rm MSSM} = -3\tfrac{10}{5}(f + \tilde{f}) - \tfrac{3}{5}4\tfrac{1}{4}(H + \tilde{h}) = -\tfrac{33}{5}$$

we get after calculation $B_{\rm th} = \frac{5}{7} = 0.74$ compared to $B_{\rm exp} \simeq 0.72$!



1. SUSY–GUTs: gauge coupling unification

Alternative view: the running couplings meet at a single point $\mathrm{M_U}$

obtained from $\log(\mathrm{M_U}/\mathrm{M_Z}) = \frac{10\pi}{28}\left[\alpha_1^{-1}(\mathrm{M_Z}) - \alpha_2^{-1}(\mathrm{M_Z})\right] \simeq 33.1$

$$\Rightarrow \mathrm{M_U} \sim 2 \cdot 10^{16}\ \mathrm{GeV}$$

Note that the small discrepancy is due to experimental errors on α_i

and also to small threshold corrections (new particles) near $\mathrm{M_U}$;

two loop corrections must be also included for more precision...

Important: for this to work, we need $\mathrm{M_{SUSY}} = \mathcal{O}(1\ \mathrm{TeV})$!!!

Note also that larger $\mathrm{M_U}$ is good to prevent proton decay:

- **In non–SUSY GUT, $\mathrm{M_{GUT}} \sim 10^{15}$ GeV, 10 times smaller...**
- **In SUSY–GUTs: additional colored Higgs/higgsino exchange**

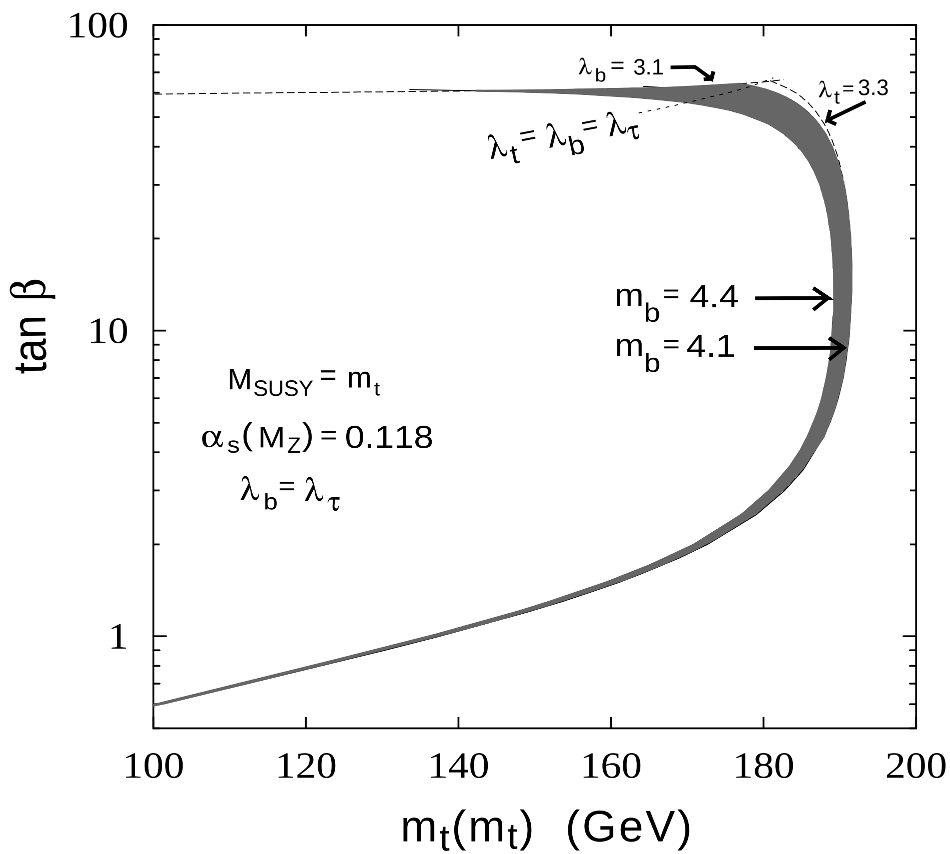
$$\tau_{\mathbf{P}}^{\mathrm{SUSY}} \propto 1/\mathrm{M_{GUT}^4} > 10^{33}\ \text{years}$$

Larger but very close to experimental bound; to be observed soon?

1. SUSY–GUTs: Yukawa coupling unification

One can also unify the (3d gen.) Yukawa couplings at M_{GUT}

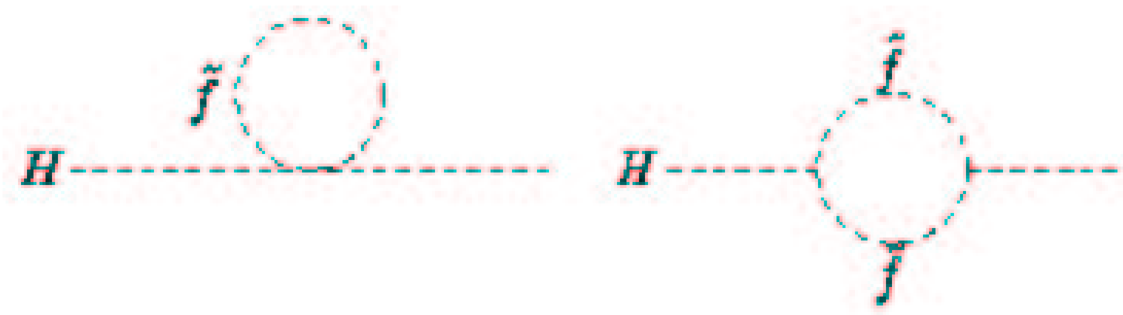
$$\frac{dh_t}{d\log Q} = \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$
$$\frac{dh_b}{d\log Q} = \frac{h_b}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right],$$
$$\frac{dh_\tau}{d\log Q} = \frac{h_\tau}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right].$$



1. SUSY–GUTs: the hierarchy problem

In SUSY, we need to add contributions of the new particles to ΔM_H^2

Ex: in the case of fermion loop, add the contributions of (2) scalars



Again, quadratic divergences for scalars (see later). But assume:

- Scalar couplings related to fermion couplings: $\lambda_f^2 = -\lambda_S$.
- Multiplicative factors are the same: $N_S = N_f$ (nb: 2 scalars).
- To simplify, the scalars have the same mass: $m_1 = m_2 = m_S$.

and add the fermionic and scalar contributions; we get:

$$\Delta M_H^2|^{\rm tot} = \frac{\lambda_f^2 N_f}{4\pi^2}\left[(m_f^2 - m_S^2)\log\left(\frac{\Lambda}{m_S}\right) + 3m_f^2\log\left(\frac{m_S}{m_f}\right)\right]$$

1. SUSY–GUTs: the hierarchy problem

The quadratic divergences have disappeared in the sum!!

