Maria Laach, 7-10 September 2005

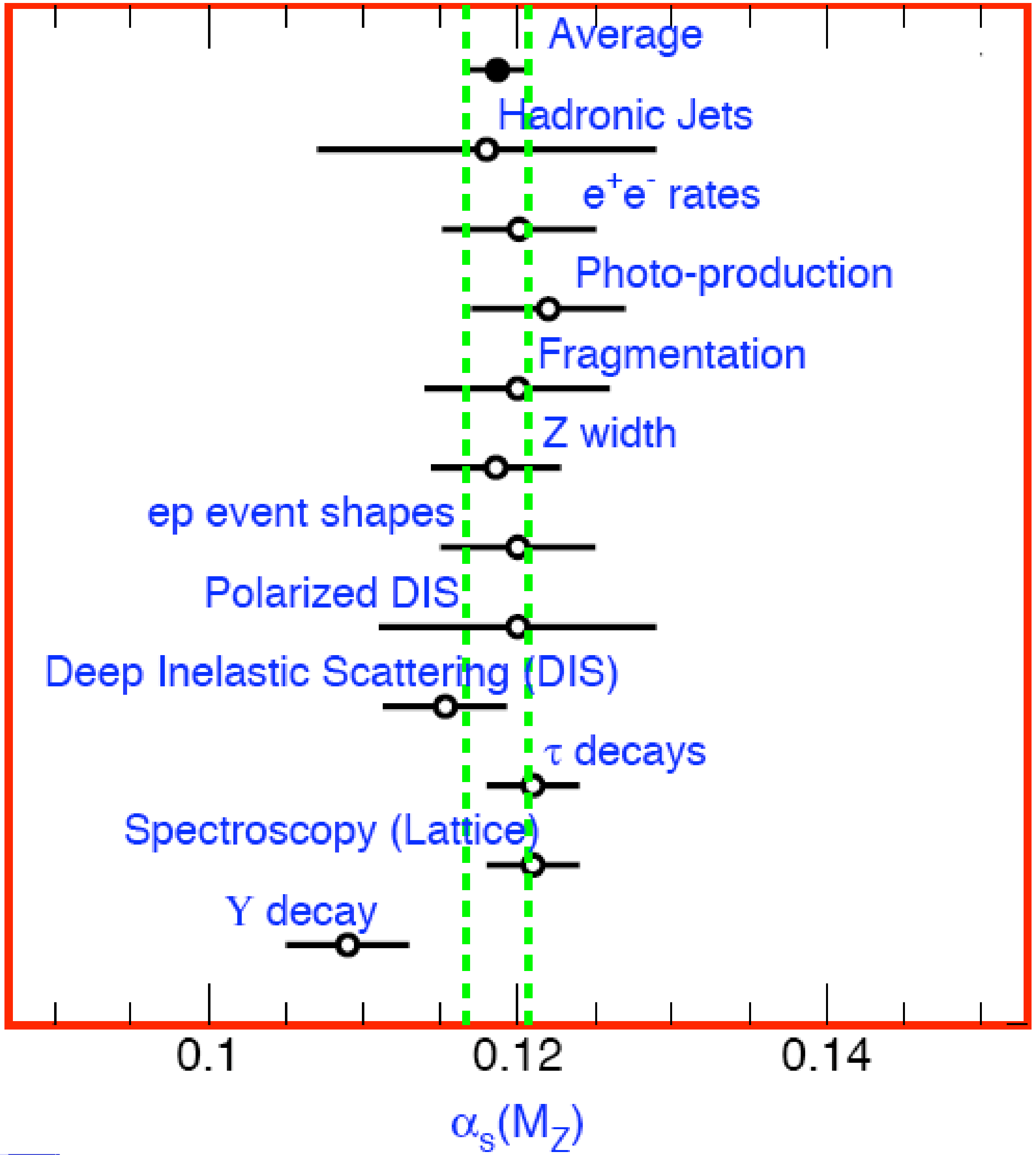
QCD

Lecture 4

G. Altarelli

Measurements of $\alpha_s(m_Z)$

PDG'04 summary on $\alpha_s(m_Z)$ $\overline{MS}$



$\alpha_s(m_Z)=0.1187\pm0.002$

$\Lambda_5 = 218\pm24$ MeV

The agreement among many different ways of measuring α_s is a strong quantitative test of QCD

The main methods for α_s at LEP/SLC are:

- inclusive Z decay, R_l, σ_l, σ_h, Γ_Z
- inclusive τ decay
- event shapes and jet rates

Inclusive:

$$R_{l,\tau} = \frac{\Gamma(Z, \tau \Rightarrow hadrons)}{\Gamma(Z, \tau \Rightarrow leptons)} \approx R^{EW}(1 + \delta_{QCD} + \delta_{NP})$$

δ_{QCD} is known to NNLO accuracy:

$$\delta_{QCD} = c_1\left(\frac{\alpha_s(Q)}{\pi}\right) + c_2\left(\frac{\alpha_s(Q)}{\pi}\right)^2 + c_3\left(\frac{\alpha_s(Q)}{\pi}\right)^3 + \ldots$$

δ_{NP} are power suppressed $(1/Q^2)^n$ terms governed by the OPE.

Here $Q = m_Z$ or m_τ

Clearly the Z case is apriori more reliable because $m_Z \gg m_\tau$.

Inclusive Z decays

(assuming the SM, m_{texp}, m_H variable):

R_l only (traditionally used for no good reason): $\alpha_s(m_Z)=0.1226\pm0.0038$
a bit large!

σ_l is more sensitive to α_s and less to m_H: $\alpha_s(m_Z)=0.1183\pm0.0030$

$$R_l = \frac{\Gamma_h}{\Gamma_l}$$

$$\Gamma_Z = (\Gamma_h + 3\Gamma_l + \Gamma_{inv})$$

$$\sigma_h = \frac{12\pi}{m_Z^2}\frac{\Gamma_l\Gamma_h}{\Gamma_Z^2}$$

$$\sigma_l = \frac{12\pi}{m_Z^2}\cdot\frac{\Gamma_l^2}{\Gamma_Z^2}$$

Better, one can use all info from R_l, Γ_Z, σ_h, σ_l ...and in general take $\alpha_s(m_Z)$ as a parameter to be fitted from the EW precision tests One obtains:

LEP1 only: $\alpha_s(m_Z)=0.1187\pm0.0027$

All EW Data: $\alpha_s(m_Z)=0.1186\pm0.0027$

In SM the dominant sources of error are m_H and higher orders in the QCD expansion. Error from power corrections very small. In addition, th. error from possible new physics.

α_s from Rτ

$$R_\tau = \frac{\Gamma(\tau \Rightarrow \nu_\tau + hadrons)}{\Gamma(\tau \Rightarrow \nu_\tau + leptons)}$$

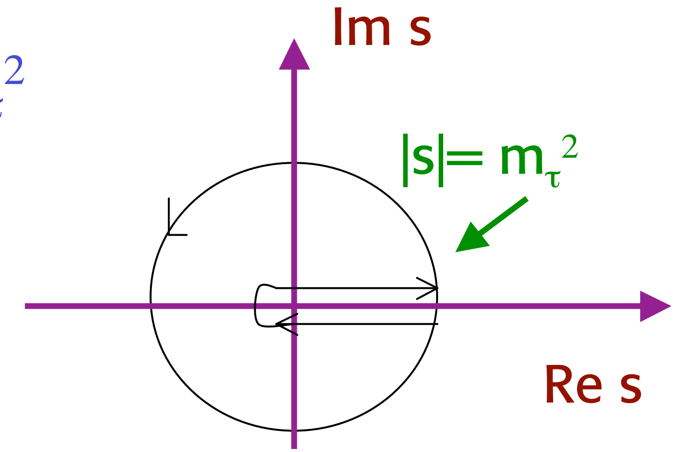
R_τ has a number of advantages that, at least in part, compensate the smallness of m_τ=1.777 GeV:

- R_τ is more inclusive than $R_{e+e-}(s)$.

$$R_\tau = \frac{1}{\pi}\int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 Im\,\Pi_\tau(s)$$

- one can use analiticity to go to $|s| = m_\tau^2$

$$R_\tau = \frac{1}{2\pi i}\oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1 - \frac{s}{m_\tau^2}\right)^2 \Pi_\tau(s)$$



- factor $(1-s/m_\tau^2)^2$ kills sensitivity to Re s= m_τ^2 (thresholds)

Still the quoted result looks a bit too precise ← higher orders, diff.procedures

PDG'04 $\alpha_s(m_Z)=0.121\pm0.0007(\mathrm{exp})\pm0.003(\mathrm{th})$

This precision is obtained by taking for granted that corrections suppressed by $1/m_\tau^2$ are negligible.

$$R\tau \sim R\tau^0[1+\delta_{pert}+\delta_{np}]$$

This is because in the massless theory:

$$\delta_{np} = \frac{ZERO}{m_\tau^2} + c_4 \cdot \frac{<O_4>}{m_\tau^4} + c_6 \cdot \frac{<O_6>}{m_\tau^6} + \ldots$$

In fact there are no dim 2 operators (e.g. $g_\mu g^\mu$ is not gauge invariant) except for light quark m^2 ($m\sim$few MeV) .

Most people believe that. I am not sure that the gap is not filled by ambiguities of $o(\Lambda^2/m_\tau^2)$ from δ_{pert}.

eg effect of ultraviolet renormalons

GA, Nason, Ridolfi '95; Chetyrkin, Narison,Zakharov '98

α_s from scaling violations in DIS

The scaling violations are clearly observed
and the (N)NLO QCD fits are remarkably good.

These fits provide

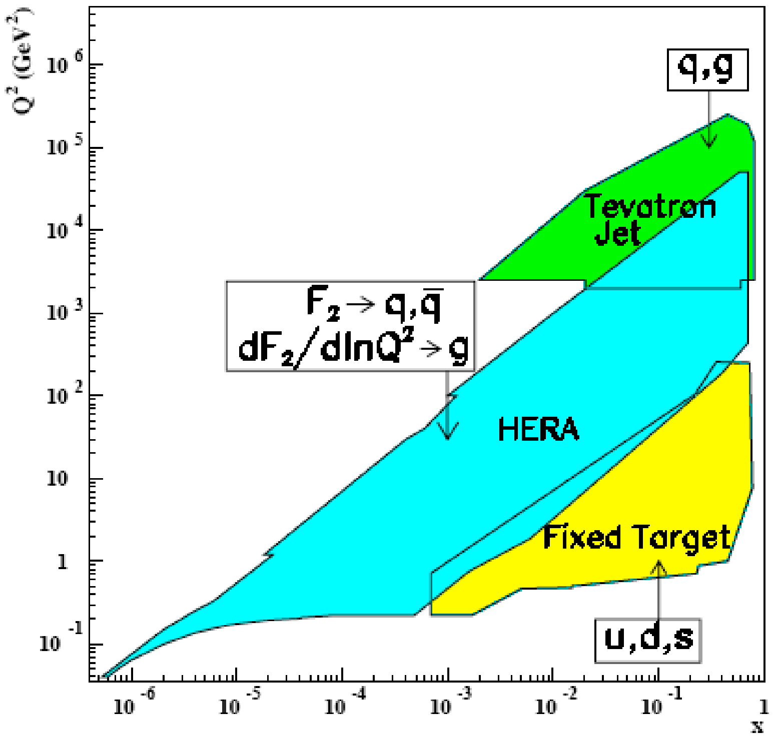
- an impressive set of QCD tests
- measurements of $q(x,Q^2)$, $g(x,Q^2)$
- measurements of $\alpha_s(Q^2)$

