Physics beyond the Standard Model

Basics and Phenomenology of Supersymmetry

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- **Problems of the SM: GUTs, SUSY, alternatives**
- **The Minimal Supersymmetric Standard Model**
- **The Higgs and SUSY particle spectrum in the MSSM**
- **Higgs and sparticles at colliders**

The SM of strong and electroweak interactions

• SM based on the gauge symmetry $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$:

Matter fields: 3 generations of fermions $\mathrm{f_{L,R} = \frac{1}{2}(1 \mp \gamma_5) f}$

$$\mathrm{I_f^{3L,3R} = \pm\frac{1}{2}, 0 \quad \Rightarrow L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L,\ R = e_R^-,\ Q = \begin{pmatrix} u \\ d \end{pmatrix}_L,\ u_R,\ d_R}$$

$$\mathrm{Y_f = 2Q_f - 2I_f^3 \Rightarrow Y_L = -1, Y_R = -2, Y_Q = \frac{1}{3}, Y_{u_R} = \frac{4}{3}, Y_{d_R} = -\frac{2}{3}}$$

Same holds for the two other generations: $\mu, \nu_\mu, \mathrm{c, s};\ \tau, \nu_\tau, \mathrm{t, b}$.

There is no ν_{R} (and neutrinos are and stay exactly massless).

Gauge fields: $\mathrm{g_\mu^{1,..,8}, W_\mu^{1,2,3}, B_\mu}$ fields, generators of SU(3), SU(2), U(1)

the corresponding gauge bosons are exactly massless at this stage.

Field str.: $\mathrm{W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c,\ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu}$

Minimal coupling via cov.der. : $\mathrm{D_\mu \psi = \left(\partial_\mu - ig T_a W_\mu^a - ig' \frac{Y}{2} B_\mu\right) \psi}$

$$\mathcal{L}_{\mathrm{SM}}^{\mathrm{EW}} = \mathrm{-\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{L}\, iD_\mu \gamma^\mu\, L + \bar{e}_R\, iD_\mu \gamma^\mu\, e_R \cdots}$$

Brutal incorporation of mass terms violates SU(2)xU(1) gauge invariance.

The Higgs mechanism

Masses through the mechanism of spontaneous symmetry breaking

$\Rightarrow$ **introduce a doublet of complex scalar fields** $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ **with** $Y_\Phi = 1$

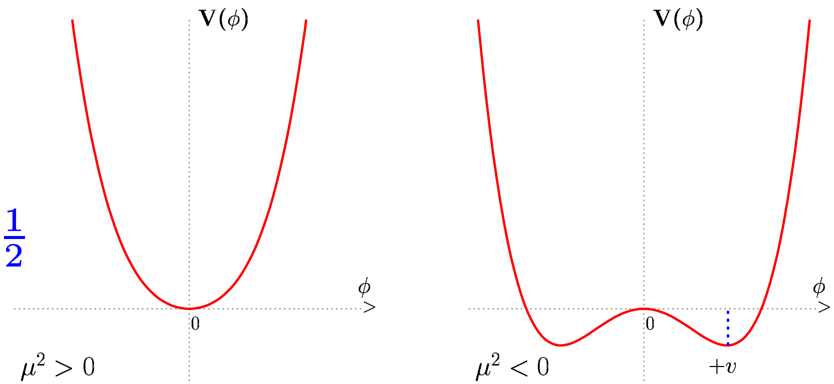
with a Lagrangian that is invariant under $\mathrm{SU(2)_L \times U(1)_Y}$

$$\mathcal{L}_S = (D^\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

$\mu^2 > 0$: **4 scalar particles.**

$\mu^2 < 0$: Φ **develops a vev:**

$$\langle 0 | \Phi | 0 \rangle = (0 / \frac{v}{\sqrt{2}}), \; v = \; (-\frac{\mu^2}{\lambda})^{\frac{1}{2}}$$



$\Rightarrow$ **3 degrees of freedom for** $W_L^\pm, Z_L$ **and thus gauge boson masses:**

$$M_W = (v/2) g \, , \; M_Z = (v/2) \sqrt{g^2 + g'^2} \, , \; M_A = 0 \, ,$$

To generate fermion masses use <u>same</u> doublet Φ **as well as** $\tilde{\Phi}$

$$\mathcal{L}_{\rm Yuk} = -f_e (\bar{e}, \bar{\nu})_L \Phi e_R - f_d (\bar{u}, \bar{d})_L \Phi d_R - f_u (\bar{u}, \bar{d})_L \tilde{\Phi} u_R ..., \, m_f = \frac{f_f \, v}{\sqrt{2}}$$

The SM of strong and electroweak interactions

The SM has many virtues:

– Gauge principle: esthetical and minimal interactions

– Theory which is unitary, perturbative and renormalisable

– Only one yet unknown parameter: the Higgs boson mass

Once M_H is fixed: everything is predictable with precision!

It passed succesfully (almost) all experimental tests:

– QCD/SU(3)$_{\mathrm{C}}$ checked at all available energies ($\mathrm{m}_\tau \to$ LEP2).

– $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{Y}}$ symmetry checked at LEP2 ($\mathrm{e}^+\mathrm{e}^- \to \mathrm{W}^+\mathrm{W}^-$).

– Flavor structure and CP checked precisley in B–factories.

– Quantum structure of the theory probed at the level of 0.1%.

However, it has also phenomenological and conceptual problems.

most theorists firmly believe that the SM is an effective theory:

a low energy, $\lesssim 1$ TeV, manifestation of more a fundamental theory.

Shortcomings of the SM

Some of the general problems of the SM:

– Too many parameters (19 and if no ν_R) which are put by hand.

– Does not explain 3-familly structure and charge assignement.

– Does not say anything about the masses and CKM of fermions.

– Does not incorporate naturally masses for the neutrinos.

– Does not explain baryon asymmetry in the universe.

– Does not have natural explanation for $\mu^2 < 0$.

– Does not have a candidate for cold dark matter.

And there are two big conceptual problems:

– Does not incorporate the fourth fundamental force: gravitation (gravity is mediated by spin–2 graviton: not renormalisable!).