(the same job can be done for (s)contributions of W,Z,H etc..).

Logarithmic divergence still there, but contribution small.

No divergences at all if in addition $m_S = m_f$ (exact SUSY)!

$\Rightarrow$ Symmetry fermions–scalars $\rightarrow$ no divergence in Λ^2

“Supersymmetry” no divergences at all: M_H is protected!

Note that if $M_S \gg 1\ \mathrm{TeV}$ the fine tunning problem is back!!!

In summary:

– gauge symmetry protects the gauge boson masses

– chiral symmetry protects the fermion masses

– SUSY is needed to protect the scalar masses.

We need SUSY at low energies, $M_{SUSY} = \mathcal{O}(1\ \mathrm{TeV})$!

(also to solve the gauge unification and Dark Matter problems..).

1. SUSY–GUTs: Dark Matter

- In minimal SUSY models, there is a symmetry called R–parity which makes that the lightest SUSY particle is absolutely stable.
- Experimental constraints show that this LSP must be quite heavy, $M_{LSP} \gtrsim 10$–50 GeV, and thus is non–relativistic (cold dark matter).
- In most areas of the SUSY parameter space, this LSP is electrically neutral and it interacts very weakly.
- For some values of its mass and couplings, this LSP can have a relic abundance which is within the range given by WMAP data.

The LSP is an ideal candidate for Dark Matter!

Again, for this to work, the particle must be lighter than $\mathcal{O}(\mathrm{TeV})$!

At least 3 good reasons to believe in SUSY!

in addition to SUSY being part of superstrings and link with gravity!

2. Supersymmetry: Basics

Here, we give only basic facts needed later in phenomenology discussion

For details on theoretical issues, see the lectures of Sven Heinemeyer.

SUSY: a symmetry relating scalars/vector bosons and fermions.

SUSY generators $\mathcal{Q}$ transform fermions into bosons and vice–versa:

$$\mathcal{Q}|\mathrm{Fermion}> |\mathrm{Boson}> \;,\;\; \mathcal{Q}|\mathrm{Boson}> |\mathrm{Fermion}>$$

$\mathcal{Q}$ must be an anti–commuting (complicated) object.

$\mathcal{Q}^\dagger$ is also a distinct symmetry generator:

$$\mathcal{Q}^\dagger|\mathrm{Fermion}> |\mathrm{Boson}> \;,\;\; \mathcal{Q}^\dagger|\mathrm{Boson}> |\mathrm{Fermion}>$$

Such theories are highly restricted [e.g., no go theorem, see SH]

and in a 4-dimension theory with chiral fermions [as in the SM]:

$\mathcal{Q}, \mathcal{Q}^\dagger$ carry spin–$\frac{1}{2}$ with L,R helicities and they should obey....

2. Supersymmetry: Basics

.... The SUSY algebra: (which schematically is given by)

$$\{\mathcal{Q},\mathcal{Q}^\dagger\} = P^\mu\ ,\ \{\mathcal{Q},\mathcal{Q}\}=0\ ,\ \{\mathcal{Q}^\dagger,\mathcal{Q}^\dagger\}=0,$$
$$[P^\mu,\mathcal{Q}] = 0, [P^\mu,\mathcal{Q}^\dagger]=0, [T^a,\mathcal{Q}]=0, [T^a,\mathcal{Q}^\dagger]=0$$

$\mathbf{P}^\mu$**: generator of space–time transformations.**

$\mathbf{T^a}$ **generators of internal (gauge) symmetries.**

$\Rightarrow$ **SUSY: unique extension of the Poincaré group of space–time symmetry to include a four–dimensional Quantum Field Theory...**

Single–particle states of the theory are in irreducible representations of the SUSY algebra above, which are called supermultiplets.

Fermions and bosons of same supermultiplet are superpartners.

They must have the same mass and gauge quantum numbers.

Three types of supermultiplets are needed...

2. Supersymmetry: Supermultiplets and Superpartners

- **Chiral (or "scalar") supermultiplet** (ζ with $\zeta^c = \zeta$ and S):

– 1 two–component Weyl fermion with spin $\pm\frac{1}{2}$ ($\mathbf{n_F = 2}$)

– 2 real spin–0 scalar = 1 complex scalar ($\mathbf{n_B = 2}$)

- **Gauge (or "vector") supermultiplet** ($\mathbf{A}_\mu^{\mathbf{a}}$ and $\lambda_{\mathbf{A}}$):

– 1 two–component Weyl gaugino–fermion with spin $\pm\frac{1}{2}$ ($\mathbf{n_F = 2}$)

– 1 real spin–1 massless gauge vector boson ($\mathbf{n_B = 2}$)

- **Gravitational supermultiplet:**

– 1 two–component Weyl gravitino–fermion with spin $\pm\frac{3}{2}$ ($\mathbf{n_F = 2}$)

– 1 real spin–2 massless graviton ($\mathbf{n_B = 2}$)

Ex: $\Psi = \binom{\mathrm{e_L}}{\mathrm{e_R}}$ with $\mathrm{e_{L/R}}$ being 2–component Weyl LH/RH fermions

Each state has a complex spin–0 superpartner noted $\tilde{\mathrm{e}}_\mathrm{L}$ and $\tilde{\mathrm{e}}_\mathrm{R}$.

One can define $\mathrm{e} \equiv \mathrm{e_L}$ and $\bar{\mathrm{e}} = \mathrm{e_R^\dagger}$ so that one has:

two LH chiral supermultiplets for the electron: $(\mathrm{e}, \tilde{\mathrm{e}}_\mathrm{L}), (\bar{\mathrm{e}}, \tilde{\mathrm{e}}_\mathrm{R}^*)$.

The same for all other leptons and quarks (except for massless ν_L).

2. Supersymmetry: Interactions

- All fields involved have the canonical kinetic energies

$$\mathcal{L}_{\rm kin} = \Sigma_{\rm i}\{(\mathbf{D}_\mu \mathbf{S}_{\rm i}^*)(\mathbf{D}^\mu \mathbf{S}_{\rm i}) + \mathbf{i}\overline{\psi}_{\rm i}\mathbf{D}_\mu\gamma^\mu\psi_{\rm i}\} + \Sigma_{\rm a}\{-\frac{1}{4}\mathbf{F}^{\rm a}_{\mu\nu}\mathbf{F}^{\mu\nu {\rm a}} + \frac{\rm i}{2}\overline{\lambda}_{\rm a}\mathbf{D}$$

with D the covariant derivative. [Note that $\psi(\lambda)$ have 4(2) comps.]