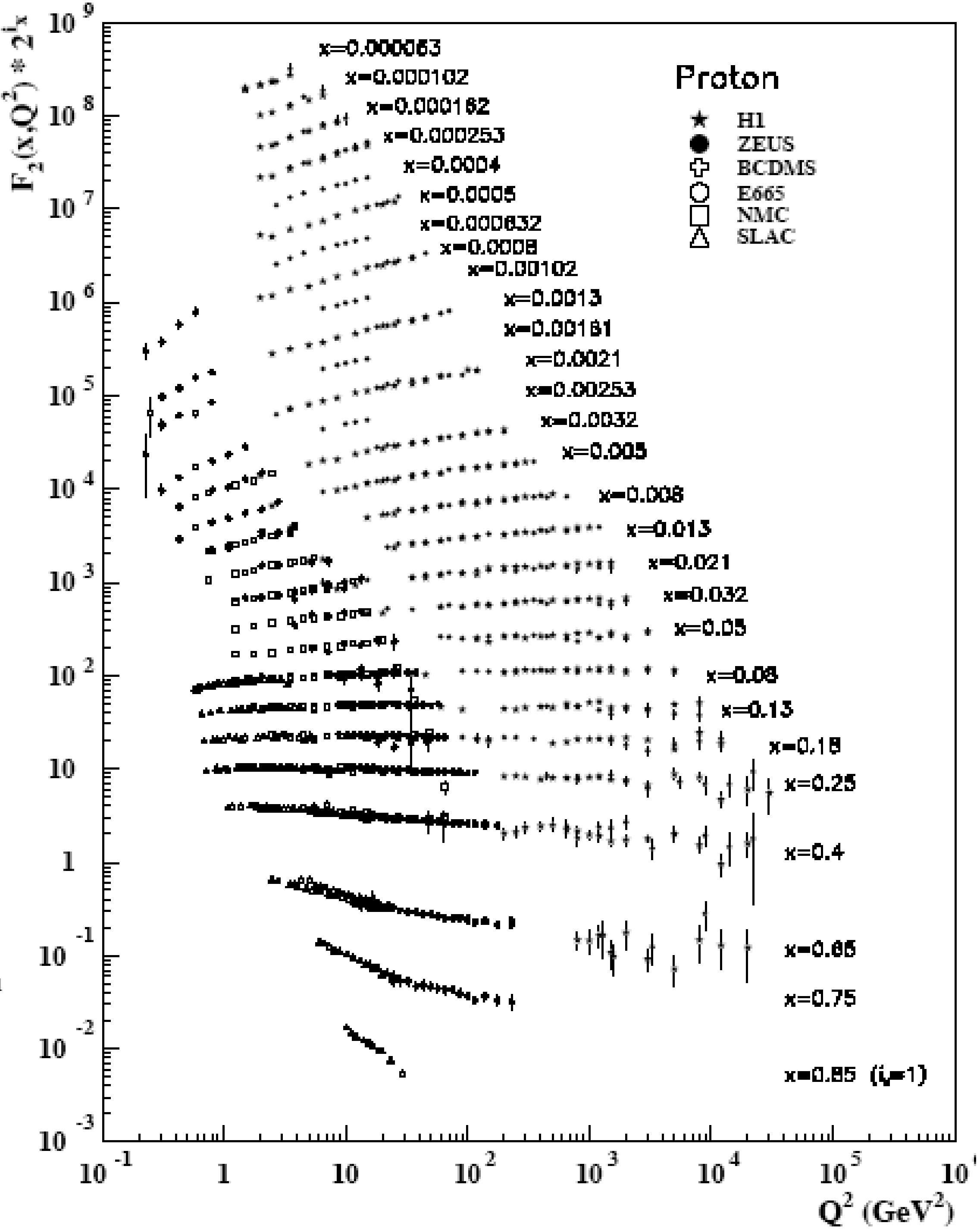
$$\frac{\partial q_i(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{q_i q_j}(y,\alpha_S)q_j(\frac{x}{y},Q^2) \; + \; P_{q_i g}(y,\alpha_S)g(\frac{x}{y},Q^2)\right\}$$

$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y}\left\{P_{g q_j}(y,\alpha_S)q_j(\frac{x}{y},Q^2) \; + \; P_{gg}(y,\alpha_S)g(\frac{x}{y},Q^2)\right\}$$

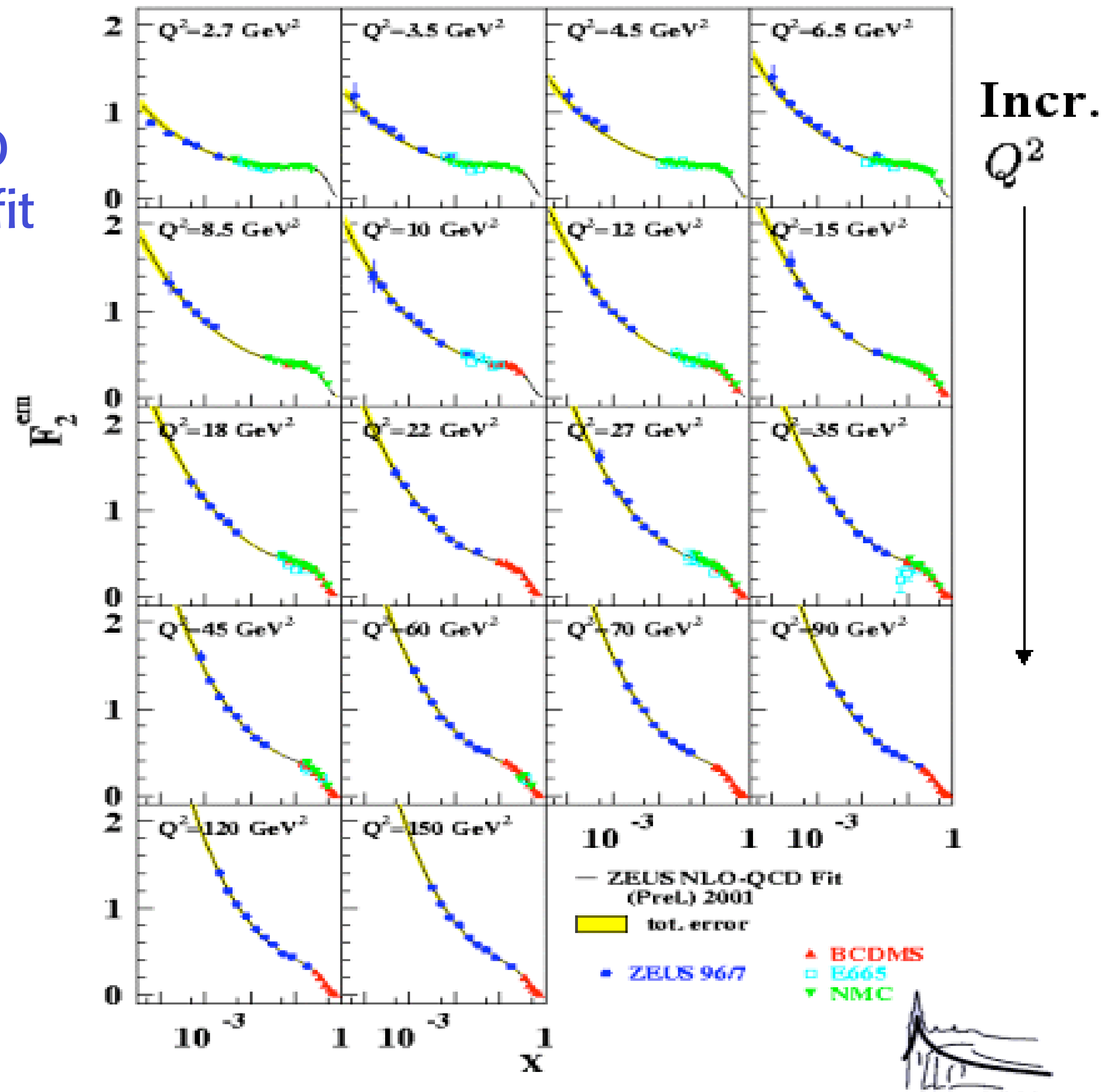
GLAP

Proton Structure Function $F_2(x,Q^2)$





Example of NLO QCD evolution fit



EPS HEP 2001

Splitting functions

For many years all splitting funct.s P have been known to NLO accuracy: $\alpha_s P \sim \alpha_s P_1 + \alpha_s^2 P_2 + \ldots\ldots$

GLAP, Floratos et al; Gonzales-Arroyo et al; Curci et al; Furmanski et al

Then the complete, analytic NNLO results have been derived for the first few moments (N<13,14).

Larin, van Ritbergen, Vermaseren+Nogueira

Finally, in 2004, the calculation of the NNLO splitting functions has been totally completed $\alpha_s P \sim \alpha_s P_1 + \alpha_s^2 P_2 + \alpha_s^3 P_3 + \ldots\ldots$

Moch, Vermaseren, Vogt

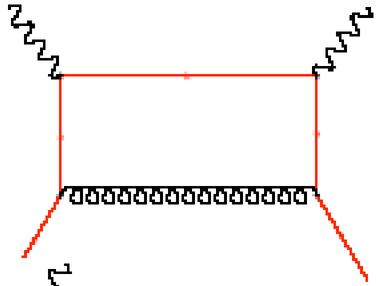
A really monumental, fully analytic, computation

The calculation (in a nut shell)

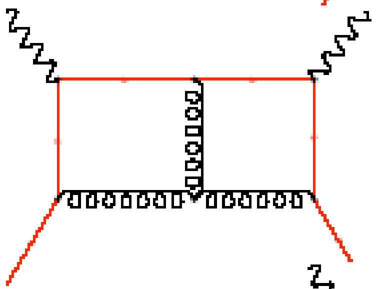
Moch

– Calculate anomalous dimensions (Mellin moments of splitting functions)
$\longrightarrow$ divergence of Feynman diagrams in dimensional regularization $D = 4 - 2\varepsilon$

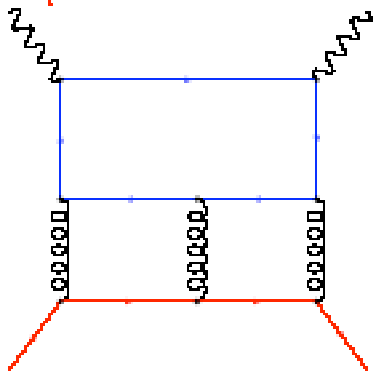
$$\gamma_{\rm ij}^{(n)}(N) = -\int_0^1 dx\, x^{N-1} P_{\rm ij}^{(n)}(x)$$



– **One-loop** Feynman diagrams
$\longrightarrow$ in total 18 for $\gamma_{\rm ij}^{(0)}$ / $P_{\rm ij}^{(0)}$
(pencil + paper)



– **Two-loop** Feynman diagrams
$\longrightarrow$ in total 350 for $\gamma_{\rm ij}^{(1)}$ / $P_{\rm ij}^{(1)}$
(simple computer algebra)



– **Three-loop** Feynman diagrams
$\longrightarrow$ in total 9607 for $\gamma_{\rm ij}^{(2)}$ / $P_{\rm ij}^{(2)}$
(cutting edge technology $\longrightarrow$ computer algebra system FORM Vermaseren '89-'04)

NLO singlet splitting functions

$$\begin{aligned}
P^{(0)}_{\rm ps}(x) &= 0 \\
P^{(0)}_{\rm qg}(x) &= 2 n_f \, p_{\rm qg}(x) \\
P^{(0)}_{\rm gq}(x) &= 2 C_F \, p_{\rm gq}(x) \\
P^{(0)}_{\rm gg}(x) &= C_A \Big(4 p_{\rm gg}(x) + \frac{11}{3} \delta(1-x) \Big) - \frac{2}{3} n_f \delta(1-x)
\end{aligned}$$