– The hierachy problem: does not explain why $M_W \ll M_P$ radiative corrections to M_H quadratically divergent: $M_H \sim M_P$.

Beyond SM: Technicolor??

What is this fundamental theory?

Not enough experimental hints for it as the SM works too well!

But theorists have several (different) well motivated ideas:

Strongly interacting theories: Technicolor, Little Higgs, etc...

Most problems due to EWSB with scalar Higgs for mass generation.

Give up the usual way to view the (perturbative) Higgs mechanism!

Technicolor: H is not a fundamental scalar but a $\bar{\psi}\psi$ condensate.

Needs a new very strong binding force: $\Lambda_{\rm new} \sim 10^3 \Lambda_{\rm QCD} \sim 1$ TeV.

Simple models are strongly disfavored by LEP/Tevatron data...

Little Higgs models: less radical option, but exper. not disfavored.

Extra symmmetries allow corrections to $\rm M_H$ at two–loop level only.

and the non–pertubative regime starts only at a scale $\Lambda_{\rm new} \sim 10$ TeV.

Works technically but does not solve problems at the high scale.

Beyond SM: Extra dimensions??

Extra dimensions. **Inspired by string theory:**

- **One assumes e.g. large compactified extra dimensions.**

The new scale is related to the compactification radius: $\Lambda = 1/\mathrm{R}$.

- **SM fields are on a brane and gravity propagates in the whole bulk.**

Gravity appears weak as many lines of force escape in extra dims.

- **2 main models: geometry of D=4 and D-4 dims independent (ADD).**

: non–factorizable (warped) geometry (RS).

Exciting scenario with attractive features and rich phenomenology!

It addresses many fundamental problems of SM with a fresh viewpoint.

(ex: Randal–Sundrum with warped metric: hierarchy, mass gen., ...)

However, precision tests show that Λ **is much higher than 1 TeV.**

In addition, theory non–renormalisable: breakdown for $\mathrm{Q} \gtrsim \Lambda$.

Can be part of the whole truth, but will not be considered here...

Beyond the SM: GUTs

In SM, we have 3 different gauge groups with 3 coupling constants:

$\Rightarrow$ **SU(3)xSU(2)xU(1) subgroup of a bigger unifying group.**

Grand Unified Theory (GUT): SU(5), SO(10), E_6 etc....

- **only one coupling constant a the GUT scale $M_{GUT} = M_U$**

Spontaneous breakdown to G_{SM} at M_U (intermdeiate scale?).

- **GUT has fundamental representation including all SM fermions.**

Ex: SO(10) has dim. 16 repr. which incorporates 15 SM fermions.

- **Space left for RH neutrinos: generation of m_ν via see–saw.**
- **Baryon asymmetry of the universe through leptogenesis**
- **Explains charge quantization (ex. in SU(5): e,d in multiplet).**
- **Can relate the masses of fermions at M_U (Yukama coupling unif.)**

This is the route that we will follow here: GUTs and SUSY–GUTs.

But there are at least three big problems in normal GUTs:

gauge coupling unification, absence of dark matter, hierarchy problem.

Problems of GUT: unification

First of all, what are the three gauge couplings?

The strong SU(3) and electroweak SU(2) gauge couplings are

$$\alpha_3 = g_S^2/(4\pi)\ ,\ \ \alpha_2 = g_2^2/(4\pi) = g^2/(4\pi)$$

More complicated for U(1) coupling g': needs proper normalisation

scale is arbitrary: no physical change in $\frac{1}{2}$ Yg' if we use Ya and g'/a!

[in SU(2) generators normalized by the relation Tr$\left(\frac{\tau^a}{2}\cdot\frac{\tau^b}{2}\right) = \frac{1}{2}\delta^{ab}$].

In a GUT, say SU(5), $Y_G = a\frac{Y}{2}$ is the hypercharge generator with

common normalisation as those of the subgroups SU(2) or SU(3)

Example of normalisation condition: $\mathrm{Tr}(Y_G)^2 = \mathrm{Tr}(a\frac{Y}{2})^2 = \mathrm{Tr}(T_3)^2$

For one generation of fermions (u, d, e, ν_e) we have:

$$\mathrm{Tr}(T_3)^2 = 3(1/4 + 1/4) + 1/4 + 1/4 = 2$$

$$\mathrm{Tr}(Y_G)^2 = a^2/4[3(1/9 + 16/9 + 1/9 + 4/9) + 1 + 1 + 4] = a^2(1$$

$$Y_G = \sqrt{3/5}(Y/2) \Rightarrow \alpha_1 = (5/3)g'^2/(4\pi) = g_1^2/(4\pi)$$

Problems of GUT: unification

In fact, this is a prediction for $\sin^2\theta_W$ at the GUT scale!

$$g_2 \tan\theta_W = g' = \sqrt{\tfrac{5}{3}} g_1 \; ; \; g_2(M_G) = g_1(M_G) \Rightarrow \sin^2\theta_W(M_G) = \tfrac{3}{8}$$

The running of the coupling constants: due to radiative corrections to the interaction term in the original Lagrangian ($\gamma f\bar{f}$ in QED).

Due to Ward identities, equivalent to ren. of two–point functions.

The logarithmic running has same coefficient as the divergence:

Evolution determined by RGE [see lectures by Matthias Jamin]

$$\frac{d\alpha_i}{d\log(Q)} = -\frac{b_i}{2\pi}\alpha_i^2$$

$b_i \equiv$ coeff. of β functions and $\propto$ divergences of two–point functions.

They depend on the relevant gauge group and particle content.

Problems of GUT: unification

Ex, for SU(N): $\mathbf{b_N = \frac{11}{3}N - \frac{2}{3}n_{\rm fermions} - \frac{1}{6}n_{\rm scalars}}$

In the SM: $\mathbf{b_3^{\rm SM} = 11 - 2 \times \frac{6}{3} = 7 \ , \ b_2^{\rm SM} = \frac{22}{3} - 4 - \frac{1}{6} = \frac{19}{6}}$

For U(1): $\mathbf{b_1 = -\frac{2}{3}\sum_f Y_f^2 - \frac{1}{3}\sum_f Y_s^2 \Rightarrow b_1^{\rm SM} = -\frac{41}{10}}$ (recal norm.)