- The interactions are specified by SUSY and gauge invariance:

$$\mathcal{L}_{\rm int.\ scal-fer.-gauginos} = -\sqrt{2}\sum_{\rm i,a}\mathbf{g}_{\rm a}\left[\mathbf{S}_{\rm i}^*\mathbf{T}^{\rm a}\overline{\psi}_{\rm iL}\lambda_{\rm a} + {\rm h.c.}\right]$$

$$\mathcal{L}_{\rm int.\ quartic\, scal.} = -\frac{1}{2}\sum_{\rm a}\left(\sum_{\rm i}\mathbf{g}_{\rm a}\mathbf{S}_{\rm i}^*\mathbf{T}^{\rm a}\mathbf{S}_{\rm i}\right)^2$$

- All interactions are given by the gauge coupling constants $\mathbf{g}_{1,2,3}$

(fundamental prediction of SUSY: same g in gauge and Yukawa int.)

- At this stage, a very simple and minimal theory:

Everything is completely specified and no adjustable parameter!

2. Supersymmetry: Superpotential

Only freedom: choice of Superpotential W (SUSY and gauge inv!).

It gives the scalar potential and Yukawa interactions (fer.–scal.).

– W $\equiv$ function of the superfields $\mathrm{z_i}$ only (not $\mathrm{z_i^*}$!).

– Analytic function: no derivative interaction.

– Renormalizability: only terms of dimension 2 and 3.

$$\Rightarrow \mathcal{L}_{\mathrm{W}} = -\sum_{\mathrm{i}} \left| \frac{\partial \mathrm{W}}{\partial \mathrm{z_i}} \right|^2 - \frac{1}{2} \sum_{\mathrm{ij}} \left[\overline{\psi}_{\mathrm{iL}} \frac{\partial^2 \mathrm{W}}{\partial \mathrm{z_i} \partial \mathrm{z_j}} \psi_{\mathrm{j}} + \mathrm{h.c.} \right]$$

To obtain the interactions explicitly: take $\partial \mathrm{W}/\partial \mathrm{z_i}|_{\mathrm{z_i}=\mathrm{S_i}}$.

The SUSY tree–level scalar potential is $\mathrm{V_{tree}} = \mathrm{V_F} + \mathrm{V_D}$.

• F–terms from W through derivatives wrt all scalars $\mathrm{S_i}$:

$$\mathrm{V_F} = \sum_{\mathrm{i}} \mathrm{F_i F_i^*} = \sum_{\mathrm{i}} |\mathrm{W^i}|^2 \text{ with } \mathrm{W^i} = \partial \mathrm{W}/\partial \mathrm{S_i}$$

• D–terms corresponding to the U(1),SU(2),SU(3) gauge groups:

$$\mathrm{V_D} = \frac{1}{2} \sum_{\mathrm{i}} \mathrm{D_i D_i^*} = \frac{1}{2} \sum_{\mathrm{a}=1}^{3} \left(\sum_{\mathrm{i}} \mathrm{g_a S_i^* T^a S_i} \right)^2$$

2. Supersymmetry: SUSY–breaking

SUSY cannot be an exact symmetry since no scalars exist with the same mass as known fermions (smultiplets) $\Rightarrow$ must be broken.

Spontaneous SUSY breaking?

Means that the Lagrangian is invariant under (global) SUSY but the ground state $|0>$ is not: $\mathcal{Q}|0> \neq 0$ and $\mathcal{Q}^\dagger|0> \neq 0$.

Recall: Hamiltonian is related to the SUSY charges: $\{\mathcal{Q}, \mathcal{Q}^\dagger\} \sim \mathrm{P}^\mu$

So that one has: $<0|\mathrm{H}|0> \equiv <0|\mathrm{P}^0|0> \propto <0|\mathcal{Q}\mathcal{Q}^\dagger|0> = \mathrm{E}_{\mathrm{vac}} \neq 0$

In fact, the vacuum energy should be positive: $\mathrm{E}_{\mathrm{vac}} > 0$.

- **$\langle 0|D|0\rangle 0$ or D–term breaking: leads to CCB minima**

$\Rightarrow$ does not work in the MSSM!

- **$\langle 0|F|0\rangle 0$ or F–term breaking): needs a linear, $\mathrm{a_i \Phi_i}$, term in W**

$\Rightarrow$ requires a singlet sfield under $\mathrm{G_{SM}}$; not in the MSSM!

2. Supersymmetry: explicit breaking

Solution: SUSY–breaking occurs in a hidden sector of particles with no (or very tiny) couplings to the visible sector of the MSSM.

If mediating interaction is flavor–blind, universal breaking terms.

Examples: gravity (mSUGRA), gauge (GMSB) mediation ...

Many breaking schemes but none is fully satisfactory at the moment:

$\Rightarrow$ Explicit breaking by hand (also with several possibilities...).

- **We need SUSY breaking at low energy to solve the problems:**

– Quadratic divergences in the Higgs sector.

– Unification of the coupling constants of $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$.

– Dark Matter problem (existence of a massive stable particle), etc.

- **In the breaking, we still need to preserve: gauge invariance, renormalizability, and no quadratic divergence (soft SUSY–breaking).**

$\Rightarrow$ "Low energy SUSY" $\equiv$ effective theory at low energy.

3. The MSSM: gauge group

As discussed earlier, in SUSY theories:

– For each SM particle, there is a SUSY partner with spin $\frac{1}{2}$ difference.

– SUSY must be broken at a low scale $M_{SUSY} = \mathcal{O}(1\ \mathbf{TeV})$

To solve the unfication, hierachy and dark matter problems of the SM.

The MSSM is the most economic low energy SUSY extension of SM.

The MSSM is based on the following assumptions:

• Minimal gauge group: $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)$.

The SM spin–1 gauge bosons [$B, \mathbf{W_{1-3}}$ and $\mathbf{g_{1-8}}$] and their spin–$\frac{1}{2}$ gaugino partners $[\tilde{b}, \tilde{\mathbf{w}}_{\mathbf{1-3}}, \tilde{\mathbf{g}}_{\mathbf{1-8}}]$ are in vector superfields.