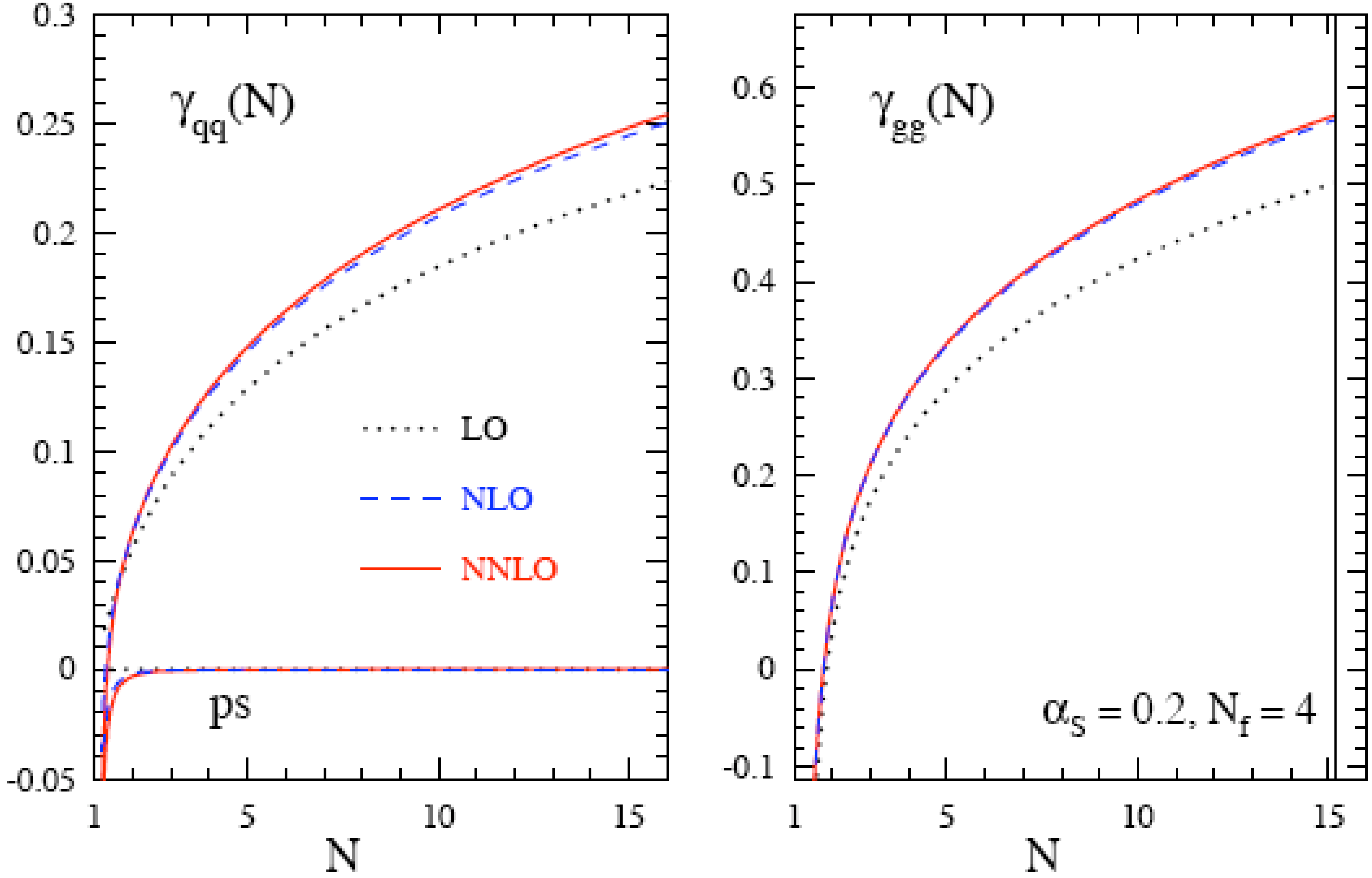
$$\begin{aligned}
P^{(1)}_{\rm ps}(x) &= 4 C_F n_f \Big(\frac{20}{9} \frac{1}{x} - 2 + 6x - 4 {\rm H}_0 + x^2 \Big[\frac{8}{3} {\rm H}_0 - \frac{56}{9} \Big] + (1+x) \Big[5 {\rm H}_0 - 2 {\rm H}_{0,0} \Big] \Big)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm qg}(x) &= 4 C_A n_f \Big(\frac{20}{9} \frac{1}{x} - 2 + 25x - 2 p_{\rm qg}(-x) {\rm H}_{-1,0} - 2 p_{\rm qg}(x) {\rm H}_{1,1} + x^2 \Big[\frac{44}{3} {\rm H}_0 - \frac{218}{9} \Big] \\
&+ 4(1-x) \Big[{\rm H}_{0,0} - 2 {\rm H}_0 + x {\rm H}_1 \Big] - 4 \zeta_2 x - 6 {\rm H}_{0,0} + 9 {\rm H}_0 \Big) + 4 C_F n_f \Big(2 p_{\rm qg}(x) \Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 \\
&- \zeta_2 \Big] + 4x^2 \Big[{\rm H}_0 + {\rm H}_{0,0} + \frac{5}{2} \Big] + 2(1-x) \Big[{\rm H}_0 + {\rm H}_{0,0} - 2x {\rm H}_1 + \frac{29}{4} \Big] - \frac{15}{2} - {\rm H}_{0,0} - \frac{1}{2} {\rm H}_0 \Big)
\end{aligned}$$

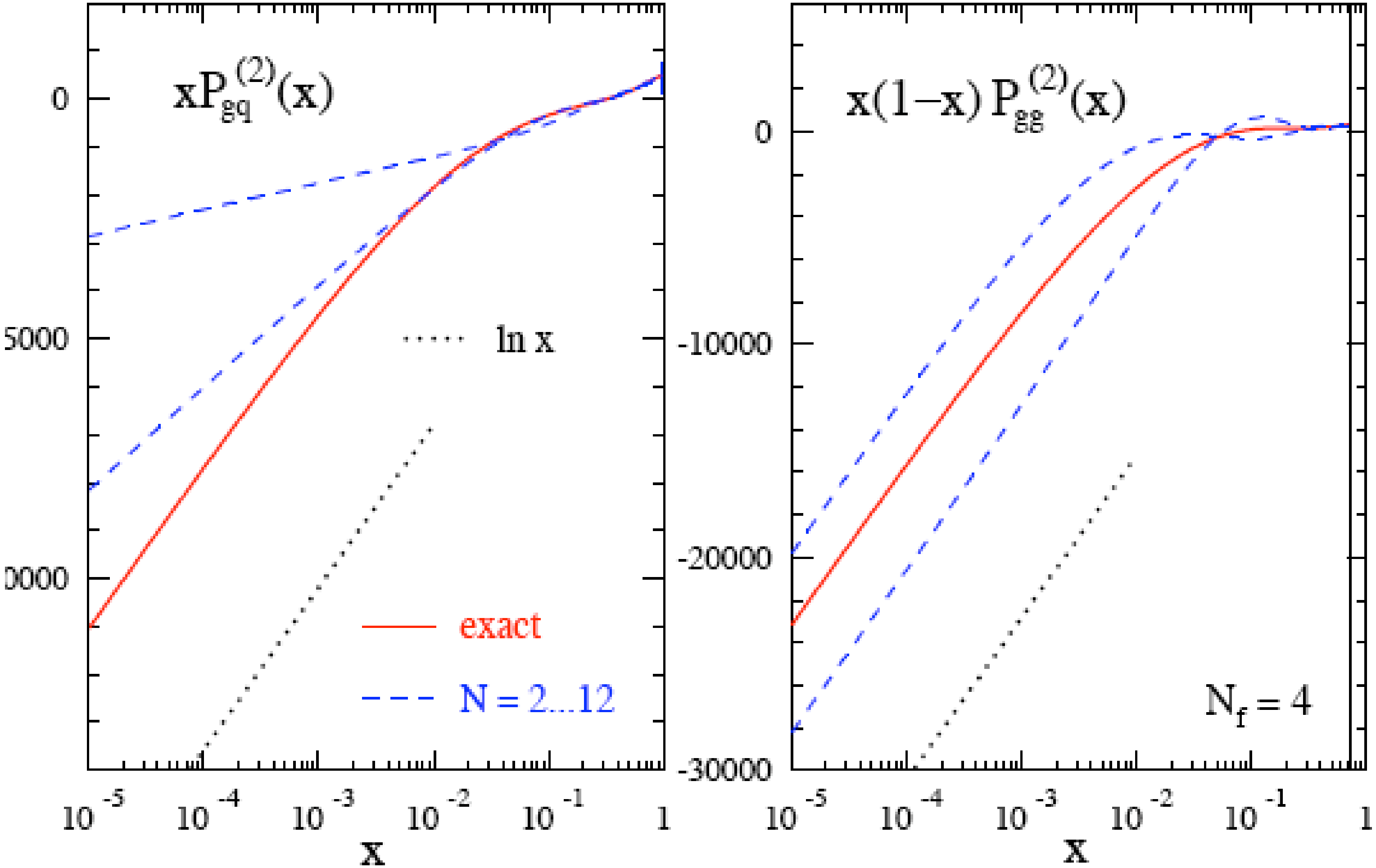
$$\begin{aligned}
P^{(1)}_{\rm gq}(x) &= 4 C_A C_F \Big(\frac{1}{x} + 2 p_{\rm gq}(x) \Big[{\rm H}_{1,0} + {\rm H}_{1,1} + {\rm H}_2 - \frac{11}{6} {\rm H}_1 \Big] - x^2 \Big[\frac{8}{3} {\rm H}_0 - \frac{44}{9} \Big] + 4 \zeta_2 - 2 \\
&- 7 {\rm H}_0 + 2 {\rm H}_{0,0} - 2 {\rm H}_1 x + (1+x) \Big[2 {\rm H}_{0,0} - 5 {\rm H}_0 + \frac{37}{9} \Big] - 2 p_{\rm gq}(-x) {\rm H}_{-1,0} \Big) - 4 C_F n_f \Big(\frac{2}{3} x \\
&- p_{\rm gq}(x) \Big[\frac{2}{3} {\rm H}_1 - \frac{10}{9} \Big] \Big) + 4 {C_F}^2 \Big(p_{\rm gq}(x) \Big[3 {\rm H}_1 - 2 {\rm H}_{1,1} \Big] + (1+x) \Big[{\rm H}_{0,0} - \frac{7}{2} + \frac{7}{2} {\rm H}_0 \Big] - 3 {\rm H}_{0,0} \\
&+ 1 - \frac{3}{2} {\rm H}_0 + 2 {\rm H}_1 x \Big)
\end{aligned}$$

$$\begin{aligned}
P^{(1)}_{\rm gg}(x) &= 4 C_A n_f \Big(1 - x - \frac{10}{9} p_{\rm gg}(x) - \frac{13}{9} \Big(\frac{1}{x} - x^2 \Big) - \frac{2}{3} (1+x) {\rm H}_0 - \frac{2}{3} \delta(1-x) \Big) + 4 {C_A}^2 \Big(27 \\
&+ (1+x) \Big[\frac{11}{3} {\rm H}_0 + 8 {\rm H}_{0,0} - \frac{27}{2} \Big] + 2 p_{\rm gg}(-x) \Big[{\rm H}_{0,0} - 2 {\rm H}_{-1,0} - \zeta_2 \Big] - \frac{67}{9} \Big(\frac{1}{x} - x^2 \Big) - 12 {\rm H}_0 \\
&- \frac{44}{3} x^2 {\rm H}_0 + 2 p_{\rm gg}(x) \Big[\frac{67}{18} - \zeta_2 + {\rm H}_{0,0} + 2 {\rm H}_{1,0} + 2 {\rm H}_2 \Big] + \delta(1-x) \Big[\frac{8}{3} + 3 \zeta_3 \Big] \Big) + 4 C_F n_f \Big(2 {\rm H}_0 \\
&+ \frac{2}{3} \frac{1}{x} + \frac{10}{3} x^2 - 12 + (1+x) \Big[4 - 5 {\rm H}_0 - 2 {\rm H}_{0,0} \Big] - \frac{1}{2} \delta(1-x) \Big) \, .
\end{aligned}$$

NNLO singlet splitting functions



$\gamma_{qq}(N)$
ps
LO
NLO
NNLO
$\gamma_{gg}(N)$
$\alpha_S = 0.2,\ N_f = 4$
N
N



– Exact result, estimates from fixed moments and leading small-x term
– Splitting function $P_{gq}^{(2)}$ (left) and $P_{gg}^{(2)}$ (right)

QCD predicts the Q^2 dependence of $F(x, Q^2)$ not the x shape. But the Q^2 dependence is related to the x shape by the QCD evolution eqs.

For each x-bin approx. a straight line in $d\log F(x, Q^2)/d\log Q^2$: the log slope.
[Q^2 span and precision of data not much sensitive to curvature]

The scaling violations of non-singlet str. functs. would be ideal: small dep. on input parton densities

$$\frac{d}{dt}\log F(x, t) = \frac{\alpha_s(t)}{2\pi}\int_x^1 dy \frac{F(y, t)}{yF(x, t)} P_{qq}\left(\frac{x}{y}\right)$$

But for F_p-F_n exp. errors add up in difference, and $F_{3\nu N}$ not terribly precise (and come essentially from only one experiment CCFR)

For xF_3, using NNLO moments for N=1,3,..,13, the following results have been derived.

Using Bernstein moments

A combination of Mellin moments which emphasizes a value of x and a given spread in order to be sensitive to the interval where the measured points are

$\alpha_s(m_Z)=0.1153\pm0.0063$

Santiago, Yndurain '01

$\alpha_s(m_Z)=0.1174\pm0.0043$

Maxwell, Mirjalili '02

Here the error from scale dep. not included (a model dep. scale fixing is chosen)

Using Mellin moments $\alpha_s(m_Z)=0.1190\pm0.0060$

Kataev, Parente, Sidorov '02

Good overall agreement. Not very precise: error ~±0.006

When one measures α_s from scaling viols. in F_2 from e or μ beams, data are abundant, exp. errors small but:

α_s ⟺ gluon correlation

Using data on p from SLAC, BCDMS, E665 and HERA, NLO kernels + NNLO for N=2,4,..,12:

$\alpha_s(m_Z)=0.1166\pm0.0013$ (!!th error?)

