Cosmology and the origin of structure

Rocky I: The universe observed

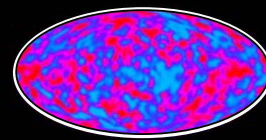
Rocky II: Perturbations

Rocky III: Inflation

http://home.fnal.gov/~rocky/maria_laach_1.pdf
http://home.fnal.gov/~rocky/maria_laach_2.pdf
http://home.fnal.gov/~rocky/maria_laach_3.pdf

Herbstschule für Hochenergiephysik Maria Laach
Rocky Kolb
Fermilab & The University of Chicago

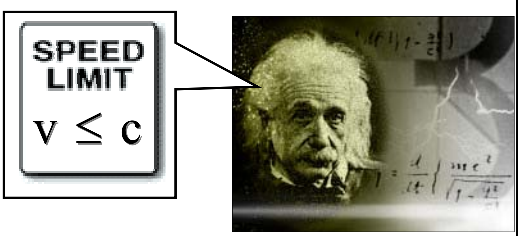
Primordial perturbations



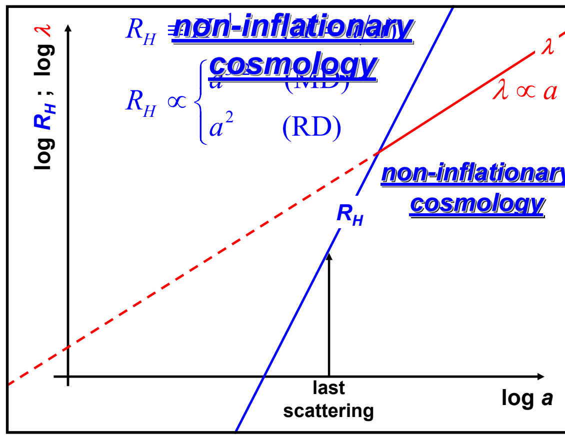
CBR: a snapshot of the universe 380,000 AB

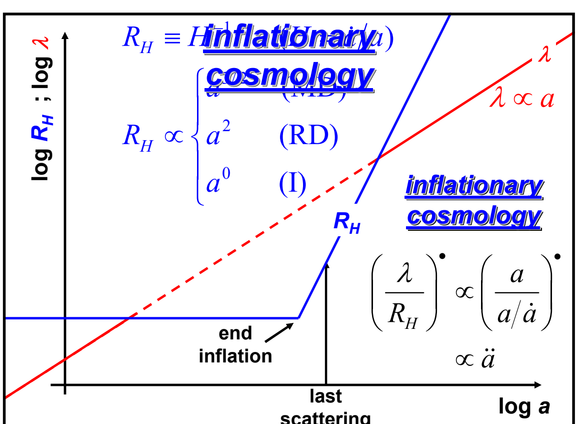
correlations on scales ≫ 380,000 light years

More than 380,000 light years in less than 380,000 years?



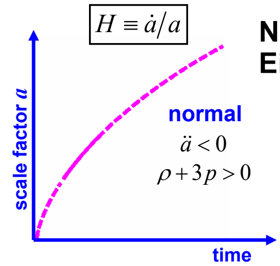
- v ≤ c for velocity *through* space
- no limit on expansion velocity *of* space
- “acausal” requires “accelerated” expansion