Superfields	$\mathbf{SU(3)_C}$	$\mathbf{SU(2)_L}$	$\mathbf{U(1)_Y}$	Particle content
$\hat{\mathbf{G}}^{\mathbf{a}}$	8	1	0	$\mathbf{G}^\mu, \tilde{\mathbf{g}}$
$\hat{\mathbf{W}}^{\mathbf{i}}$	1	3	0	$\mathbf{W}^\mu_{\mathbf{i}}, \tilde{\omega}_{\mathbf{i}}$
$\hat{\mathbf{B}}$	1	1	0	$\mathbf{B}^\mu, \tilde{\mathbf{b}}$

3. The MSSM: particle content

- **Minimal particle content:**

– Three fermion generations [as in SM no $\nu_{\rm R}$...] and their spin–0 SUSY partners, the sfermions $\tilde{\rm f}_{\rm L}, \tilde{\rm f}_{\rm R}$, combined in chiral supermultiplets.

– No chiral anomalies ($\sum_{\rm f} {\rm Q_f} \equiv 0$) and fermion mass generation in a SUSY invariant way (no conjugate ${\rm H}^*$ field for u–quarks), we need: two chiral superfields with ${\rm Y}=+1$ and ${\rm Y}=-1$.

Superfield	$\rm SU(3)_C$	$\rm SU(2)_L$	$\rm U(1)_Y$	Particle content
$\hat{Q}$	3	2	$\frac{1}{3}$	(u_L, d_L), $(\tilde{u}_L, \tilde{d}_L)$
$\hat{U}^c$	$\overline{3}$	1	$-\frac{4}{3}$	$\overline{u}_R$, $\tilde{u}^*_R$
$\hat{D}^c$	$\overline{3}$	1	$\frac{2}{3}$	$\overline{d}_R$, $\tilde{d}^*_R$
$\hat{L}$	1	2	-1	(ν_L, e_L), $(\tilde{\nu}_L, \tilde{e}_L)$
$\hat{E}^c$	1	1	2	$\overline{e}_R$, $\tilde{e}^*_R$
$\hat{H}_1$	1	2	-1	$(H_1, \tilde{h}_1)$
$\hat{H}_2$	1	2	1	$(H_2, \tilde{h}_2)$

3. The MSSM: R–parity

• R–parity conservation:

To eliminate terms violating B and L numbers (and proton decay):

Discrete and multiplicative symmetry called R-parity or $\mathbf{R_p}$:

$$\mathbf{R_p = (-1)^{2s+3B+L}}$$

Then
$$R = +1 \text{ for all ordinary SMparticles}$$
$$R = -1 \text{ for all the SUSY particles}$$

The consequences of $\mathbf{R_p}$ conservation are very important:

– SUSY particles always produced in pairs.

– SUSY particles decay into an odd number of SUSY particles.

– The lightest SUSY particle (LSP) is absolutely stable.

3. The MSSM: Superpotential

At this stage, we have a globally supersymmetric Lagrangian.

- **Everything is specified by SUSY and gauge invariance.**
- **No additional parameter compared to SM.**
- **Only freedom, the choice of the Superpotential.**

The most general Superpotential compatible with SUSY, gauge invariance, renormalizability and R–parity conservation is:

$$\mathbf{W} = \sum_{\rm i,j=gen} Y^u_{ij}\, \hat{u}^i_R \hat{H}_2 . \hat{Q}^j + Y^d_{ij}\, \hat{d}^i_R \hat{H}_1 . \hat{Q}^j + Y^l_{ij}\, \hat{l}^i_R \hat{H}_1 . \hat{L}^j + \mu \hat{H}_1 . \hat{H}_2$$

- $Y^{u,d,l}_{ij}$ **denote the Yukawa couplings among the three generations (and which simply a generalisation of the SM Yukawa interaction).**
- μ **supersymmetric Higgs–higgsino parameter with dimension of mass (it is thus a supersymmetric parameter, see later....).**

3. The MSSM: soft–SUSY breaking

- **Soft SUSY breaking:**

To explicitely break Supersymmetry without reintroducing the quadratic divergences (the so–called soft SUSY–breaking), we add by hand a collection of soft terms (of dimension two and three):

$$\mathcal{L}_{\rm gaugino} = \frac{1}{2}\left[M_1 \tilde{b}\tilde{b} + M_2 \Sigma_{a=1}^{3} \tilde{w}^a \tilde{w}_a + M_3 \Sigma_{a=1}^{8} \tilde{g}^a \tilde{g}_a \ + \ {\rm h.c.}\right]$$

$$\mathcal{L}_{\rm sf.} = \Sigma_i m^2_{\tilde{Q},i} \tilde{Q}_i^\dagger \tilde{Q}_i + m^2_{\tilde{L},i} \tilde{L}_i^\dagger \tilde{L}_i + m^2_{\tilde{u},i} |\tilde{u}_{R_i}|^2 + m^2_{\tilde{d},i} |\tilde{d}_{R_i}|^2 + m^2_{\tilde{l},i} |\tilde{l}_{R}$$

$$\mathcal{L}_{\rm Higgs} = m_2^2 H_2^\dagger H_2 + m_1^2 H_1^\dagger H_1 + B\mu (H_2.H_1 + {\rm h.c.})$$

$$\mathcal{L}_{\rm tr.} = \Sigma_{i,j}\left[A^u_{ij} Y^u_{ij} \tilde{u}_{R_i} H_2.\tilde{Q}_j + A^d_{ij} Y^d_{ij} \tilde{d}_{R_i} H_1.\tilde{Q}_j + A^l_{ij} Y^l_{ij} \tilde{l}_{R_i} H_1.\tilde{L}_j + \right.$$

A rather complicated and problematic potential indeed!

- **Too many parameters and thus not very predictive.**
- **Leads generically to a problematic phenomenology.**

3. The MSSM: the parameters

In the most general case (mixing and phases): 105 free parameters!

- **complex gaugino masses M_1, M_2, M_3 : 6**
- **3×3 hermitian mass matrices $m_{\tilde{F}}$: 45**
- **3×3 complex trilinear coupling matrices A_f : 54**
- **2×2 matrix for the bilinear B coupling : 4**
- **Higgs masses squared, $m^2_{H_1}, m^2_{H_2}$: 2**

111–6 (due to constraints from symmetries and Higgs sector)=105.