Santiago, Yndurain '01 [Bernstein moments]

Or using data on p from SLAC, BCDMS, NMC and HERA, NLO kernels + NNLO for N=2,4,..,12:

$\alpha_s(m_Z)=0.1143\pm0.0013(\text{exp})+\text{th error}$

Alekhin '02 [Mellin moments]

The difference in central value between these nominally most precise determinations makes clear that the total error ~±0.003

This estimate of TH errors is confirmeded by dispersion of results from other analyses

- Using data on p from BCDMS and NMC, NLO kernels, truncated moments

Moments from x_0 to 1 in measured range, coupled eqs.

$$\alpha_s(m_Z)=0.122\pm0.006$$

Forte, Latorre, Magnea, Piccione '02

- All lepton data, including HERA and CCFR, NLO evol. eqs.

$$\alpha_s(m_Z)=0.119\pm0.004$$

Martin, Roberts, Stirling, Thorne '01

- Proton data, Nachtmann moments including soft gluon resumm. at large x and estimate of higher twist

$$\alpha_s(m_Z)=0.1188\pm0.0017$$

Simula, Osipenko '02

Compare with e^+e^- ->Z -> hadrons: $\alpha_s(m_Z)=0.118\pm0.003$

Singlet splitting function at small x

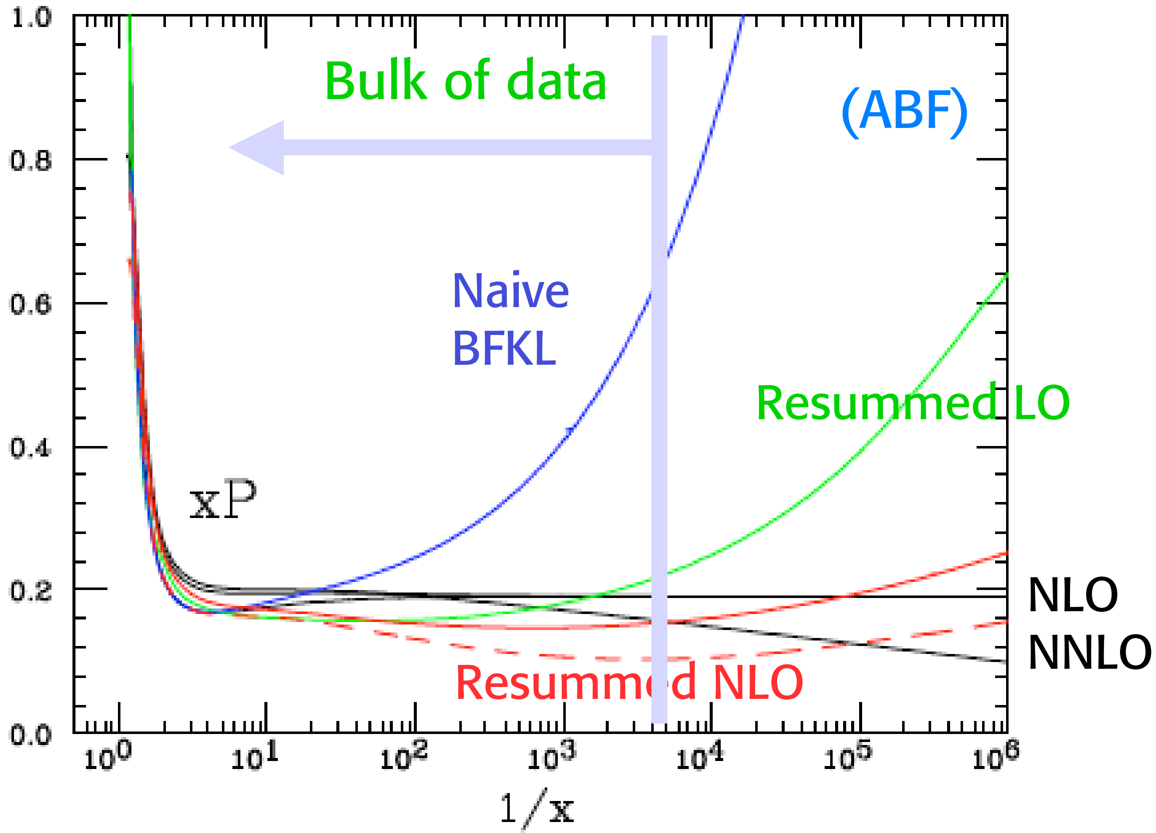
The problem of correctly including BFKL at small x has been solved

Ciafaloni, Colferai, Salam, Stasto
Altarelli, Ball, Forte (ABF)

MOMENTUM CONS.+SYMMETRY+ R.G. RESUMMATION

$\Rightarrow$ SOFT SIMPLE POLE IN ANOMALOUS DIMENSION (REGGE BEHAVIOUR)

- BFKL RISE OF SPLITTING FUNCTION TAMED BY RUNNING COUPLING
- RESUMMED RESULT CLOSE TO NLO GLAP
- PERTURBATIVE RESUMMED EXPANSION STABLE



Polarized Structure Functions

Who carries the proton spin?

$$\frac{1}{2}\Delta\Sigma + \Delta g + \Delta L_z = \frac{1}{2}$$

typically $\Delta\Sigma_{exp} \sim 0.2$

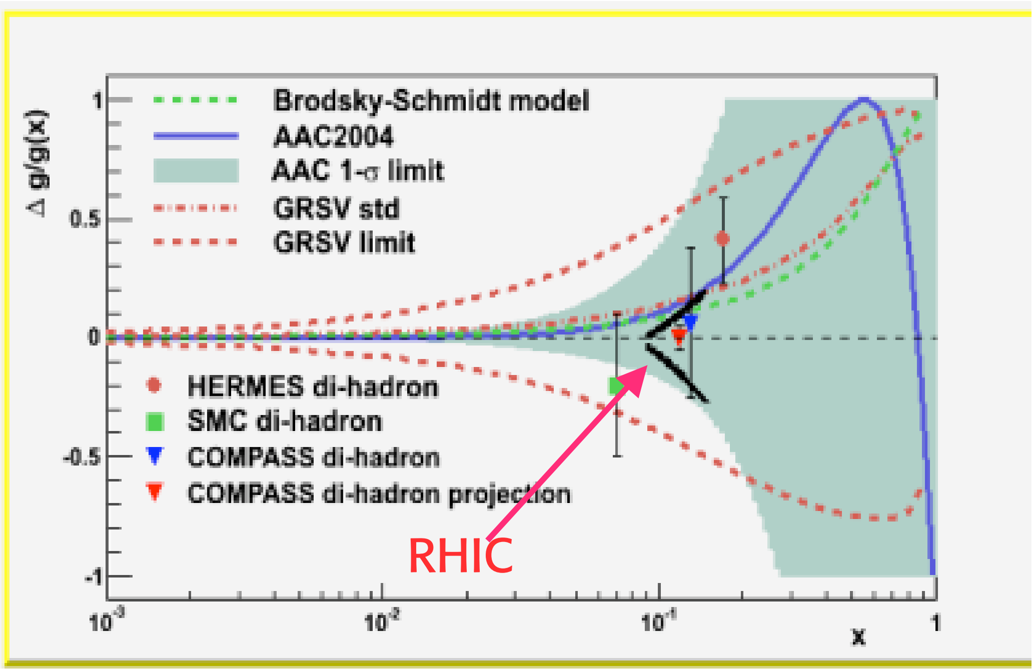
It must be either $\Delta g + \Delta L_z$ or $\Delta\Sigma$ terms at small x

recall: $\Delta g \sim \log Q^2$

x below the measured range

Δg measured indirectly from scaling violations,
directly from asymmetries, e.g. cc production

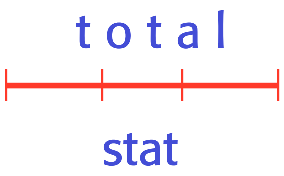
Existing direct measurements
Hermes, COMPASS, RHIC
still very crude.
No hint of large Δg.
Good future perspective at
COMPASS and RHIC