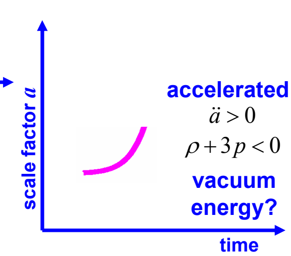


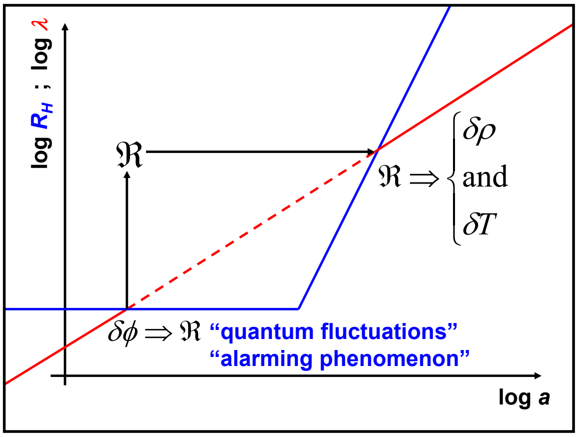


$H \equiv \dot{a}/a$

Newton / Einstein: $\ddot{a} \propto -G\left(\rho + 3p\right)$







Inflation, as a whole, can be divided into three parts

1. **Beginning**
   eternal inflation, wave function of the universe, did the universe have a beginning ????
2. **Middle**
   density perturbations, gravitational waves, (particle production in the expanding universe)
3. **End**
   defrosting, heating, preheating, reheating, baryogenesis, phase transitions, dark matter, (particle production in the expanding universe)

The proper vibrations of the expanding universe

Erwin Schrödinger, *Physica* **6**, 899 (1939)

Introduction:

"… proper vibrations [positive and negative frequencies] cannot be rigorously separated in the expanding universe. … this is a phenomenon of outstanding importance. With particles it would mean production or annihilation of matter, merely by expansion,… Alarmed by these prospects, I have examined the matter in more detail."

Conclusion:

"… There will be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to … the 'alarming phenomenon'…"

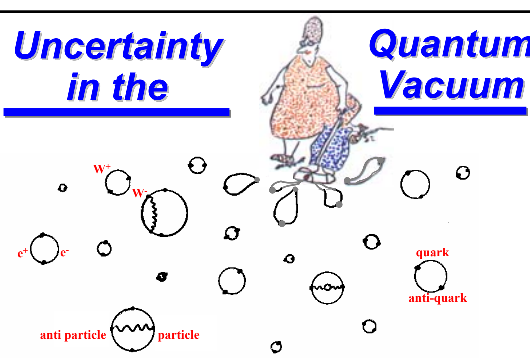
The proper vibrations of the expanding universe

Erwin Schrödinger, *Physica* **6**, 899 (1939)

Creation of a *single* pair of particles	$H \simeq 60\ \text{km s}^{-1}\text{Mpc}^{-1}$
per Hubble volume	$V_H \equiv (c/H)^3 \simeq 10^{12}\,\text{Mpc}^3$
per Hubble time	$t_H \equiv H^{-1} \simeq 10^{10}$ years
with "Hubble energy"	$E_H \equiv \hbar H \simeq 10^{-33}$ eV

Alarming?

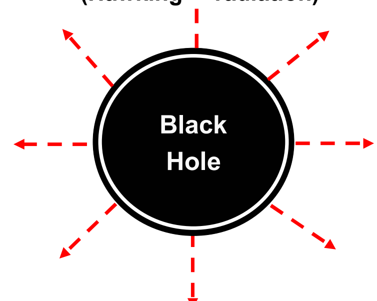
Uncertainty in the Quantum Vacuum

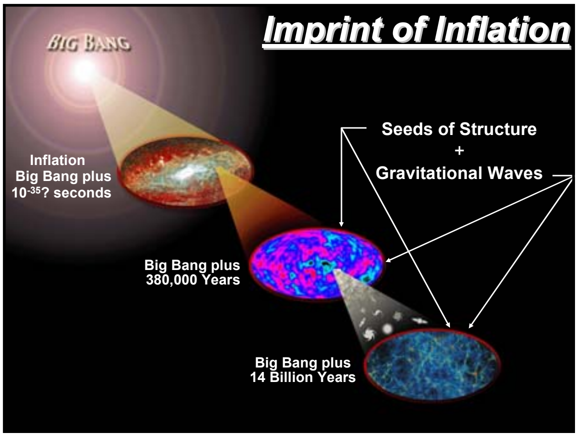


Nothing is something!