For “generic” sets of these parameters, leads to severe problems:

- **large flavor changing neutral currents [FCNC]**
- **unacceptable amount of additional CP–violation**
- **color and/or charge breaking minima**
- **an incorrect value of the Z boson mass, etc......**

We need more constrained MSSMs

4. Constrained MSSMs: pMSSM

A phenomenologically viable MSSM is defined by assuming:

- **all soft SUSY–breaking parameters are real (no new CP viol).**
- **Mass and trilinear cpls. for sfermions diagonal (no FCNC)**
- **1st/2d sfermion generation universality (no pb. with Kaons)**

Phenomenological MSSM (pMSSM) with 22 free parameters:

$\tan\beta$: **the ratio of the vevs of the two–Higgs doublet fields.**

$m^2_{H_u}, m^2_{H_d}$: **the Higgs mass parameters squared.**

M_1, M_2, M_3: **the bino, wino and gluino mass parameters.**

$m_{\tilde{q}}, m_{\tilde{u}_R}, m_{\tilde{d}_R}, m_{\tilde{l}}, m_{\tilde{e}_R}$: **1st/2d generation sfermion mass para.**

$m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}, m_{\tilde{L}}, m_{\tilde{\tau}_R}$: **third generation sfermion mass para.**

A_t, A_b, A_τ: **the third generation trilinear couplings.**

A_u, A_d, A_e: **the first/second generation trilinear couplings.**

4. Constrained MSSMs: pMSSM

In fact:

- You can trade $m^2_{H_u}, m^2_{H_d}$ with more "physical" μ and M_A

(in fact: μ^2 and $B\mu$ can be determined from ESWB, see later).

- A_u, A_d, A_e in general not relevant for phenomenology.

(enter only in "light" flavor physics: $(g-2)_\mu$, neutron edm, ...).

- If you focus on a given sector (Higgs, gauginos, sfermions):

only few parameters to deal with and model indep. analyses....

$\Rightarrow$ phenomenologically more viable model than general MSSM

- You can also use common soft–SUSY breaking terms in many cases

($m_{\tilde{q}} = m_{\tilde{u}_R} = m_{\tilde{d}_R}$; $m_{\tilde{Q}}, m_{\tilde{t}_R}, m_{\tilde{b}_R}$; A_t, A_b, A_τ; etc..)

and one ends with an even more restrictive set of parameters, $\lesssim 10$.

$\Rightarrow$ much more predictive model than general MSSM

4. Constrained MSSMs: mSUGRA

Almost all problems of MSSM solved at once if soft SUSY–breaking parameters obey a set of universal boundary conditions at $M_{\rm GUT}$.

Underlying assumption: SUSY–breaking occurs in a hidden sector communicating with visible sector through gravitational interactions.

$\Rightarrow$ Universal soft terms emerge if interactions are "flavor–blind":

Besides $g_{1,2,3}$ unification which fix the scale $\mathrm{M_{GUT}} \sim 2 \cdot 10^{16}$ GeV:

Unification of gaugino, scalar masses and trili. couplings at $\mathrm{Q = M_{GUT}}$

Universal gaugino masses: $\mathrm{M_1 = M_2 = M_3 \equiv m_{1/2}}$

Universal scalar masses: $\mathrm{M_{\tilde{Q}_i} = M_{\tilde{L}_i} = M_{H_i} \equiv m_0}$

Universal trilinear couplings: $\mathrm{A^u_{ij} = A^d_{ij} = A^l_{ij} \equiv A_0\, \delta_{ij}}$

Also: B and μ^2 from requiring of EWSB and minimization of $V_{\rm Higgs}$

$$\mu^2 = \frac{1}{2}[\tan 2\beta (\mathrm{m^2_{H_u}} \tan\beta - \mathrm{m^2_{H_d}} \cot\beta) - \mathrm{M_Z^2}]$$

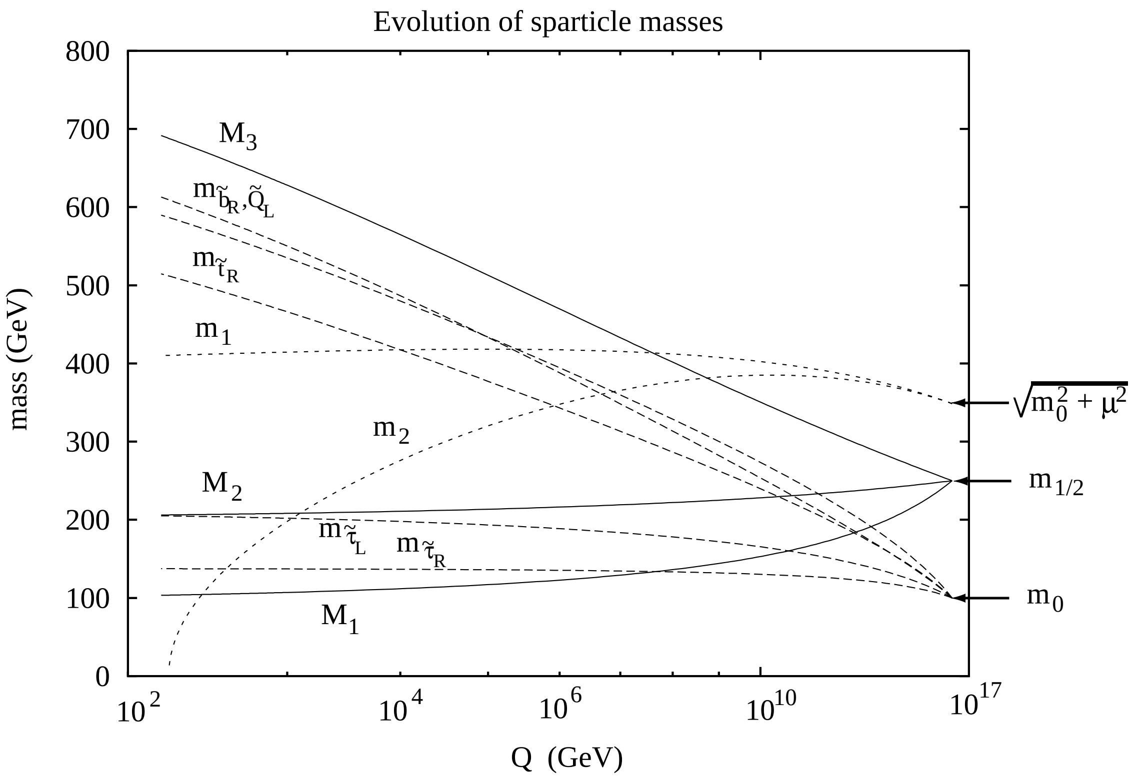
$$\mathrm{B}\mu = \frac{1}{2} \sin 2\beta [\mathrm{m^2_{H_u}} + \mathrm{m^2_{H_d}} + 2\mu^2]$$

4. Constrained MSSMs: mSUGRA

Only 4.5 param: $\tan\beta$, $\mathbf{m_{1/2}}$, $\mathbf{m_0}$, $\mathbf{A_0}$, $\mathrm{sign}(\mu)$

All soft breaking parameters at M_S are obtained through RGEs.