[note that $\mathbf{b_3, b_2 > 0}$ and $\mathbf{b_1 < 0}$: asymptotic freedom of SU(N)

$\Rightarrow \mathbf{g_3, g_2}$ decrease with energy Q and $\mathbf{g_1}$ increases with Q!].

For unification, more convenient to deal with $\mathbf{\alpha_i^{-1}}$ and RGEs

$$\mathbf{\frac{d\alpha_i^{-1}}{d\log(Q)} = +\frac{b_i}{2\pi}\alpha_i^2 \Rightarrow \alpha_i^{-1}(Q) = \alpha_i^{-1}(Q_0) + \frac{b_i}{2\pi}\log(Q/Q_0)}$$

At $\mathbf{Q = M_U}$ we need $\mathbf{\alpha_1(M_U) = \alpha_2(M_U) = \alpha_3(M_U) \equiv \alpha_U}$

with $\mathbf{Q_0 = M_Z}$, this implies the 3 relations between couplings

$$\mathbf{\alpha_U^{-1} = \alpha_{1,2,3}^{-1}(M_Z) + \frac{b_{1,2,3}}{2\pi}\log\frac{M_U}{M_Z}}$$

$$\mathbf{B_{\rm exp} = \frac{\alpha_3^{-1}(M_Z) - \alpha_2^{-1}(M_Z)}{\alpha_2^{-1}(M_Z) - \alpha_1^{-1}(M_Z)} = \frac{b_2 - b_3}{b_1 - b_2} = B_{\rm th}}$$

we can immediately calculate using the $\mathbf{b_i}$'s, $\mathbf{B_{\rm th} = \frac{115}{218} = 0.528}$

[Note: there also threshold corrections near $\mathbf{M_U}$ but they are small...]

Problems of GUT: unification

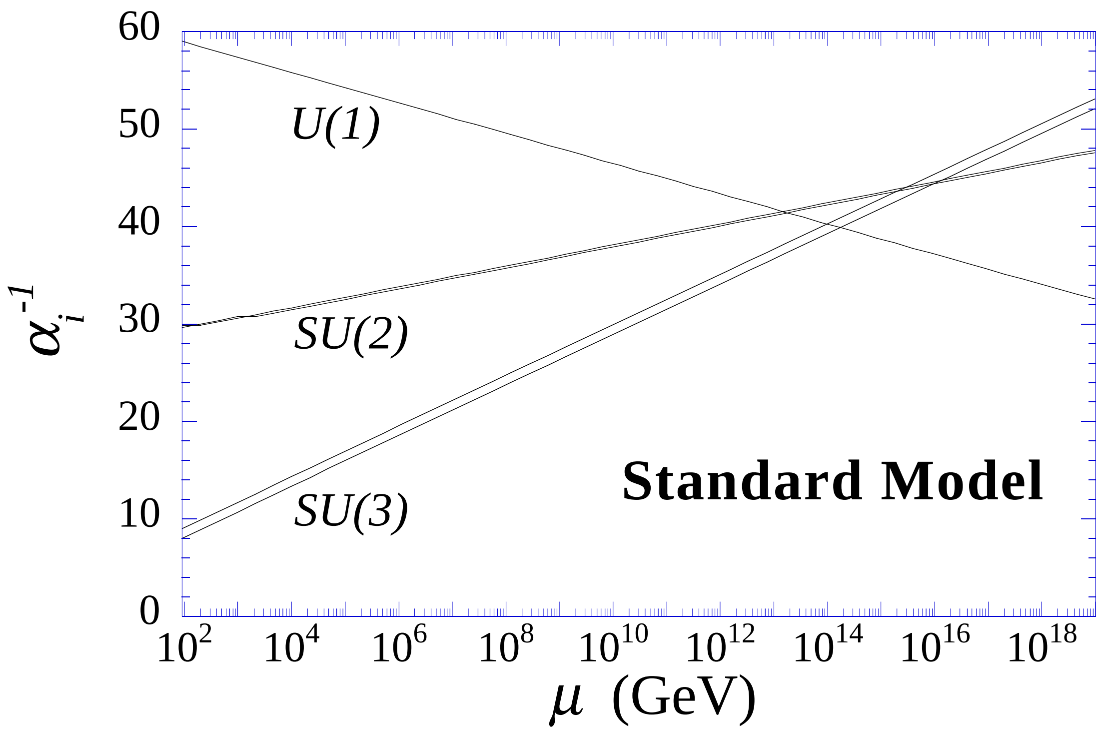
To calculate $\mathrm{B}_{\rm exp}$, use experimental values measured at LEP:

$$\sin^2\theta_W \simeq 0.231\ ,\ \alpha_s \simeq 0.119\ ,\ \alpha_{\rm EM} \simeq 1/128 \Rightarrow$$

$$\alpha_3^{-1} \simeq 8.40\ ,\ \alpha_2^{-1} = \alpha_{\rm EM}^{-1}\sin^2\theta_W \simeq 29.6\ ,\ \alpha_1^{-1} = \frac{3}{5}\alpha_2^{-1}\cot^2\theta_W \simeq 59.1$$

and we end up (up to rather small exp. errors) to $\mathrm{B}_{\rm exp} \simeq 0.72$.

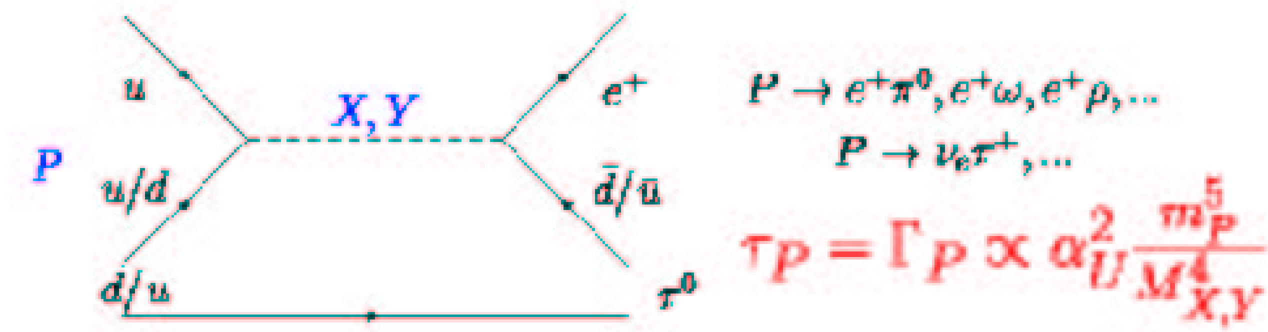
No real unification of the three gauge couplings, and by far!!!