Disturbing the vacuum

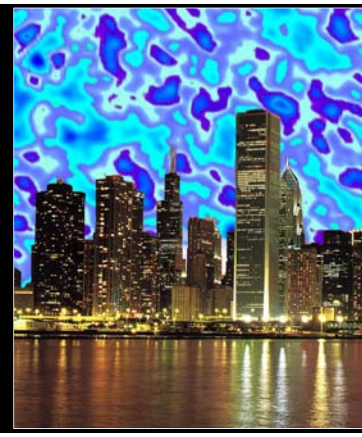
Strong gravitational field → particle production (Hawking radiation)





A pattern of vacuum quantum fluctuations

(the alarming phenomenon)



Scalar field ϕ of mass M

Fourier modes [$a(\tau)$ = expansion scale factor]

$$\phi(\vec{x},\tau)=\int\frac{d^3x}{(2\pi)^{3/2}a(\tau)}\left[h_k(\tau)e^{i\vec{k}\cdot\vec{x}}a_k+h_k^*(\tau)e^{-i\vec{k}\cdot\vec{x}}a_k^\dagger\right]$$

Mode equation (τ = conformal time)

Scalar field ϕ of mass M

Fourier modes [$a(\tau)$ = expansion scale factor]

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Mode equation (τ = conformal time)

$$h_k''(\tau)+\left[k^2+M^2a^2+(6\xi-1)a''/a\right]h_k(\tau)=0$$

$$h_k''(\tau)+\omega_k^2(\tau)h_k(\tau)=0$$

Particle creation in nonadiabatic region ω_k'/ω_k

Inflaton also determines mass density
(gauge freedom)

Gauge invariant variable $\Re$

Scalar metric perturbations:

$$ds^2=a^2(\tau)\left\{(1+\psi)d\tau^2-(1-\psi)d\vec{x}^2\right\}$$

Inflaton field perturbations:

$$\phi(\vec{x},t)=\phi_0(t)+\delta\phi(\vec{x},t)$$

Gauge invariant variable:

$$\Re=\psi-\frac{H}{\dot{\phi}_0}\delta\phi$$

intrinsic curvature perturbations on comoving hypersurfaces

Variational formalism for quantization:

$$S=\int d^4x\sqrt{-g}\left[-\frac{M_{Pl}^2}{16\pi}R+\tfrac{1}{2}(\partial\phi)^2-V(\phi)\right]$$

Inflaton field

$$g_{\mu\nu}(\vec{x},t)=g_{\mu\nu}^{\mathrm{FRW}}(t)+\delta g_{\mu\nu}^{\mathrm{scalar}}(\vec{x},t)$$

$$\phi(\vec{x},t)=\phi_0(t)+\delta\phi(\vec{x},t)$$

Scalar perturbations in terms of a field u

$$u=a\delta\phi+z\psi=-z\Re \qquad z=a\dot{\phi}_0/H$$

$$S=\int d^4x\left(\partial_\mu u\partial^\mu u-\tfrac{1}{2}m_u^2(\tau)u^2\right)$$

Minkowski space (conformal time)

$$\frac{1}{2}m_u^2(\tau)=-\frac{1}{z}\frac{d^2z}{d\tau^2}$$

mass changes with time

Variational formalism for quantization:

$$S=\int d^4x\sqrt{-g}\;\left[-\frac{M_{Pl}^2}{16\pi}R+\tfrac{1}{2}\left(\partial\phi\right)^2-V(\phi)\right]$$

$$g_{\mu\nu}(\vec{x},t)=g_{\mu\nu}^{\mathrm{FRW}}(t)+\delta g_{\mu\nu}^{\mathrm{tensor}}(\vec{x},t)$$

Inflaton field

Tensor perturbations in terms of a field v

$v\propto h_{ij}$ $v=$ **gravitons**

$$S=\int d^4x\;\left(\partial_\mu v\partial^\mu v-\tfrac{1}{2}m_v^2\left(\tau\right)v^2\right)$$ **Minkowski space (conformal time)**

$$\frac{1}{2}m_v^2\left(\tau\right)=-\frac{1}{a}\frac{d^2a}{d\tau^2}$$ **mass changes with time**

Quantum generation of perturbations:

- **Expand u in Fourier modes**
- **Wave equation for mode functions for u**

$$u_k''\left(\tau\right)+\left(k^2-\frac{1}{z}\frac{d^2z}{d\tau^2}\right)u_k\left(\tau\right)=0\qquad\left(z=a\dot{\phi}_0/H\right)$$

- **Initially only homogeneous $(k=0)$ mode.**
- **As evolve, mass is complicated function of time.**
- **Create nonzero momentum modes (perturbations).**

$$u_k/z\rightarrow P_{\Re}(k)\rightarrow\text{ Power Spectrum}$$

Slow-roll parameters

Slow-roll parameters:

$$\varepsilon(\phi)\equiv\frac{m_{Pl}^2}{4\pi}\left(\frac{H'(\phi)}{H(\phi)}\right)^2;\;\;\eta(\phi)\equiv\frac{m_{Pl}^2}{4\pi}\frac{H''(\phi)}{H(\phi)};\;\;\xi(\phi)\equiv\frac{m_{Pl}^2}{4\pi}\left(\frac{H'(\phi)H'''(\phi)}{H^2(\phi)}\right)^{1/2}$$

$$\frac{1}{z}\frac{d^2z}{d\tau^2}=2a^2H^2\left(1+\varepsilon-\frac{3}{2}\eta+\varepsilon^2-2\varepsilon\eta+\frac{1}{2}\eta^2+\frac{1}{2}\xi^2\right)$$

- $\varepsilon<1$ for inflation to occur
- expect η and ξ also to be less than unity
- so expect slow-roll part of $\frac{1}{z}\frac{d^2z}{d\tau^2}$ to be slowly varying

$$\frac{1}{z}\frac{d^2z}{d\tau^2}=2a^2H^2\left(1+\text{something small and not wildly varying}\right)$$

Quantum generation of perturbations:

- **Perturbations model-dependent function of H and how H changes during inflation.**

$$V(\phi)\leftrightarrow H(\phi)\leftrightarrow\left\{\varepsilon(\phi),\;\eta(\phi),\;\xi(\phi),\ldots\right\}$$

- **Characterize perturbations in terms of:**

$P_{\Re}(k_*)$ scalar perturbation at $k=k_*$

$n\equiv\dfrac{d\ln P_{\Re}(k_*)}{d\ln k}$ scalar spectral index

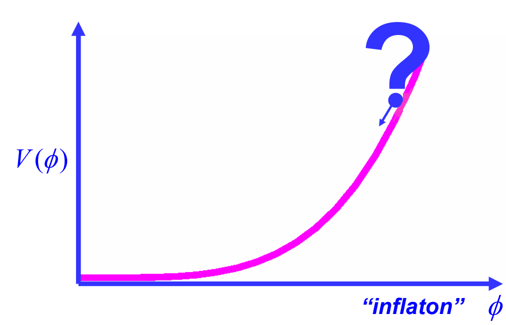
$P_{\Im}(k_*)$ tensor perturbation at $k=k_*$

$n_T\equiv\dfrac{d\ln P_{\Im}(k_*)}{d\ln k}$ tensor spectral index

Some simple questions:

1. Was inflation “normal” (i.e., 3-D FRW)?
2. Can dynamics of inflation be described in terms of a single scalar field?
3. What was the expansion rate during inflation?
4. What was the general shape of the inflaton potential?
5. What was the more or less exact shape of the inflaton potential?
6. Can inflation tell us anything about physics at very high energy scales (unification, string, Planck)?

Who is the inflaton?



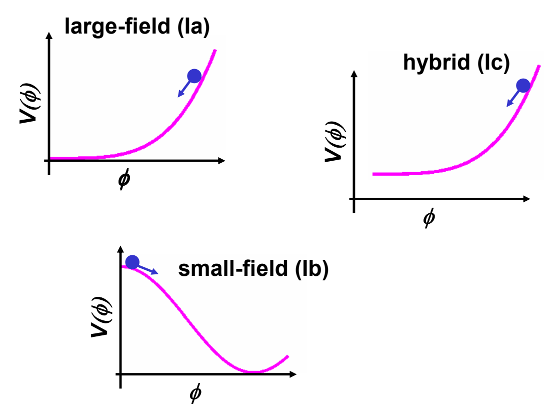
Model Classification*

Type I: single-field, slow-roll models
(or models that can be expressed as such)

- **Type Ia:** large-field models
- **Type Ib:** small-field models
- **Type Ic:** hybrid models

Type II: anything else
(branes, pre-big-bang, etc.)