With $\mathrm{M_{GUT}} \sim \mathbf{2 \cdot 10^{16}}$ GeV and $\mathrm{M_{SUSY}} \sim \sqrt{\mathbf{m_{\tilde{t}_L} m_{\tilde{t}_R}}}$:



Radiative EWSB occurs since $\mathrm{M^2_{H_2}} < 0$ at scale $\mathrm{M_Z}$ $(\mathbf{t}/\tilde{\mathbf{t}}$ loops$)$

$\Rightarrow$ EWSB more natural in MSSM ($\mu^2 < 0$ from RGEs) than in SM!

4. Constrained MSSMs: GMSB

In GMSB, SSB transmitted to MSSM fields via SM gauge interactions.

- Hidden sector for SUSY–break. contains messengers fields, $n_{\hat{q}}/n_{\hat{l}}$ quark/lepton-like pairs coupled to a gauge singlet chiral superfield $\hat{S}$.
- The spotential is $W = \lambda \hat{S}\hat{q}\hat{\bar{q}} + \lambda \hat{S}\hat{l}\hat{\bar{l}}$ with $\hat{S}$ havings vevs. s and f_S
- SSB are generated by (1or2) loop corrections at scale $M_{\rm mes} = \lambda s$

$$\mathbf{M_G(M_{mes}) = \frac{\alpha_G(M_{mes})}{4\pi} \Lambda\, g(\frac{\Lambda}{M_{mes}}) \sum_m N_R^G(m)}$$

$$\mathbf{m_s^2(M_{mes}) = 2\Lambda^2 f(\frac{\Lambda}{M_{mes}}) \sum_{m,G} (\frac{\alpha_G(M_{mes})}{4\pi})^2 N_R^G(m) C_R^G(s)}$$

$\mathbf{A_f(M_{mes}) \simeq 0}$ (generated at two–loops).

with $\mathbf{\Lambda = f_s/s, G = U(1), SU(2), SU(3)}$, m and s label messengers and scalars; f/g are one/two loop functions; N/C are Dynkin/Casimirs..

Thus, in the GMSB model there are six basic input parameters

$$\mathbf{\tan\beta\ ,\ sign(\mu)\ ,\ M_{mes}\ ,\ \Lambda\ ,\ n_{\hat{q}}\ ,\ n_{\hat{l}}}$$

plus the mass of the very light gravitino (which is the LSP).

4. Constrained MSSMs: AMSB

In AMSB, SUSY breaking occurs also in hidden sector (e.g. extra dims) and is transmitted to visible sector via (e.g. super–Weyl) anomalies.

Gaugino, scalar masses and trilinear couplings are simply related to the scale dependence of the gauge and matter kinetic functions.

In terms of gravitino mass $\mathbf{m_{3/2}}$, β functions for $\mathbf{g_a}$ and $\mathbf{Y_i}$ couplings and anomalous dimensions $\gamma_{\mathbf{i}}$ of chiral superfields, SSB terms are:

$$\mathbf{M_a} = \frac{\beta_{\mathbf{g_a}}}{\mathbf{g_a}} \mathbf{m_{3/2}} \;,\; \mathbf{A_i} = \frac{\beta_{\mathbf{Y_i}}}{\mathbf{Y_i}} \mathbf{m_{3/2}}$$

$$\mathbf{m_i^2} = -\frac{1}{4} \Big(\Sigma_{\mathbf{a}} \frac{\partial \gamma_{\mathbf{i}}}{\partial \mathbf{g_a}} \beta_{\mathbf{g_a}} + \Sigma_{\mathbf{k}} \frac{\partial \gamma_{\mathbf{i}}}{\partial \mathbf{Y_k}} \beta_{\mathbf{Y_k}} \Big) \mathbf{m_{3/2}^2}$$

RG invariant equations valid at any scale (make a predictive model).

(μ^2 and $B\mu$ terms are obtained as usual by requiring EWSB).

However, picture spoiled by tachyonic sleptons $m^2_{\tilde{L}} < 0$ in general!

$\Rightarrow$ add a non anomalous contribution to soft masses $c_i m_0^2$ to m_i^2

In minimal AMSB with a universal m_0, $c_i = 1$, the inputs are:

$$\mathbf{m_0} \;,\; \mathbf{m_{3/2}} \;,\; \tan\beta \;,\; \mathrm{sign}(\mu) \text{ and } \mathbf{c_i}$$

5. The SUSY spectrum:

The basic Lagrangian, gives the currents and corresponding states:

we need to turn the current eigenstates into the mass eigenstates

- **Charginos: mixtures of the charged higgsinos and gauginos**

$$\tilde{\mathbf{W}}^{\pm}, \tilde{\mathbf{h}}^{\pm}_{\mathbf{2/1}} \longrightarrow \chi^{\pm}_{\mathbf{1}}, \chi^{\pm}_{\mathbf{2}}$$

- **Neutralinos: mixtures of the neutral higgsinos and gauginos**

$$\tilde{\mathbf{B}}, \tilde{\mathbf{W}}^{\mathbf{0}}, \tilde{\mathbf{h}}^{\mathbf{0}}_{\mathbf{2}}, \tilde{\mathbf{h}}^{\mathbf{0}}_{\mathbf{2}}, \longrightarrow \tilde{\chi}^{\mathbf{0}}_{\mathbf{1}}, \tilde{\chi}^{\mathbf{0}}_{\mathbf{2}}, \tilde{\chi}^{\mathbf{0}}_{\mathbf{2}}, \tilde{\chi}^{\mathbf{0}}_{\mathbf{4}}$$

- **Sfermions: mixing between LH and RH sfermions of same flavor**

$$\tilde{\mathbf{f}}_{\mathbf{L}}, \tilde{\mathbf{f}}_{\mathbf{R}} \longrightarrow \tilde{\mathbf{f}}_{\mathbf{1}}, \tilde{\mathbf{f}}_{\mathbf{2}}$$

- **Higgs bosons: 2 doublets of complex fields $\mathbf{H_1, H_2 \equiv 4}$ dof**

⇒ 3 degrees of freedom to generate $\mathbf{M_{W^+}, M_{W^-}, M_Z}$

⇒ 5 degrees of freedom left: $\mathbf{M_h, M_H, M_A, M_{H^+}, M_{H^-}}$

To determine the mass and the mixing angle of the physical states

find relevant mass matrices and diagonalize them.... include RC?