Alternative view: couplings do not meet at a single point near $\mathrm{M}_{\rm GUT}$.

Problems of GUTs: proton decay

P decay occurs via exchange of the heavy SU(5) gauge bosons X,Y:



– Compute the effective 4–fermion interaction (CKM... dependent).

– Run down vertices from high scale $\mathrm{M_{X,Y} \sim M_{GUT}}$ to $\mathrm{m_P}$.

– Calculate hadronic ME of the 4–fermion operator (model dep..).

With the input GUT scale from $\mathrm{g_i's}$, $\mathrm{M_{GUT} \sim 10^{15}}$ GeV, one has:

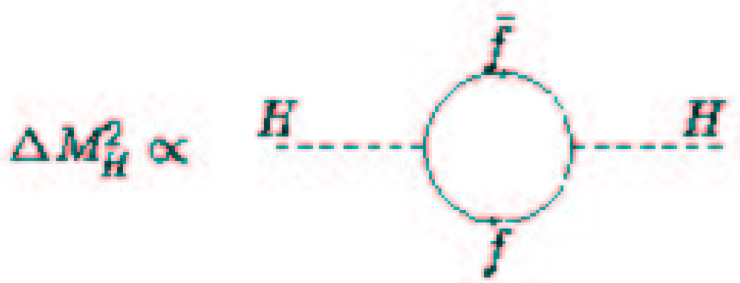
$$\tau_{\mathbf{P}}^{\rm non-SUSY\ GUT} = \mathbf{10^{30\pm1.7}}\ \rm years$$

To be compared to $\tau_{\mathbf{P}}^{\rm exp} \gtrsim 10^{33}\ \rm years$: P decay is far too fast!!!

Problem of GUTs: the hierarchy problem

Radiative corrections to the Higgs boson mass in the SM

Let us first consider the fermion loop contribution to $\mathbf{M_H^2}$



Using a cut–off Λ (see excercises later) one obtains:

$$\mathbf{\Delta M_H^2 = N_f \frac{\lambda_f^2}{8\pi^2}\left[-\Lambda^2 + 6m_f^2 \log\frac{\Lambda}{m_f} - 2m_f^2\right] + \mathcal{O}(1/\Lambda^2)}$$

We have thus a quadratic divergence, $\mathbf{\Delta M_H^2 \sim \Lambda^2}$.

Divergence is independent of $\mathbf{M_H}$, and does not disappear if $\mathbf{M_H = 0}$:

The choice $\mathbf{M_H = 0}$ does not increase the symmetry of $\mathcal{L}_{\rm SM}$.

If we fix the cut–off Λ to $\mathbf{M_{GUT}}$ or $\mathbf{M_P}$: $\Rightarrow \mathbf{M_H} \sim 10^{14}$ to 10^{17} GeV!

Problem of GUTs: the hierarchy problem

The Higgs boson mass prefers to be close to the very high scale:

This is the hierarchy problem.

But we want a light Higgs ($M_H \lesssim 1$ TeV) for unitarity etc... reasons.

We need thus to make: $M_H^2|^{\rm Physical} = M_H^2|^0 + \Delta M_H^2$+ counterterm

And adjust this counterterm with a precision of 10^{-30} (30 digits)!

This is the naturalness problem.

In a complete theory, no problem formally: we adjust the bare M_H and counterterm which are infinite, to have the physical finite mass.

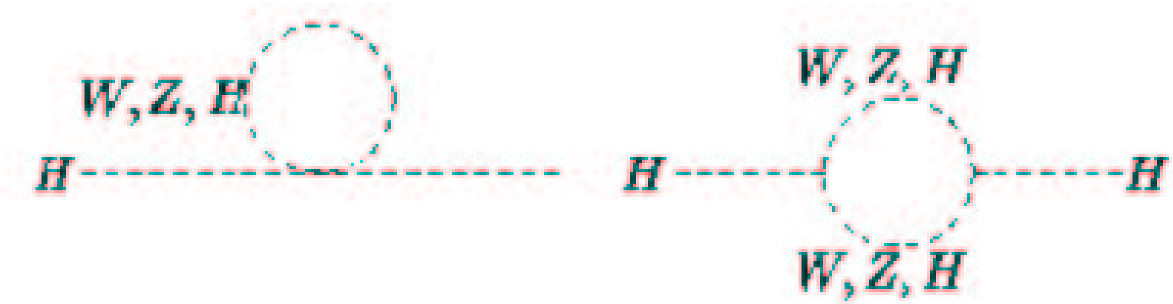
This is the case of the log divergence of m_e in QED for instance (the photon mass in QED is protected by gauge invariance....).

However, we want to give a physical meaning to the cut–off, $\Lambda = \Lambda_{\rm new}$, and the logarithmic and quadratic divergences are of different nature.

Problem of GUTs: the hierarchy problem

In the complete Standard Model:

besides the fermion loops, there are also contributions to M_H from the massive gauge bosons and from the Higgs boson itself:



Total contributions of fermions and bosons in the SM at one–loop:

$$\Delta M_H^2 \propto [3(M_W^2 + M_Z^2 + M_H^2)/4 - \sum m_f^2](\Lambda^2/M_W^2)$$

We can adjust the unknown M_H so that the quadratic divergence disappears (would be a prediction for Higgs mass, $M_H \sim 200$ GeV).