**Used for superstrings, supernovae, superconductors, …*



Quantum generation of perturbations:

- **Input inflation potential** $V(\phi)$ **:**

  $\{P_{\Re}(k_*) \quad n \quad P_{\Im}(k_*) \quad n_T\}$

- **Observer-friendly parameters:**

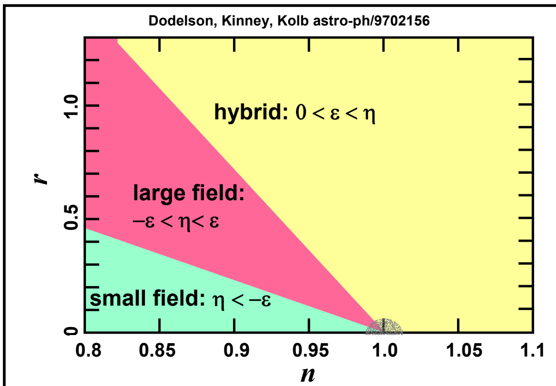
  $Q \equiv P_{\Re}(k_*) + P_{\Im}(k_*) \qquad n \qquad r \equiv \dfrac{P_{\Im}(k_*)}{P_{\Re}(k_*)}$

- **Consistency relation:**

  $n_T = -2r\left[1 - r + (1-n)\right]$

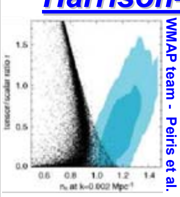
- **Inflaton potential** $V(\phi)$**:**

  $\{Q \quad n \quad r\}$ or if free parameter $\{n \quad r\}$

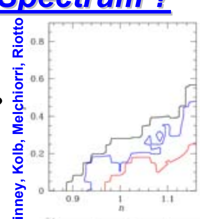


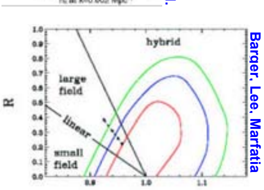
$r = (\text{tensor/scalar})_{l=2}$ $\qquad$ $n =$ scalar spectral index

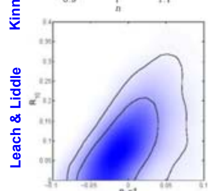
Harrison-Zel'dovich Spectrum ?



1. priors?
2. chain vs. grid?
3. marginalize?
4. data sets used?
5. definition of r?
6. …

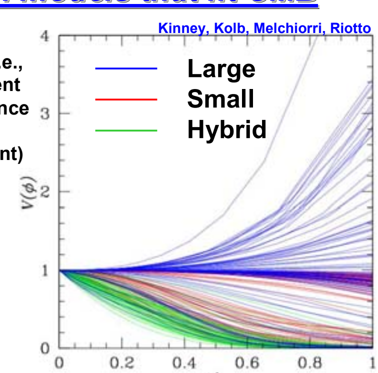


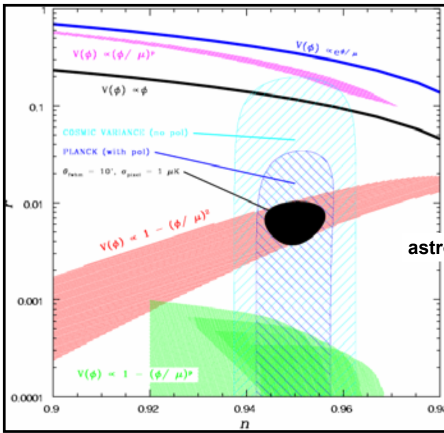




Inflation models that fit CMB

1. H-Z consistent, i.e.,
2. r=0 still consistent
3. (no strong evidence for running)
4. (n=1 still consistent)
5. ϕ^4 still OK?