5. The SUSY spectrum: charginos

The general chargino mass matrix, in terms of M_2, μ and $\tan\beta$, is

$$\mathcal{M}_\mathbf{C} = \begin{bmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{bmatrix} , \ \mathbf{s}_\beta \equiv \sin\beta \ \mathbf{etc}$$

diagonalized by: $\mathbf{U}\mathcal{M}_\mathbf{C}\mathbf{V}^{-1} \to \mathbf{U} = \mathcal{O}_-, \mathbf{V} = \left\{ \begin{array}{ll} \mathcal{O}_+ & \text{if } \det \mathcal{M}_C > \\ \sigma_3 \mathcal{O}_+ & \text{if } \det \mathcal{M}_C < \end{array} \right.$

(Pauli σ_3 to make the $\chi^\pm$ masses positive and $\mathcal{O}_\pm$ rotation matrices)

Simple analytical formulae for the masses $m^\pm_{\chi_{1,2}}$ and mixing angles.

For limiting cases, interpretation much simpler. $\mu \gg \mathbf{M_2}, \mathbf{M_W}$:

$$\mathbf{m}_{\chi_1^\pm} \simeq \mathbf{M_2} - \mathbf{M_W^2} \mu^{-\mathbf{2}} \left(\mathbf{M_2} + \mu \mathbf{s_{2\beta}} \right)$$

$$\mathbf{m}_{\chi_2^\pm} \simeq |\mu| + \mathbf{M_W^2} \mu^{-\mathbf{2}} \epsilon_\mu \left(\mathbf{M_2} \mathbf{s_{2\beta}} + \mu \right)$$

$$|\mu| \to \infty : \chi_1^\pm \ \textbf{wino with } \mathbf{m}_{\chi_1^\pm} \simeq \mathbf{M_2}; \chi_2^\pm \ \textbf{higgsino with } \mathbf{m}_{\chi_2^\pm} = |\mu$$

In the opposite limit, $M_2 \gg |\mu|, M_Z$, the roles of $\chi_1^\pm, \chi_2^\pm$ are reversed.

5. The SUSY spectrum: neutralinos

For neutralinos, the 4x4 mass matrix depends on $\mu, \mathbf{M_2}, \tan\beta, \mathbf{M_1}$.

In the $(-\mathbf{i}\tilde{\mathbf{B}}, -\mathbf{i}\tilde{\mathbf{W}}_\mathbf{3}, \tilde{\mathbf{H}}^\mathbf{0}_\mathbf{1}, \tilde{\mathbf{H}}^\mathbf{0}_\mathbf{2})$ basis, it is given by

$$\mathcal{M}_\mathbf{N} = \left[\begin{array}{cccc} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{array} \right]$$

Diagonalized by a single real matrix Z. Again for $|\mu| \gg \mathbf{M_{1,2}} \gg \mathbf{M_Z}$:

$$\mathbf{m}_{\chi^0_1} \simeq \mathbf{M_1} - \frac{\mathbf{M_Z^2}}{\mu^\mathbf{2}} \left(\mathbf{M_1} + \mu \mathbf{s}_{\mathbf{2}\beta}\right) \mathbf{s_W^2}$$

$$\mathbf{m}_{\chi^0_2} \simeq \mathbf{M_2} - \frac{\mathbf{M_Z^2}}{\mu^\mathbf{2}} \left(\mathbf{M_2} + \mu \mathbf{s}_{\mathbf{2}\beta}\right) \mathbf{c_W^2}$$

$$\mathbf{m}_{\chi^0_{3/4}} \simeq |\mu| + \frac{\mathbf{1}}{\mathbf{2}} \frac{\mathbf{M_Z^2}}{\mu^\mathbf{2}} \epsilon_\mu (\mathbf{1} \mp \mathbf{s}_{\mathbf{2}\beta}) \left(\mu \pm \mathbf{M_2 s_W^2} \mp \mathbf{M_1 c_W^2}\right)$$

For $|\mu| \to \infty$, χ^0_1 is bino (M_1), χ^0_2 wino (M_2) and χ^0_3, χ^0_4 higgsinos (μ).

In the opposite limit, $M_1, M_2 \to \infty$, the roles are again reversed.

5. The SUSY spectrum: gluinos and RGEs

Finally, the gluino mass is identified with M_3 at the tree–level

$$m_{\tilde{g}} = M_3$$

In constrained models with boundary conditions at the high energy scale M_U, the evolution of the gaugino masses given by RGEs

$$\frac{dM_i}{d\log(M_U/Q^2)} = -\frac{g_i^2 M_i}{16\pi^2} b_i \ , \ b_1 = \frac{33}{5} \ , \ b_2 = 1 \ , b_3 = -3$$

where in b_i all sparticles contribute to the evolution from Q to M_U.

Equations are related to those of the gauge couplings $\alpha_i = g_i^2/(4\pi)$.

With inputs at scale M_Z and common value at $M_U \sim 2 \times 10^{16}$ GeV, one has for gaugino mass parameters at the weak or SUSY scale M_S:

$$M_3 : M_2 : M_1 \sim \alpha_3 : \alpha_2 : \alpha_1 \sim 6 : 2 : 1$$

With norm.factor $\frac{5}{3}$ in α_1, we have $M_1 = \frac{5}{3} \tan^2\theta_W M_2 \simeq \frac{1}{2} M_2$.

$\mu \gg M_2 \Rightarrow m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim 2m_{\chi_1^0} \sim M_2, \ m_{\chi_3^0} \sim m_{\chi_4^0} \sim m_{\chi_2^\pm} \sim \mu.$

$\mu \ll M_2 \Rightarrow m_{\chi_2^0} \sim m_{\chi_1^\pm} \sim m_{\chi_1^0} \sim \mu, \ m_{\chi_4^0} \sim 2m_{\chi_3^0} \sim m_{\chi_2^\pm} \sim M_2.$

5. The SUSY spectrum: sfermions

Sfermion system described by $\tan\beta, \mu$ and 3 param.for each species: $m_{\tilde{f}_L}, m_{\tilde{f}_R}$ and A_f. For 3d generation, mixing $\propto m_f$ to be included.

$$\mathcal{M}^2_{\tilde{\mathbf{f}}} = \begin{pmatrix} m_f^2 + m_{LL}^2 & m_f X_f \\ m_f X_f & m_f^2 + m_{RR}^2 \end{pmatrix}$$

with the various entries given by

$$m_{LL}^2 = m_{\tilde{f}_L}^2 + (I_f^{3L} - Q_f s_W^2)\, M_Z^2\, c_{2\beta}$$
$$m_{RR}^2 = m_{\tilde{f}_R}^2 + Q_f s_W^2\, M_Z^2\, c_{2\beta}$$
$$X_f = A_f - \mu(\tan\beta)^{-2I_f^{3L}}$$

They are diagonalized by 2×2 rotation matrices of angle θ_f, which turn the current eigenstates $\tilde{f}_L, \tilde{f}_R$ into the mass eigenstates $\tilde{f}_1, \tilde{f}_2$.

$$\mathbf{m^2_{\tilde{f}_{1,2}} = m_f^2 + \frac{1}{2}\left[m_{LL}^2 + m_{RR}^2 \mp \sqrt{(m_{LL}^2 - m_{RR}^2)^2 + 4m_f^2 X_f^2}\,\right]}$$

5. The SUSY spectrum: sfermions

Note: mixing very strong in stop sector, $X_t = A_t - \mu \cot\beta$ and generates mass splitting between $\tilde{t}_1, \tilde{t}_2$, leading to light $\tilde{t}_1$; mixing in sbottom/stau sectors also for large $X_{b,\tau} = A_{b,\tau} - \mu \tan\beta$.