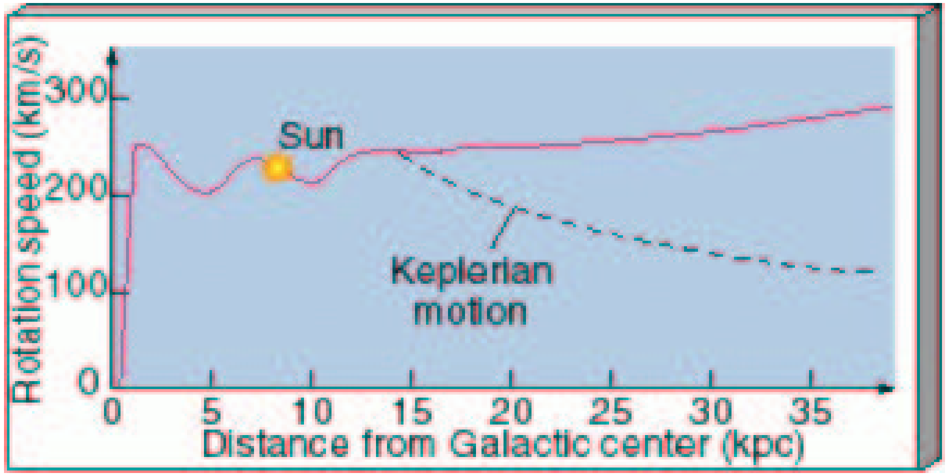
However: does not work at two–loop level or at higher orders....

Summary: the problem of the quadratic divergences to M_H is there.

There is no symmetry which protects M_H in the SM.

Problems of GUTs: no cold dark matter

The experimental measurement of the galaxy rotation curve:



shows that some dark matter should be present in universe.

From large structure formation: DM should be cold (non relativistic)

The WMAP satelitte has shown that there is 25% of CDM:

$$\Omega_{\rm DM}\, h^2 \simeq 0.113 \pm 0.009 \Rightarrow 0.09 \leq \Omega_{\rm DM}\, h^2 \leq 0.14 \text{ at } 99\% \text{ CL}$$

Needs a particle that fullfils the following conditions:

electrically neutral, weakly interacting, rather massive and stable!

There is no such a particle in the SM and also in non–SUSY GUTs!

SUSY–GUTs: general features

Low–energy SUSY comes to the rescue!!

SUSY-GUTs are the most attractive extensions of the SM

For each SM particle, there is a SUSY partner with spin–$\frac{1}{2}$ diff.

(there are also two Higgs doublets, see the discussions later).

Some theoretical advantages of SUSY theory, are:

- **It is the largest symmetry that an S–matrix can have**

(so, we go further in our quest for/use of symmetries in Nature..).

- **If SUSY gauged, we get a spin–$\frac{3}{2}$ gravitino and spin–2 graviton**

(and therefore can include gravitational interaction: conceptual SM pb!).

- **It is a natural part of Superstrings (theory of everything!?).**

If SUSY is realized at low energies (which means $\mathrm{M_{SUSY}} \sim 1$ TeV),

it solves the problems discussed in the case of non–SUSY GUTs!!!

Let us take the example of the MSSM, for instance (details later):

SUSY–GUTs: gauge coupling unification

The new SUSY particles will contribute to the running of $g_{1,2,3}$

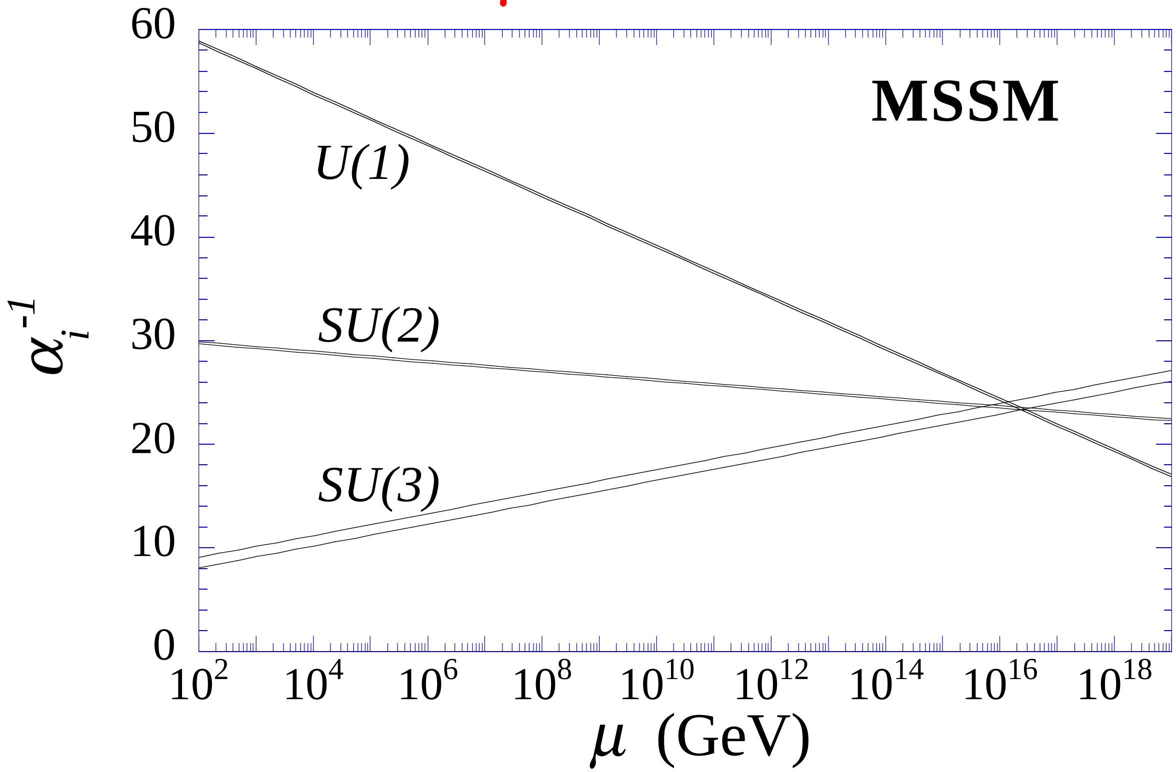
(one has to add a contribution of $b_N = -2N/3$ for gauginos)

$$b_3^{\rm MSSM} = (11 - 2\frac{6}{3} - 2 - \frac{12}{6} = 7 - 5 = 3$$

$$b_2^{\rm MSSM} = (\frac{22}{3} - 4 - \frac{1}{6} + (-\frac{4}{3} - \frac{12}{6} - \frac{2}{3} - \frac{1}{6}) = \frac{19}{6} - \frac{25}{6} = -1$$

$$b_1^{\rm MSSM} = -3\frac{10}{5}(f + \tilde{f}) - \frac{3}{5}4\frac{1}{4}(H + \tilde{h}) = -\frac{33}{5}$$

we get after calculation $B_{\rm th} = \frac{5}{7} = 0.74$ compared to $B_{\rm exp} \simeq 0.72$!