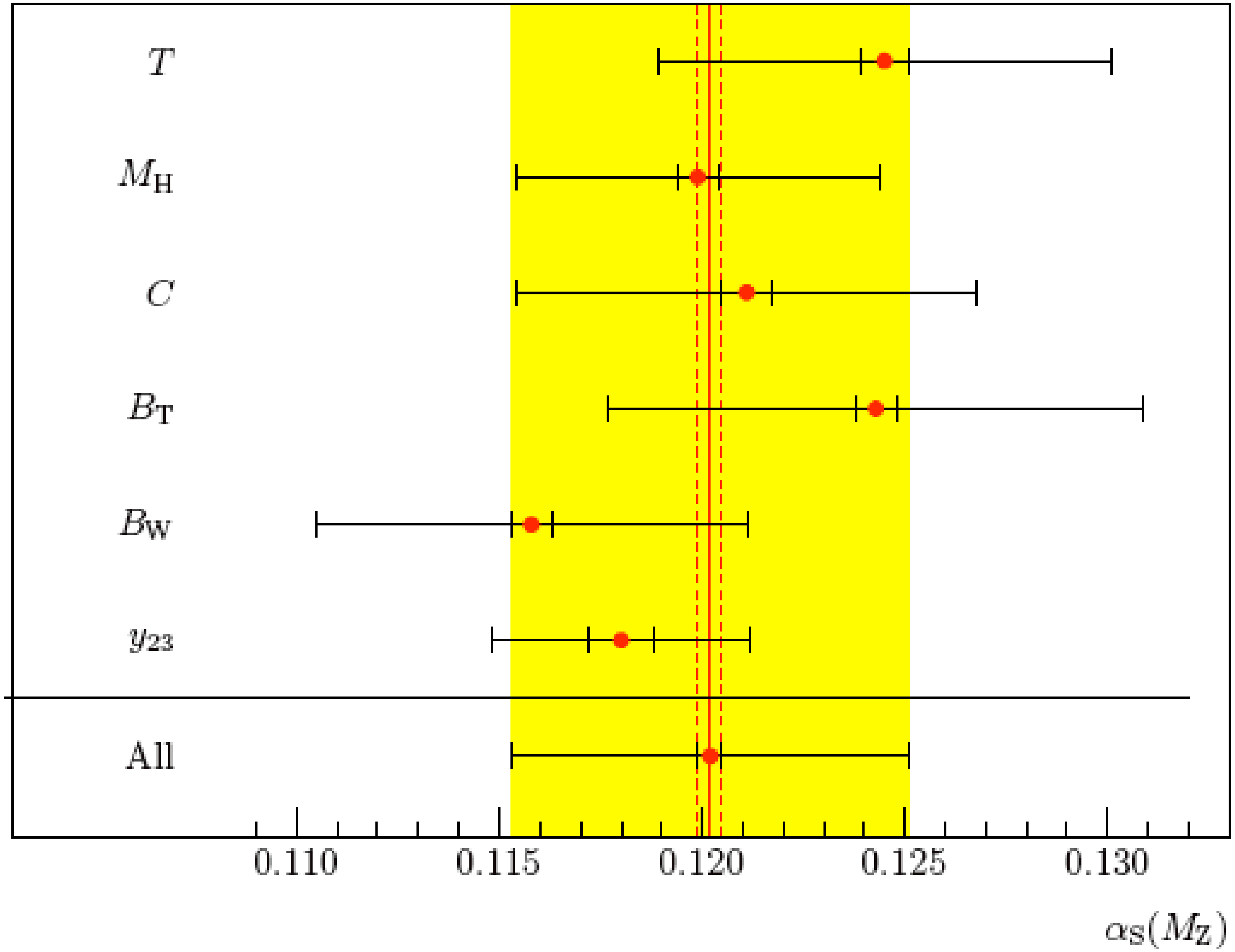


We hope the challenge of
measuring Δg well will be
soon realized

α_s from event shape and jet rates in e^+e^-

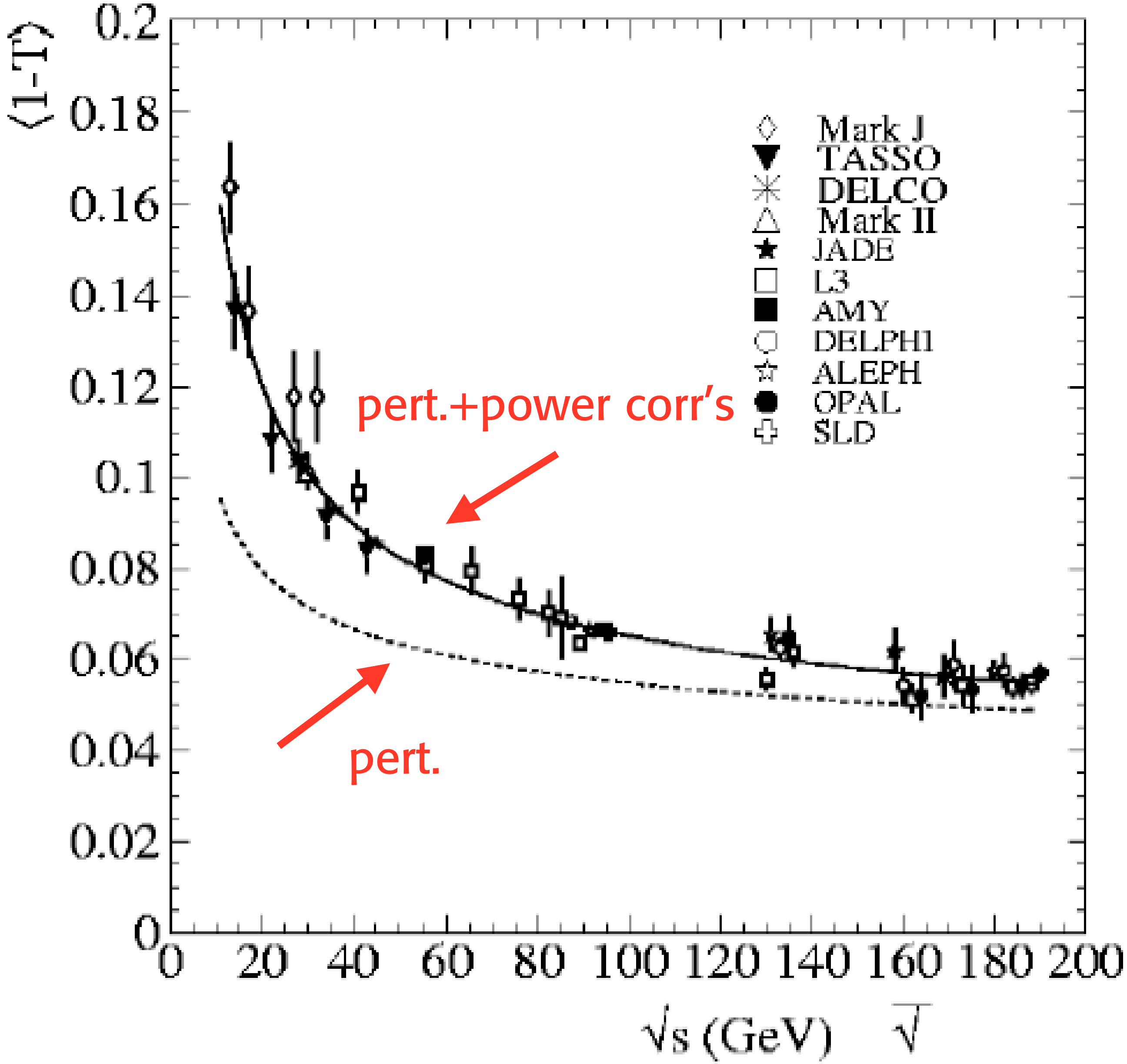
Many infrared safe observables (Thrust, Heavy jet mass,....)





Main source of error: power corr's, hadronization

Mean value of thrust vs energy



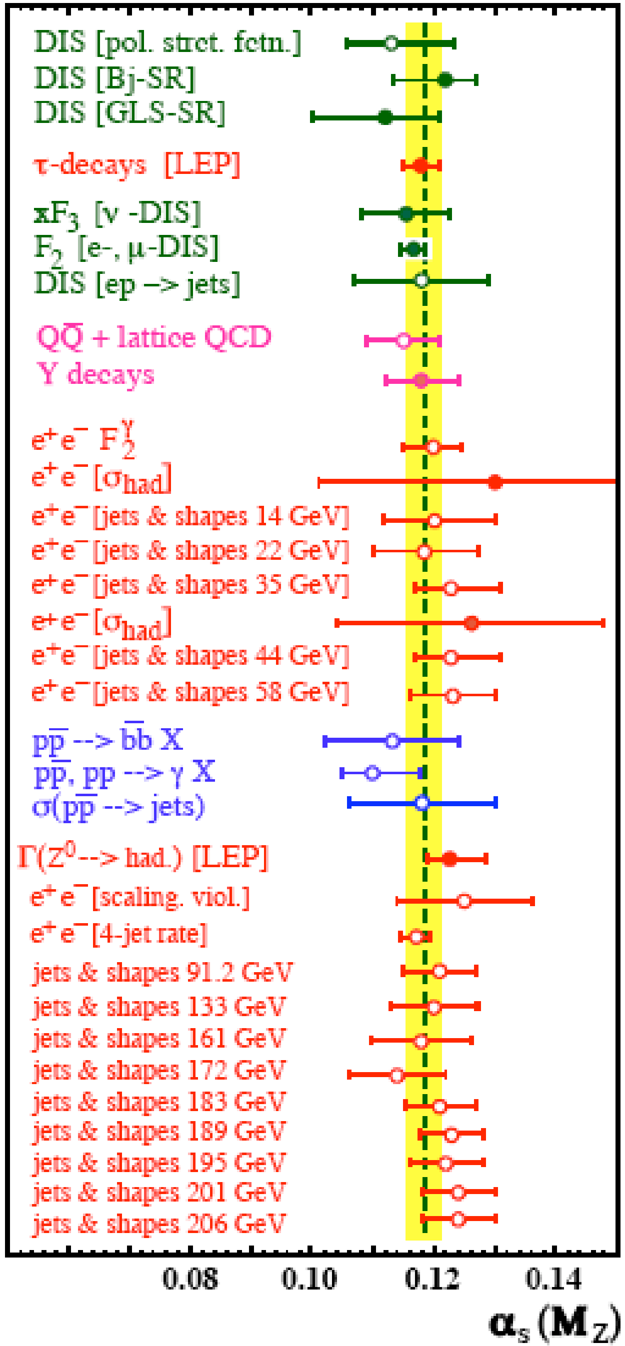
Overall combination of hadronic event shapes at LEP

$\alpha_s(m_Z)=0.120\pm0.005$

In his typical figure Bethke clearly overemphasizes jets&shapes (correlated, affected by hadronization corrections....)

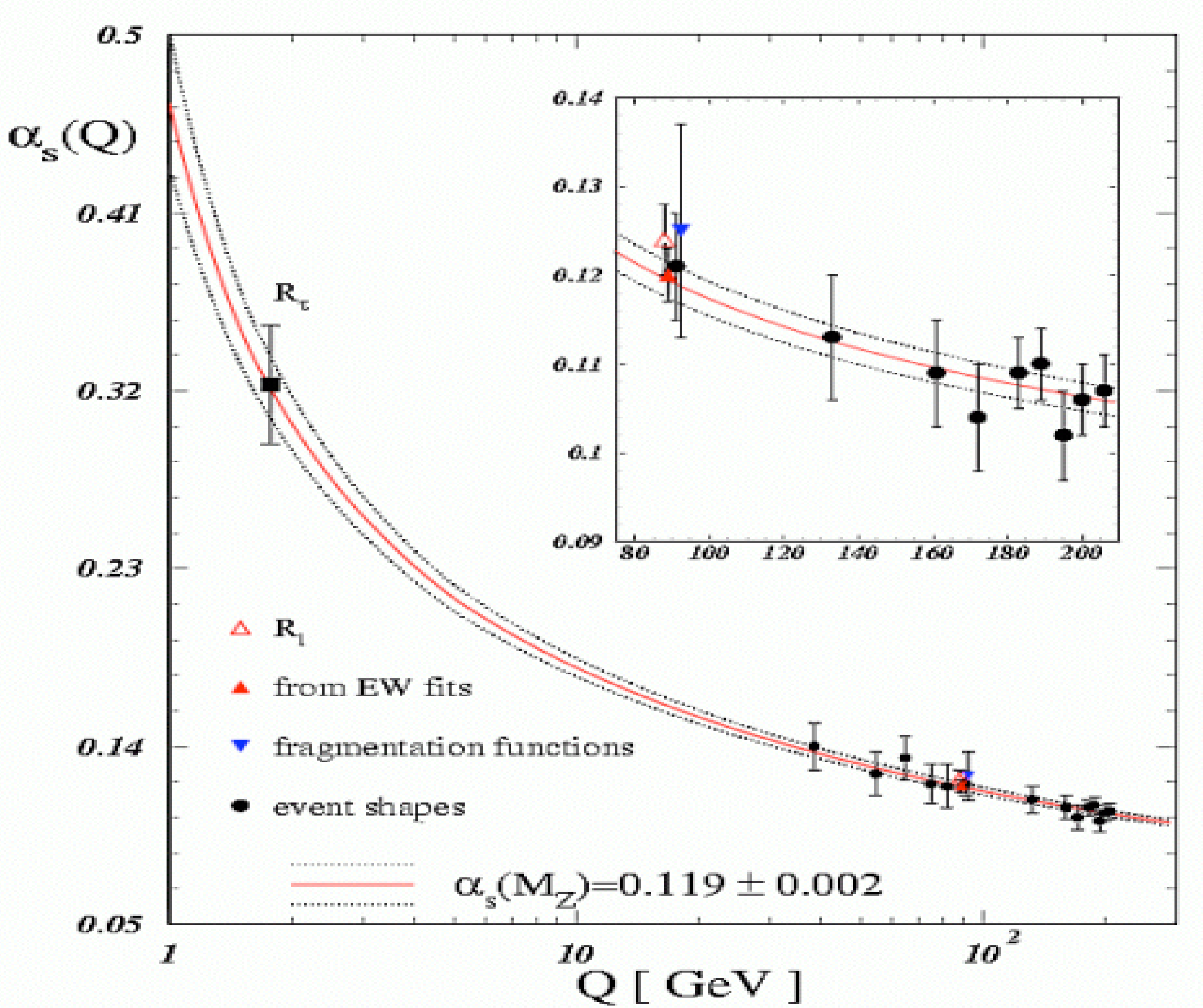
Bethke average quite in agreement with PDG:

$\alpha_s(m_Z)=0.1182\pm0.0027$



LEP Measurements

Before LEP, in '89 $\alpha_s(m_Z) \sim 0.11 \pm 0.01$



There are many powerful QCD tests beyond measurements of α_s.

- Jet rates and distributions in e^+e^-
- Jets or γ's at large p_T in pp
- W and Z cross-sections and p_T distributions in pp
- Heavy quark cross-sections and distributions in ep or pp
- Quarkonium decays

••••

Overall the agreement is spectacular

Parton densities extracted from DIS (with feedback from other hard processes) are available for further use.

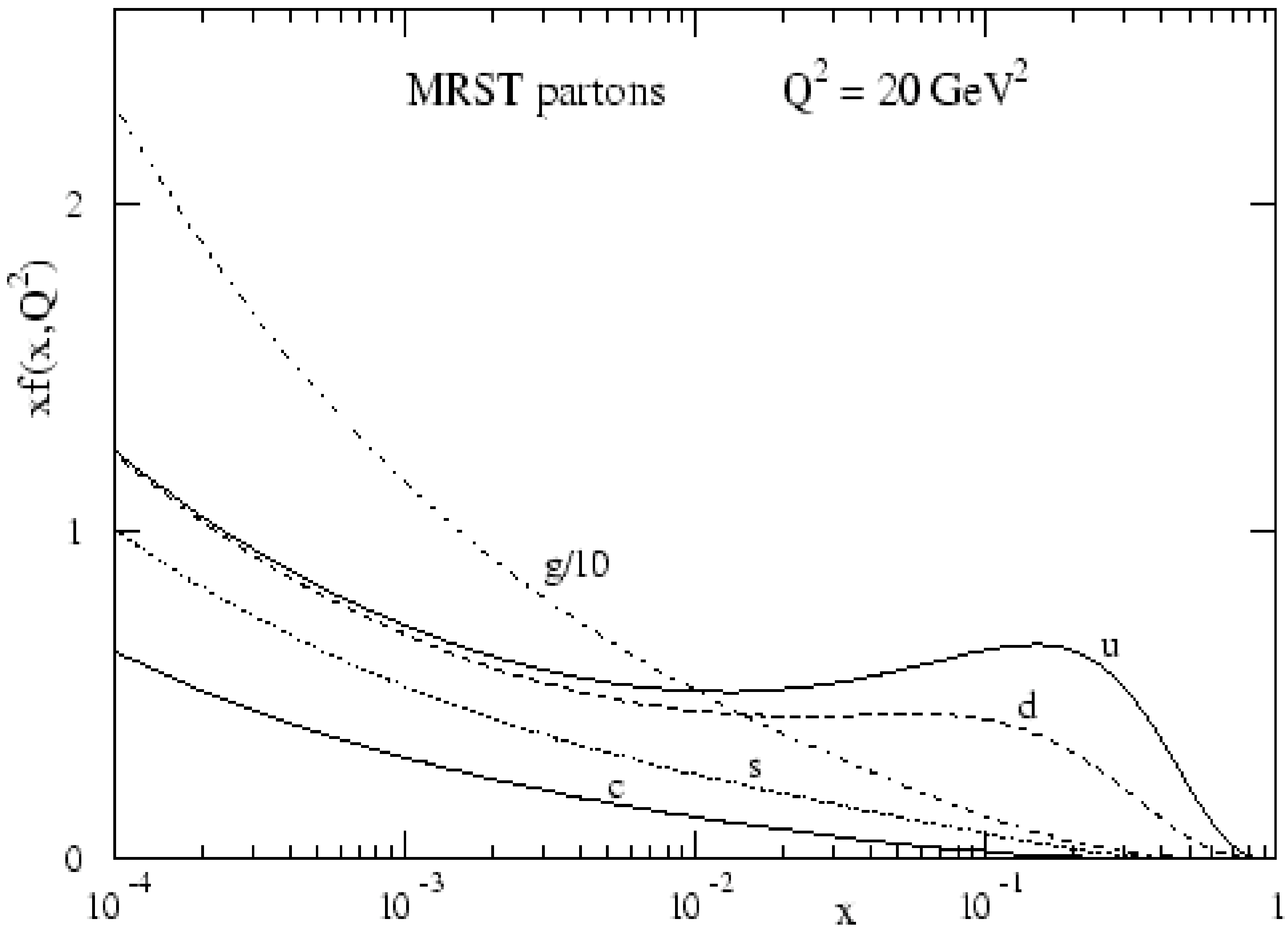


Fig. 19: Parton distributions by the MRST group.