In cMSSM with universal m_0 and $m_{1/2}$ at M_{GUT}, the RGEs for scalar masses are simple if Yukawas are small ($c(\tilde{f})$ depend on I, Y, color):

$$m^2_{\tilde{f}_{L,R}} = m_0^2 + \sum_{i=1}^{3} F_i(f) m^2_{1/2} \,, \quad F_i = \frac{c_i(f)}{b_i}\left[1 - \left(1 - \frac{\alpha_U}{4\pi} b_i \log\frac{Q^2}{M_U^2}\right)^{-2}\right]$$

$$\tilde{L}: \begin{pmatrix} \frac{3}{10} \\ \frac{3}{2} \\ 0 \end{pmatrix}, \tilde{l}_R: \begin{pmatrix} \frac{6}{5} \\ 0 \\ 0 \end{pmatrix}, \tilde{Q}: \begin{pmatrix} \frac{1}{30} \\ \frac{3}{2} \\ \frac{8}{3} \end{pmatrix}, \tilde{u}_R: \begin{pmatrix} \frac{8}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}, \tilde{d}_R: \begin{pmatrix} \frac{2}{15} \\ 0 \\ \frac{8}{3} \end{pmatrix}$$

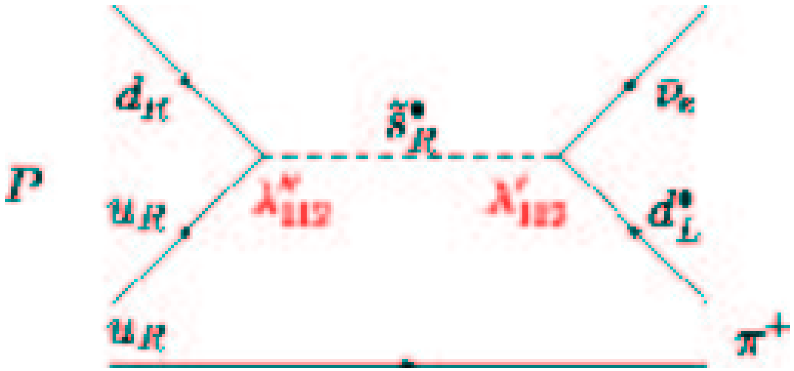
With inputs at M_Z, $\alpha_U \simeq 0.041$ and M_U, one obtains

$$m^2_{\tilde{q}_i} \sim m_0^2 + 6m^2_{1/2} \,, \quad m^2_{\tilde{\ell}_L} \sim m_0^2 + 0.52 m^2_{1/2} \,, \quad m^2_{\tilde{e}_R} \sim m_0^2 + 0.15$$

For 3d generation squarks, Yukawa couplings to be included!!

6. Extensions of MSSM: Rp violation

To avoid fast P decay, we do not need both L and B conservation



In most general W, include ΔL=1 or ΔB=1 interactions:

$$\mathbf{W_{\Delta L=1} = \frac{1}{2}\lambda_{ijk}L_iL_j\bar{e}_k + \lambda'_{ijk}L_iQ_j\bar{d}_k + \mu'_iL_iH_u}$$
$$\mathbf{W_{\Delta B=1} = \frac{1}{2}\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k}$$

P decay modes and experimental limits on $\not B$ and $\not L$ imply $\lambda^{,',''}_{ijk} \ll 1$.

- **However, at least 45 new parameters in the general case.**
- **no stable LSP and thus no SUSY DM candidate...**
- **But, rich phenomenology (e.g. s channel sfermion production)**
- **enters in neutrino phenomenology and adresses small ν masses**

6. Extensions of the MSSM: CP violation

One can allow for some CP–violating parameters, in particular:

- Complex $\mathbf{M_1, M_2, M_3}$ (some phases rotated away) and μ
- Complex trilinear $\mathbf{A_f}$ couplings, in particular $\mathbf{A_f}$.

The MSSM Higgs sector stays CP–conserving at the tree–level but complex parameters enter at the one–loop level through μ and $\mathbf{A_t}$.

- CP violation is needed for (direct) baryogenesis in MSSM
- However, many new parameters will enter in the general case
- Complicates the determination of spectrum but less fine-tunning!
- Strongly constrained by data ($n_{\rm edm}..$) and needs cancelations
- No sign yet of any additionnaly from $\not{C}$P in B–factories etc...

One can also allow for flavor non–diagonal interactions, howver:

- Parameters strongly constrained from FCNC, K, B physics...
- Only adds complications/parameters (no theory motivation)...

6. Extensions of the MSSM: NMSSM

The μ problem: μ enters EWSB and the determination of $\mathrm{M_Z}$.

It must be of order SUSY–breaking parameters such as $\mathrm{M_{H_1}, M_{H_2}}$.

But μ is a SUSY preserving parameter, comes from $\mathbf{W} \propto \mu \mathbf{\hat{H}_1 \hat{H}_2}$,

and, a priori, no reason for having $\mu \propto \mathrm{M_Z, M_{SUSY}} \ll \mathrm{M_{GUT}}$....

Solution: μ is related to a vev of an additional field S with $\langle S \rangle = s$

NMSSM: introduce a gauge singlet superfield $\hat{S}$ into superpotential

$$\mathbf{W} = \mathbf{W}_{\rm MSSM} + \lambda \mathbf{\hat{H}_1 \hat{H}_2 \hat{S}} + \frac{\mathbf{1}}{\mathbf{3}} \kappa \mathbf{\hat{S}}$$

Extended spectrum in NMSSM compared to MSSM:

- one additional neutralino state: $\Rightarrow \chi^0_{1,...,5}$
- two additional Higgs particles $\Rightarrow \mathbf{H_1, H_2, H_3,\ A_1, A_2,\ H^+, H^-}$

$\Rightarrow$ less constrained and fine tuned model, richer phenomenology...

Ex: upper bound on h mass is $\mathrm{M_h^{NMSSM}} = \mathrm{M_h^{MSSM}} + \mathbf{20}$–40 GeV.

LEP searches bounds are not valid and h lighter than 100 GeV.