SUSY–GUTs: gauge coupling unification

Alternative view: the running couplings meet at a single point $\mathrm{M_U}$

obtained from $\log(\mathrm{M_U}/\mathrm{M_Z}) = \frac{10\pi}{28}\left[\alpha_1^{-1}(\mathrm{M_Z}) - \alpha_2^{-1}(\mathrm{M_Z})\right] \simeq 33.1$

$$\Rightarrow \mathrm{M_U} \sim 2 \cdot 10^{16}\ \mathrm{GeV}$$

Note that the small discrepancy is due to experimental errors on α_i

and also to small threshold corrections (new particles) near $\mathrm{M_U}$;

two loop corrections must be also included for more precision...

Important: for this to work, we need $\mathrm{M_{SUSY}} = \mathcal{O}(1\ \mathrm{TeV})$!!!

Note also that larger $\mathrm{M_U}$ is good to prevent proton decay:

- **In non–SUSY GUT, $\mathrm{M_{GUT}} \sim 10^{15}$ GeV, 10 times smaller...**
- **In SUSY–GUTs: additional colored Higgs/higgsino exchange**

$$\tau_{\mathbf{P}}^{\mathrm{SUSY}} \propto 1/\mathrm{M_{GUT}^4} > 10^{33}\ \text{years}$$

Larger but very close to experimental bound; to be observed soon?

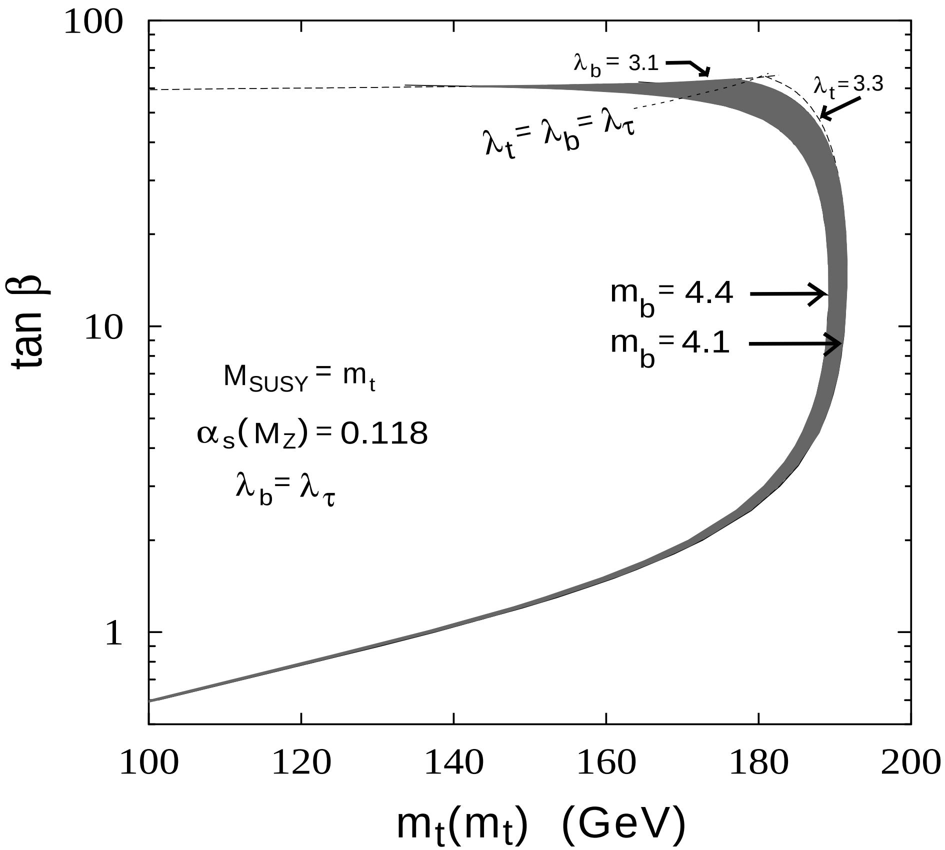
SUSY–GUTs: Yukawa coupling unification

One can also unify the (3d gen.) Yukawa couplings at M_{GUT}

$$\frac{dh_t}{d\log Q} = \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$

$$\frac{dh_b}{d\log Q} = \frac{h_b}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right],$$

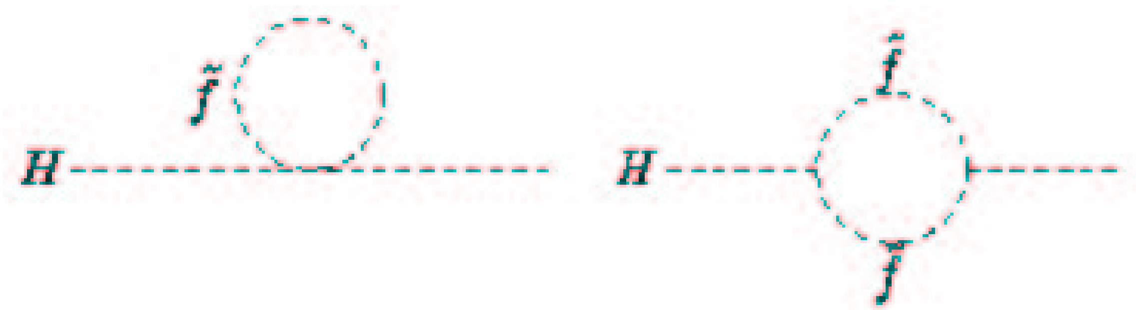
$$\frac{dh_\tau}{d\log Q} = \frac{h_\tau}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right].$$