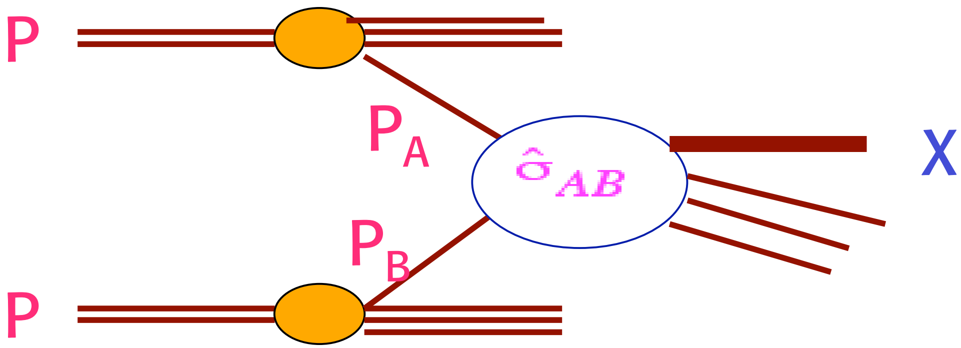
Parton densities extracted from DIS are used to compute hard processes, via the Factorisation Theorem:

density of parton A

$$\sigma(s) = \sum_{A,B} \int dx_1 dx_2 p_A(x_1, s) p_B(x_2, s) \hat{\sigma}_{AB}(x_1 x_2 s)$$

reduced X-section

For example, at hadron colliders



V=γ*,W,Z

X=V, jets, QQ, H.....

Q=b,c,t

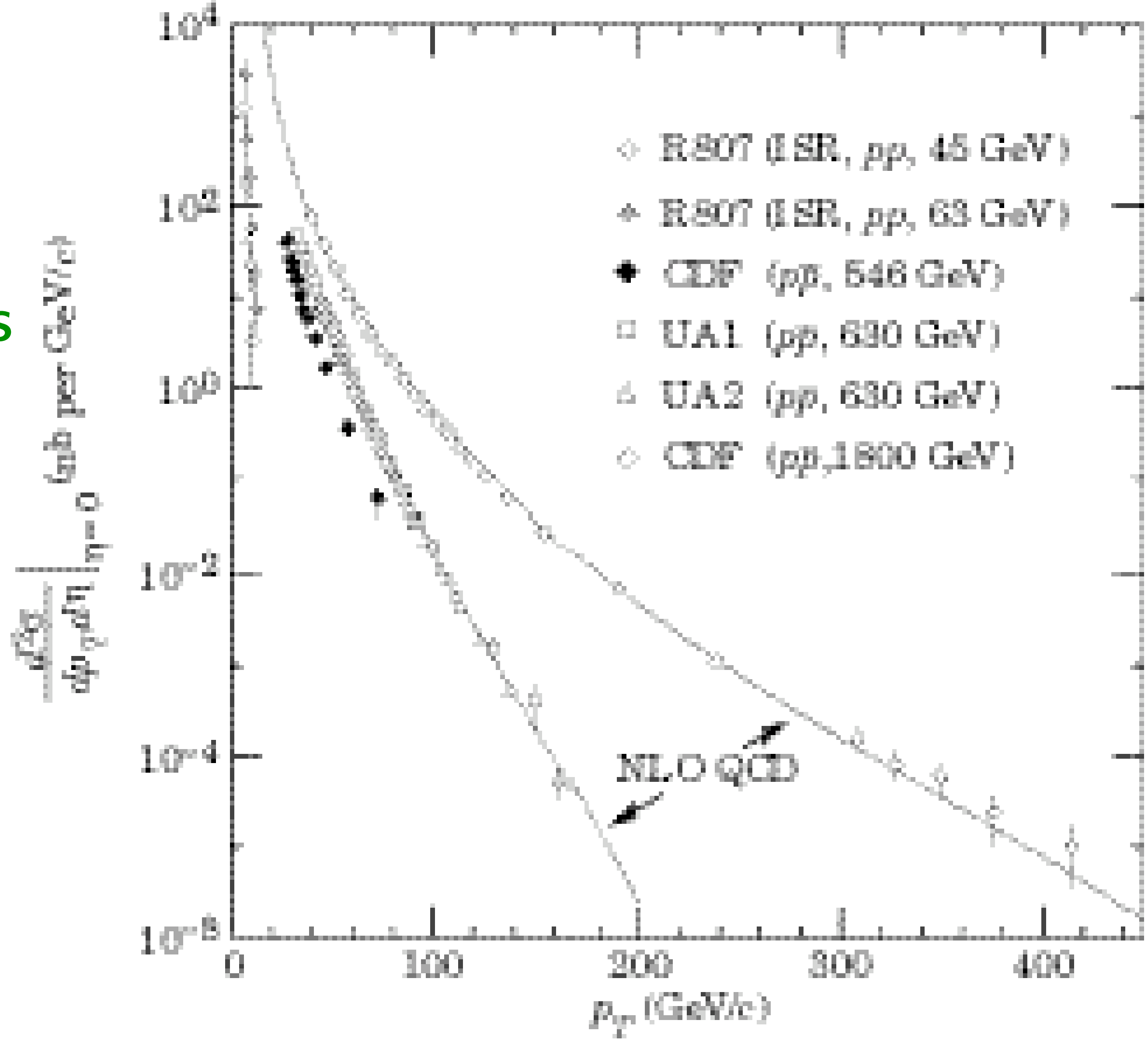
•Very stringent tests of QCD
•Feedback on constraining parton densities

Jet Production in pp or pp^{bar} interactions

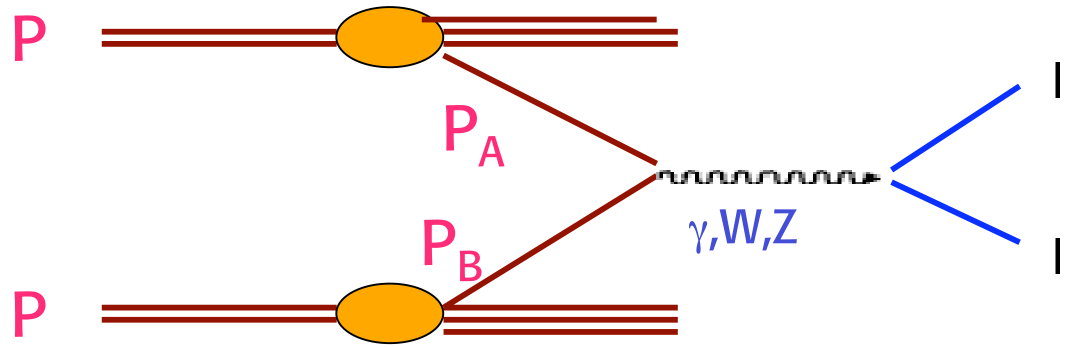
p_1p_2-> jet +X: all scalar products large

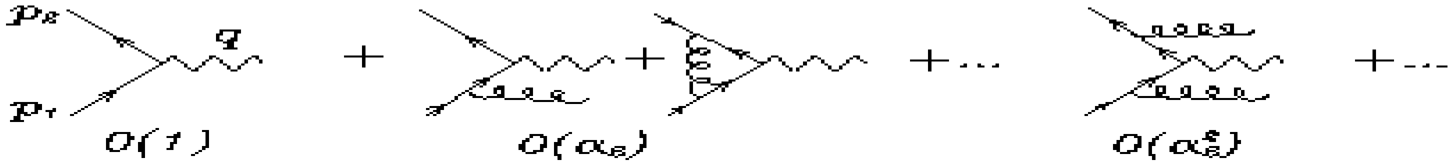
$(p_1+p_2)^2$=s; $(p_{1,2}$-jet$)^2$=t,u also large -> the jet must be at large p_T

NLO QCD fits
no free parameters
except exp.
norm' n
Note: many orders
of magnitude!



W, Z and Drell-Yan lepton pair production at hadron colliders.

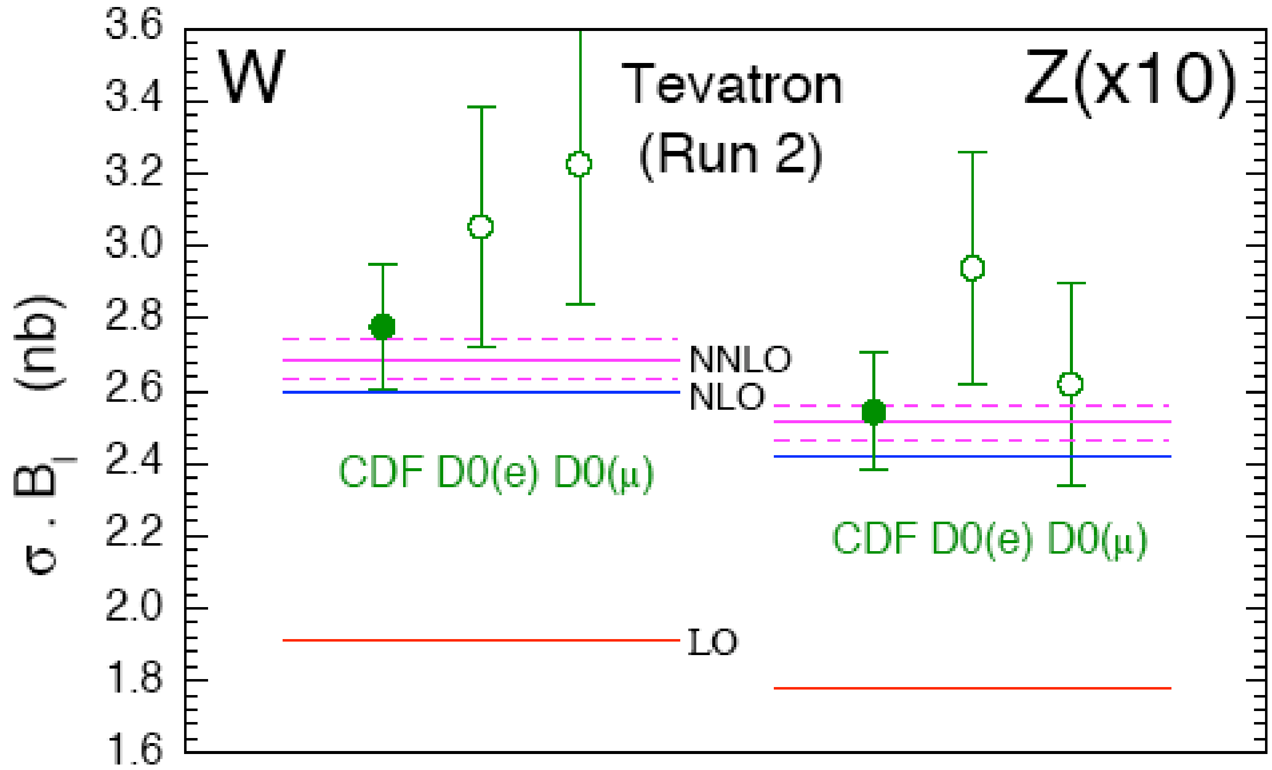




o(1): Drell, Yan; o(α_s): Altarelli, K.Ellis, Martinelli; Kubar-Andre, Paige; o($\alpha_s{}^2$): Hamberg, van Neerven, Matsuura+Zijestra

The prediction for $\sigma B_{W,Z}$ is obtained using parton densities from DIS, the measured Λ and B from the EW theory

Can be used as luminosity monitor at the LHC!