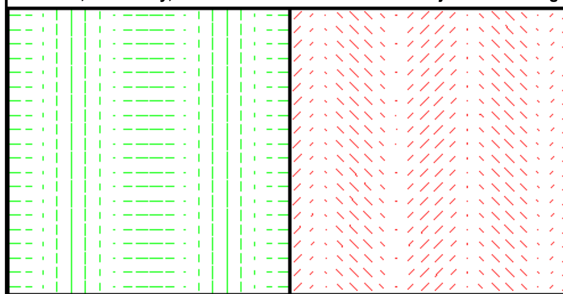


Kinney
astro-ph/9806259

Polarization pattern

Stebbins, Kosowsky, Kamionkowski

Seljak & Zaldarriaga



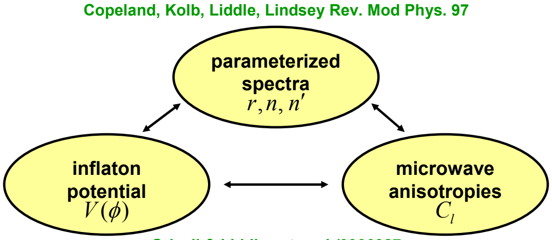
E modes

B modes
(gravitational waves)

Reconstruction

Bond, Abney, Copeland, Grivell, Kolb, Liddle, Lindsey, Turner, Sourdeep

Copeland, Kolb, Liddle, Lindsey Rev. Mod Phys. 97



Grivell & Liddle astro-ph/9906327

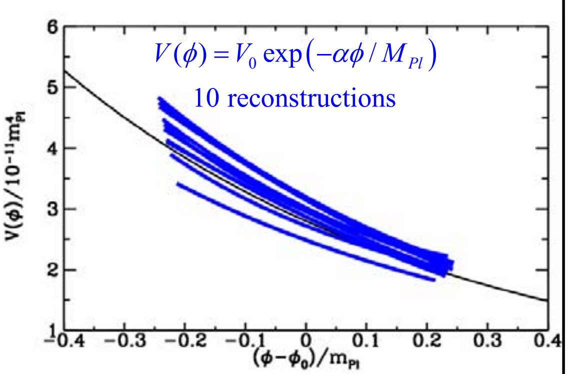
Reconstruction

$$\text{scalar} \sim \frac{V'(\phi)}{V(\phi)}$$

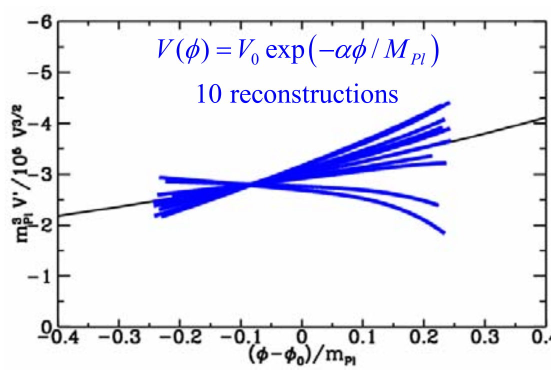
$$\text{tensor} \sim V(\phi)$$

1. tensor spectral index in terms of scalar & tensor (consistency relation)
2. knowledge of the scale of *V* requires tensor

Power-law spectrum



Power-law spectrum



Comparison to observation:

1. a (nearly exact) *power-law*
2. spectrum of *gaussian*
3. *super-Hubble-radius*
4. *scalar* perturbations (seeds of structure) &
5. *tensor* perturbations (gravitational waves)
6. related by a *consistency relation*
7. in their *growing mode*
8. in a *spatially flat* universe.

Issues

1. Transplanckian physics
   - probe of short-distance physics?
2. Defrosting
   - preheating, reheating, ….
3. Particle production
   - WIMPZILLAS, gravitons, ….
4. Why only one inflaton?
   - isocurvature perturbations
5. Extra dimensions, brane, bulk, etc.?
   - new dynamics

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