SUSY–GUTs: the hierarchy problem

In SUSY, we need to add contributions of the new particles to ΔM_H^2

Ex: in the case of fermion loop, add the contributions of (2) scalars



Again, quadratic divergences for scalars (see later). But assume:

- **Scalar couplings related to fermion couplings:** $\lambda_f^2 = -\lambda_S$.
- **Multiplicative factors are the same:** $N_S = N_f$ **(nb: 2 scalars).**
- **To simplify, the scalars have the same mass:** $m_1 = m_2 = m_S$.

and add the fermionic and scalar contributions; we get:

$$\Delta M_H^2|^{\rm tot} = \frac{\lambda_f^2 N_f}{4\pi^2}\left[(m_f^2 - m_S^2)\log\left(\frac{\Lambda}{m_S}\right) + 3m_f^2\log\left(\frac{m_S}{m_f}\right)\right]$$

SUSY–GUTs: the hierarchy problem

The quadratic divergences have disappeared in the sum!!

(the same job can be done for (s)contributions of W,Z,H etc..).

Logarithmic divergence still there, but contribution small.

No divergences at all if in addition $m_S = m_f$ (exact SUSY)!

$\Rightarrow$ Symmetry fermions–scalars $\rightarrow$ no divergence in Λ^2

"Supersymmetry" no divergences at all: M_H is protected!

Note that if $M_S \gg 1\ \mathrm{TeV}$ the fine tunning problem is back!!!

In summary:

– gauge symmetry protects the gauge boson masses

– chiral symmetry protects the fermion masses

– SUSY is needed to protect the scalar masses.

We need SUSY at low energies, $M_{SUSY} = \mathcal{O}(1\ \mathrm{TeV})$!

(also to solve the gauge unification and Dark Matter problems..).

SUSY–GUTs: Dark Matter

- In minimal SUSY models, there is a symmetry called R–parity which makes that the lightest SUSY particle is absolutely stable.
- Experimental constraints show that this LSP must be quite heavy, $M_{\rm LSP} \gtrsim 10$–50 GeV, and thus is non–relativistic (cold dark matter).
- In most areas of the SUSY parameter space, this LSP is electrically neutral and it interacts very weakly.
- For some values of its mass and couplings, this LSP can have a relic abundance which is within the range given by WMAP data.

The LSP is an ideal candidate for Dark Matter!

Again, for this to work, the particle must be lighter than $\mathcal{O}(\rm TeV)$!

At least 3 good reasons to believe in SUSY!

in addition to SUSY being part of superstrings and link with gravity!

Supersymmetry: Basics

Here, we give only basic facts needed later in phenomenology discussion

For details on theoretical issues, see the lectures of Sven Heinemeyer.

SUSY: a symmetry relating scalars/vector bosons and fermions.

SUSY generators $\mathcal{Q}$ transform fermions into bosons and vice–versa:

$$\mathcal{Q}|\mathrm{Fermion}> \; |\mathrm{Boson}> \;\; , \;\; \mathcal{Q}|\mathrm{Boson}> \; |\mathrm{Fermion}>$$

$\mathcal{Q}$ must be an anti–commuting (complicated) object.

$\mathcal{Q}^\dagger$ is also a distinct symmetry generator:

$$\mathcal{Q}^\dagger|\mathrm{Fermion}> \; |\mathrm{Boson}> \;\; , \;\; \mathcal{Q}^\dagger|\mathrm{Boson}> \; |\mathrm{Fermion}>$$

Such theories are highly restricted [e.g., no go theorem, see SH]

and in a 4-dimension theory with chiral fermions [as in the SM]:

$\mathcal{Q}, \mathcal{Q}^\dagger$ carry spin–$\frac{1}{2}$ with L,R helicities and they should obey....

Supersymmetry: Basics

.... The SUSY algebra: (which schematically is given by)

$$\{\mathcal{Q}, \mathcal{Q}^\dagger\} = P^\mu \,,\ \{\mathcal{Q}, \mathcal{Q}\} = 0 \,,\ \{\mathcal{Q}^\dagger, \mathcal{Q}^\dagger\} = 0,$$

$$[P^\mu, \mathcal{Q}] = 0, [P^\mu, \mathcal{Q}^\dagger] = 0, [T^a, \mathcal{Q}] = 0, [T^a, \mathcal{Q}^\dagger] = 0$$

$\mathbf{P}^\mu$**: generator of space–time transformations.**

$\mathbf{T^a}$ **generators of internal (gauge) symmetries.**

$\Rightarrow$ **SUSY: unique extension of the Poincaré group of space–time symmetry to include a four–dimensional Quantum Field Theory...**

Single–particle states of the theory are in irreducible representations of the SUSY algebra above, which are called supermultiplets.

Fermions and bosons of same supermultiplet are superpartners.

They must have the same mass and gauge quantum numbers.

Three types of supermultiplets are needed...

Supersymmetry: Supermultiplets and Superpartners

- **Chiral (or "scalar") supermultiplet** (ζ with $\zeta^c = \zeta$ and S):

– 1 two–component Weyl fermion with spin $\pm\frac{1}{2}$ ($\mathbf{n_F = 2}$)

– 2 real spin–0 scalar = 1 complex scalar ($\mathbf{n_B = 2}$)

- **Gauge (or "vector") supermultiplet** ($\mathbf{A_\mu^a}$ and $\lambda_\mathbf{A}$):

– 1 two–component Weyl gaugino–fermion with spin $\pm\frac{1}{2}$ ($\mathbf{n_F = 2}$)

– 1 real spin–1 masseless gauge vector boson ($\mathbf{n_B = 2}$)

- **Gravitational supermultiplet:**

– 1 two–component Weyl gravitino–fermion with spin $\pm\frac{3}{2}$ ($\mathbf{n_F = 2}$)

– 1 real spin–2 masseless graviton ($\mathbf{n_B = 2}$)

Ex: $\Psi = \binom{\mathrm{e_L}}{\mathrm{e_R}}$ with $\mathrm{e_{L/R}}$ being 2–component Weyl LH/RH fermions

Each state has a complex spin–0 superpartner noted $\tilde{\mathrm{e}}_\mathrm{L}$ and $\tilde{\mathrm{e}}_\mathrm{R}$.