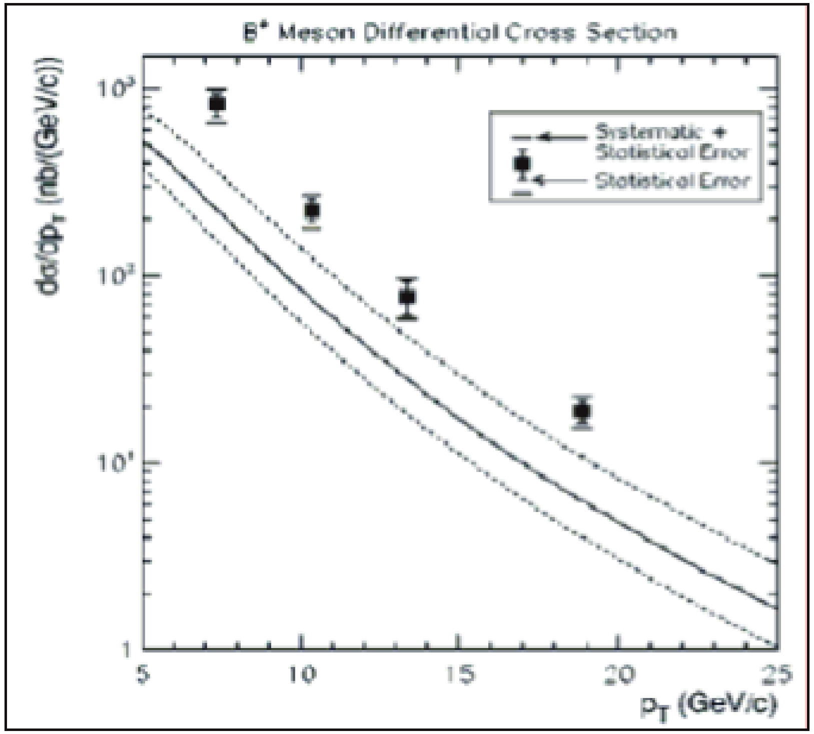


p_T distribution has also been a classic laboratory

Differential rapidity distributions at NNLO recently computed

Anastasiou et al

There was a problem on b-prod'n at the Tevatron

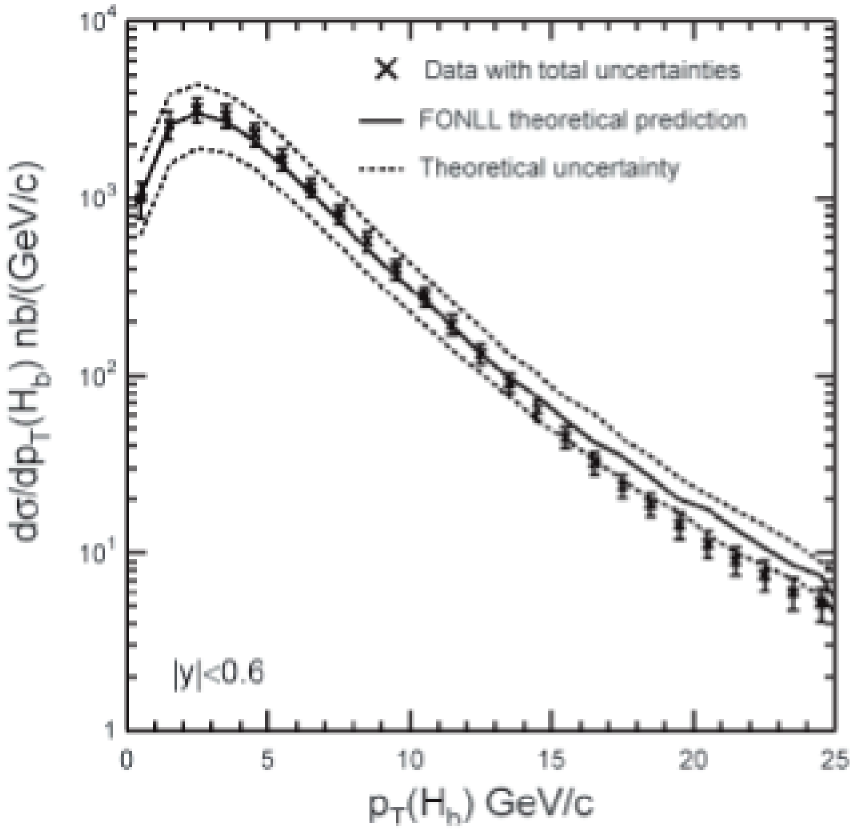


CDF Run I

- log resummation
- better fragmentation fncts
- b hadrons not b quarks
- better pdf's

The problem appears now to be solved

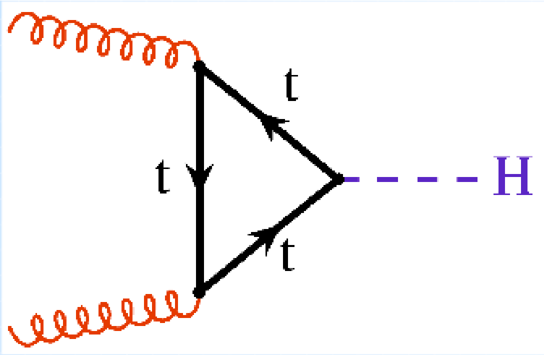
Nason, Dawson,K.Ellis
Cacciari, Greco, Nason
Frixione, Nason, Webber
Cacciari, Frixione, Mangano
Nason, Ridolfi



Predictions for future tests

Higgs production via g+g -> H

Very important for the LHC



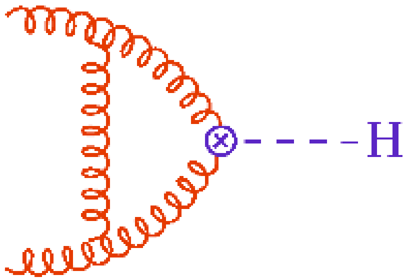
Effective lagrangian (m_t -> infinity)

$$\mathscr{L} = C_1 H G^{\mu\nu} G_{\mu\nu}$$

C_1 known to α_s^4

Chetyrkin, Kniehl, Steinhauser

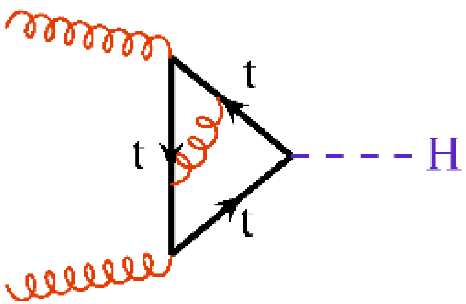
NLO corr.s computed with effective lagrangian

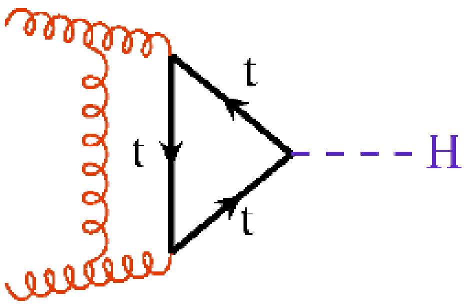


Dawson
Djouadi, Spira, Graudenz, Zerwas

AND the full theory

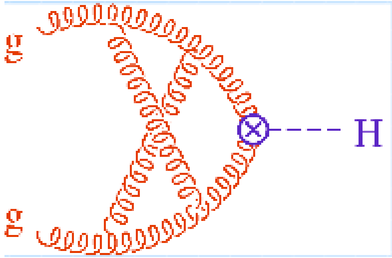
Djouadi, Spira, Graudenz, Zerwas

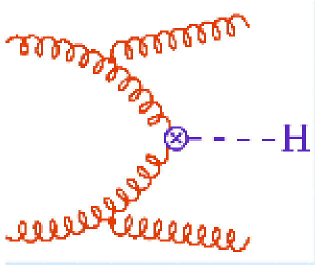




They agree very well

Recently the NNLO calculation has been completed (analytic)

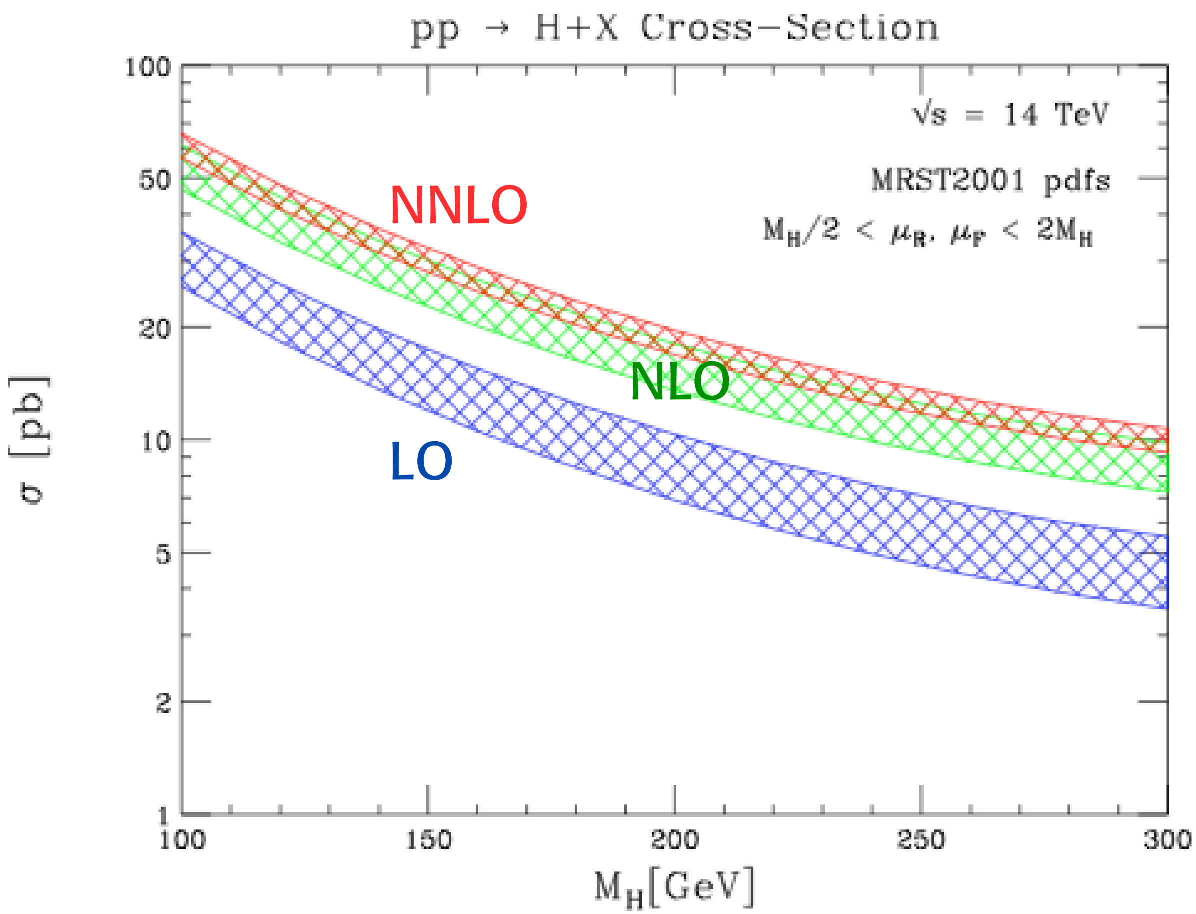




Harlander, Kilgore
Ravindran, Smith, van Nerven
Anastasiou, Melnikov

Also NLO y and p_T distributions have been computed

Anastasiou et al
De Florian, Grazzini, Kunszt
Ravindran, Smith, van Nerven
Glosser, Schmidt



Higgs p_T distribution: $[\log(p_T/m_H)]^n$ resummed

Bozzi, Catani, De Florian, Grazzini

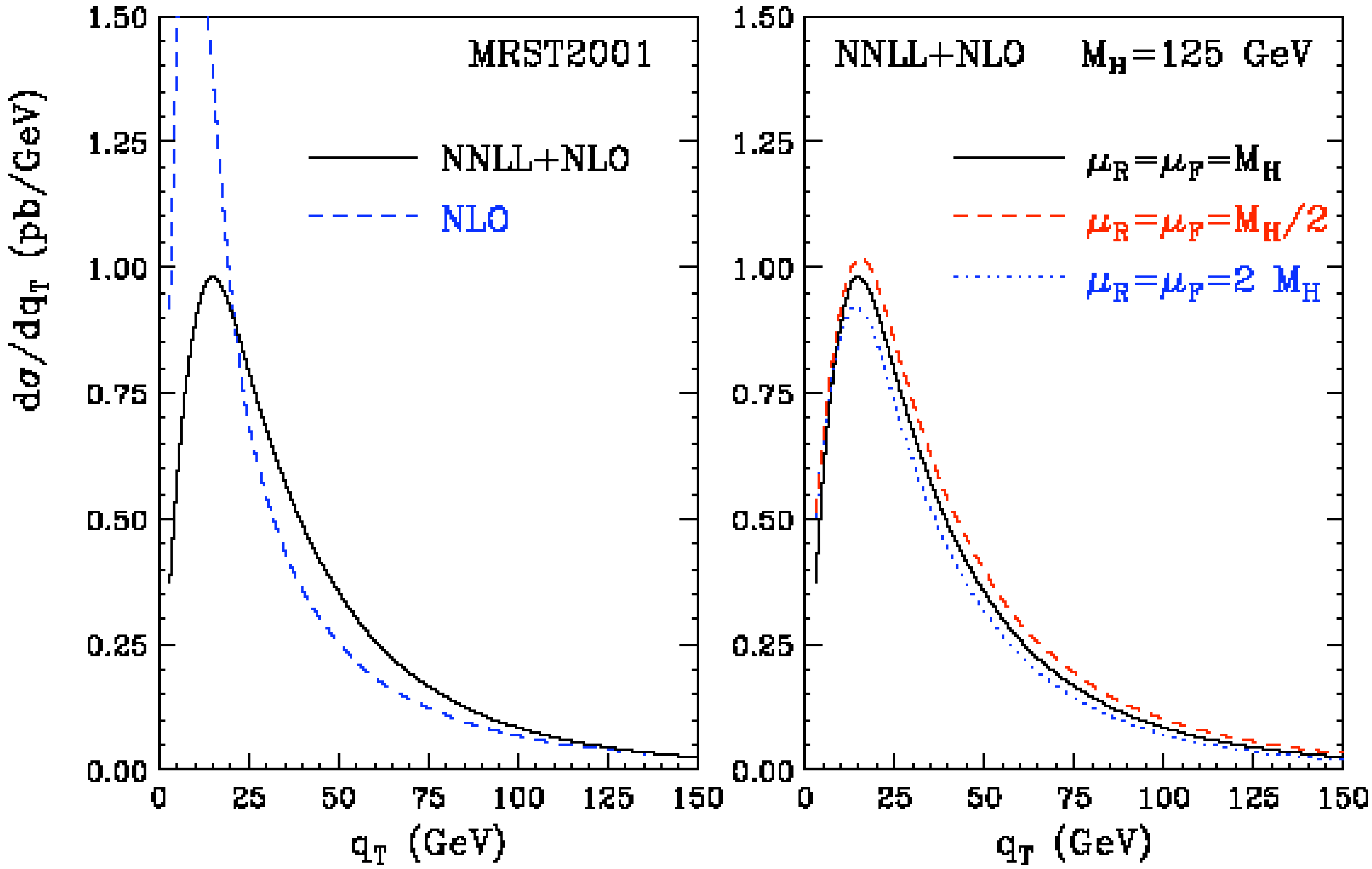


Figure 7. Resummed pQCD prediction for the Higgs transverse momentum distribution at the LHC, from Bozzi *et al.*[25]

QCD event simulation

A big boost in the preparation to LHC experiments

General algorithms for computer NLO calculations
eg the dipole formalism Catani, Seymour,..
Matching matrix elements and parton showers
e.g. MC@NLO-based on HERWIG Frixione, Nason, Webber

Perturbative (+ resumm.s)

$$d\sigma = A\alpha_S^N [\, 1 + (c_{1,1}L + c_{1,0})\alpha_S + (c_{2,2}L^2 + c_{2,1}L + +c_{2,0})\alpha_S^2 + \dots \,]$$

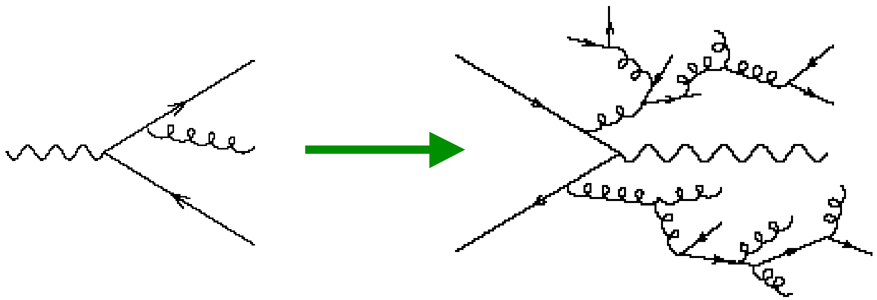
L= large log eg L=log(p_T/m)

Complementary virtues:
the hard skeleton plus
the shower development
and hadronization

Parton showers

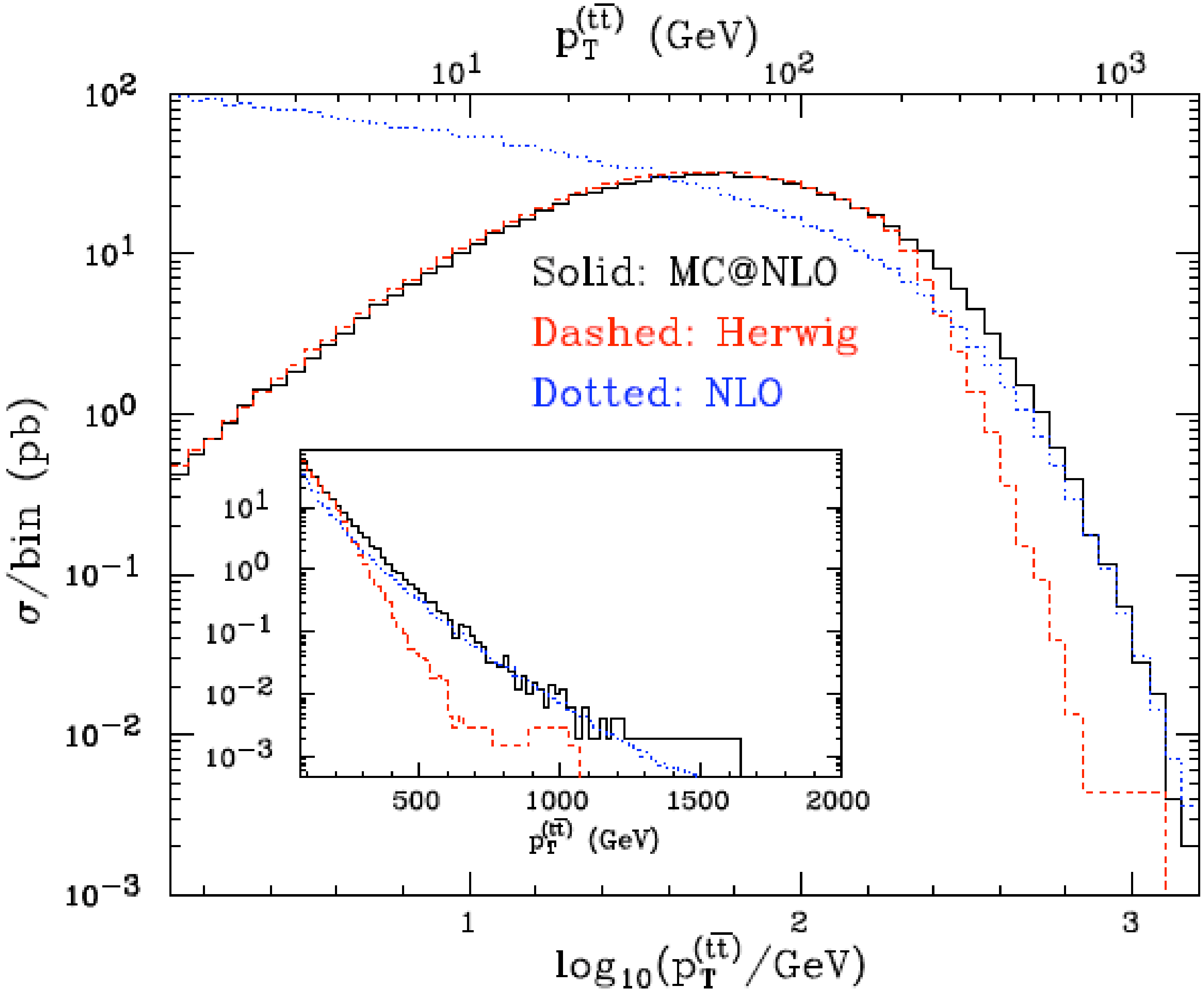
collinear emissions factorize

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \times \frac{\alpha_S}{2\pi}\frac{dt}{t}P_{qq}(z)dz\frac{d\varphi}{2\pi}$$
$$t = (p_q + p_g)^2 \longrightarrow 0$$

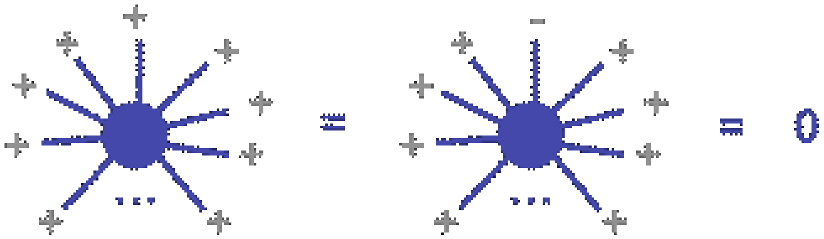


hadronization added

p_T distribution of tt at the Tevatron



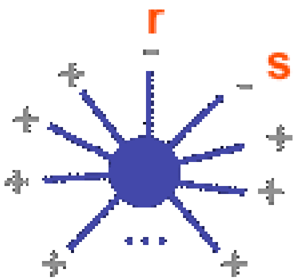
String theory improved QCD: a powerful breakthrough



Amplitudes of n incoming gluons with ±helicities

Parke, Taylor '86

Maximum
Helicity
Violating



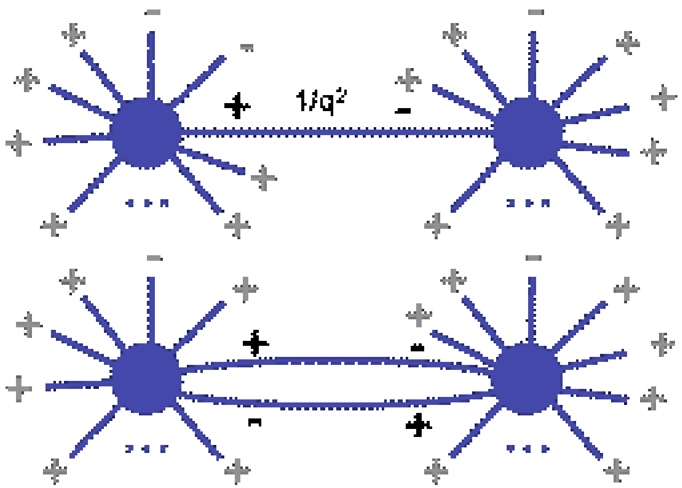
$$= i\, g_S^{n-2} \frac{\langle r, s\rangle^4}{\prod_{j=1}^{n} \langle j, j+1\rangle} \qquad \begin{cases} \langle i, j\rangle = u_-(p_i)\bar{u}_+(p_j) \\ |\langle i, j\rangle| = \sqrt{2 p_i \cdot p_j} \end{cases}$$

Relation between gauge th and string th in twistor space

Witten '03

allows to compute all helicity amplitudes (effective vertices and propagators).

Very compact results much faster than Feynman diagrams



Cachazo, Svrecek, Witten '04

Rapid progress: at tree level

Powerful recursion relations — Britto, Cachazo, Feng; BCF, Witten

Inclusion of massless fermions — Georgiou, Khoze

of external EW vector bosons — Berne et al

of external Higgs — Dixon, Glover, Khoze; Badger +GK

Already important for multijet events at the LHC

and also loops: QCD 1-loop

Bedford, Brandhuber, Spence and Travaglini; Bern, Dixon and Kosower; Bidder, Bjerrum-Bohr, Dunbar and Perkins

Looks very promising

Conclusion

QCD is a non abelian unbroken gauge quantum field theory of fundamental physical relevance

Its physics content is very large and our knowledge esp. in the non perturbative domain is still very limited but progress both from experiment (HERA, Tevatron, RHIC, LHC) and from theory is continuous

Very good agreement with experiment

I most warmly thank the Organisers for their invitation to this very exciting School in this interesting place.