One can define $\mathrm{e} \equiv \mathrm{e_L}$ and $\bar{\mathrm{e}} = \mathrm{e_R^\dagger}$ so that one has:

two LH chiral supermultiplets for the electron: $(\mathrm{e}, \tilde{\mathrm{e}}_\mathrm{L}), (\bar{\mathrm{e}}, \tilde{\mathrm{e}}^*_\mathrm{R})$.

The same for all other leptons and quarks (except for massless ν_L).

Supersymmetry: Interactions

- All fields involved have the canonical kinetic energies

$$\mathcal{L}_{\rm kin} = \Sigma_i\{(D_\mu S_i^*)(D^\mu S_i) + i\overline{\psi}_i D_\mu \gamma^\mu \psi_i\} + \Sigma_a\{-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{i}{2}\overline{\lambda}_a D$$

with D the covariant derivative. [Note that $\psi(\lambda)$ have 4(2) comps.]

- The interactions are specified by SUSY and gauge invariance:

$$\mathcal{L}_{\rm int.\ scal-fer.-gauginos} = -\sqrt{2}\sum_{i,a} g_a \left[S_i^* T^a \overline{\psi}_{iL} \lambda_a + {\rm h.c.} \right]$$

$$\mathcal{L}_{\rm int.\ quartic\,scal.} = -\frac{1}{2}\sum_a \left(\sum_i g_a S_i^* T^a S_i \right)^2$$

- All interactions are given by the gauge coupling constants $g_{1,2,3}$

(fundamental prediction of SUSY: same g in gauge and Yukawa int.)

- At this stage, a very simple and minimal theory:

Everything is completely specified and no adjustable parameter!

Supersymmetry: Superpotential

Only freedom: choice of Superpotential W (SUSY and gauge inv!).

It gives the scalar potential and Yukawa interactions (fer.–scal.).

– $\mathbf{W} \equiv$ function of the superfields $\mathbf{z_i}$ only (not $\mathbf{z_i^*}$!).

– Analytic function: no derivative interaction.

– Renormalizability: only terms of dimension 2 and 3.

$$\Rightarrow \mathcal{L}_{\mathbf{W}} = -\sum_{\mathbf{i}} \left| \frac{\partial \mathbf{W}}{\partial \mathbf{z_i}} \right|^{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}} \sum_{\mathbf{ij}} \left[\overline{\psi}_{\mathbf{iL}} \frac{\partial^{\mathbf{2}} \mathbf{W}}{\partial \mathbf{z_i} \partial \mathbf{z_j}} \psi_{\mathbf{j}} + \mathrm{h.c.} \right]$$

To obtain the interactions explicitly: take $\partial \mathbf{W} / \partial \mathbf{z_i} |_{\mathbf{z_i} = \mathbf{S_i}}$.

The SUSY tree–level scalar potential is $\mathbf{V}_{\mathrm{tree}} = \mathbf{V_F} + \mathbf{V_D}$.

- **F–terms from W through derivatives wrt all scalars $\mathbf{S_i}$:**

$$\mathbf{V_F} = \sum_{\mathbf{i}} \mathbf{F_i F_i^*} = \sum_{\mathbf{i}} |\mathbf{W^i}|^{\mathbf{2}} \ \mathrm{with}\ \mathbf{W^i} = \partial \mathbf{W} / \partial \mathbf{S_i}$$

- **D–terms corresponding to the U(1),SU(2),SU(3) gauge groups:**

$$\mathbf{V_D} = \frac{\mathbf{1}}{\mathbf{2}} \sum_{\mathbf{i}} \mathbf{D_i D_i^*} = \frac{\mathbf{1}}{\mathbf{2}} \sum_{\mathbf{a=1}}^{\mathbf{3}} \left(\sum_{\mathbf{i}} \mathbf{g_a S_i^* T^a S_i} \right)^{\mathbf{2}}$$

Supersymmetry: SUSY–breaking

SUSY cannot be an exact symmetry since no scalars exist with the same mass as known fermions (smultiplets) $\Rightarrow$ must be broken.

Spontaneous SUSY breaking?

Means that the Lagrangian is invariant under (global) SUSY but the ground state $|0>$ is not: $\mathcal{Q}|0> \neq 0$ and $\mathcal{Q}^\dagger|0> \neq 0$.

Recall: Hamiltonian is related to the SUSY charges: $\{\mathcal{Q}, \mathcal{Q}^\dagger\} \sim \mathrm{P}^\mu$

So that one has: $<0|\mathrm{H}|0> \equiv <0|\mathrm{P^0}|0> \propto <0|\mathcal{Q}\mathcal{Q}^\dagger|0> = \mathrm{E_{vac}} \neq 0$

In fact, the vacuum energy should be positive: $\mathrm{E_{vac}} > 0$.

- **$\langle 0|D|0\rangle 0$ or D–term breaking: leads to CCB minima**

$\Rightarrow$ does not work in the MSSM!

- **$\langle 0|F|0\rangle 0$ or F–term breaking): needs a linear, $\mathrm{a_i \Phi_i}$, term in W**

$\Rightarrow$ requires a singlet sfield under $\mathrm{G_{SM}}$; not in the MSSM!

Supersymmetry: Basics

Solution: SUSY–breaking occurs in a hidden sector of particles with no (or very tiny) couplings to the visible sector of the MSSM.

If mediating interaction is flavor–blind, universal breaking terms.

Examples: gravity (mSUGRA), gauge (GMSB) mediation ...

Many breaking schemes but none is fully satisfactory at the moment:

$\Rightarrow$ Explicit breaking by hand (also with several possibilities...).

- **We need SUSY breaking at low energy to solve the problems:**

– Quadratic divergences in the Higgs sector.

– Unification of the coupling constants of $\mathrm{SU(3)_C \times SU(2)_L \times U(1)_Y}$.

– Dark Matter problem (existence of a massive stable particle), etc.

- **In the breaking, we still need to preserve: gauge invariance, renormalizability, and no quadratic divergence (soft SUSY–breaking).**

$\Rightarrow$ "Low energy SUSY" $\equiv$ effective theory at low energy.