Radiative Corrections in the Electroweak Standard Model

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02. - 12. September 2003

- Relevance of radiative corrections
- Radiative corrections to muon decay
- Radiative corrections to Z physics
- Precision tests of the electroweak Standard Model
- Electroweak radiative corrections at high energies

Radiative Corrections

Electromagnetic and weak couplings are small

$\hookrightarrow$ Electroweak Standard Model can be evaluated in perturbation theory

= expansion about theory without interaction

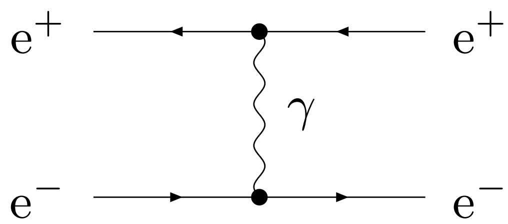
perturbation series

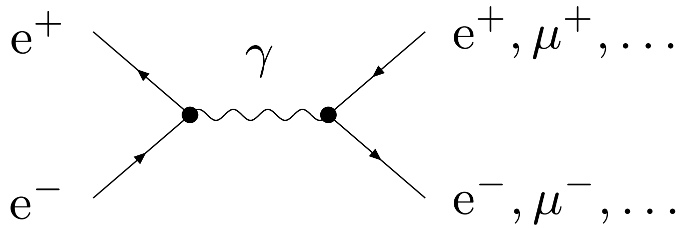
= power series in coupling e = power series in Planck constant $\hbar$

= expansion in # of loops of Feynman graphs

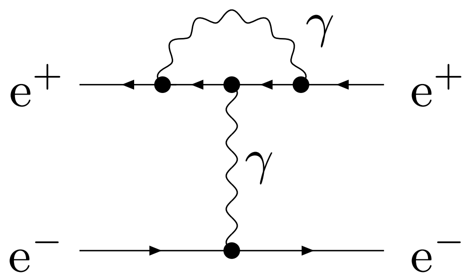
$\Rightarrow$ loop diagrams = quantum corrections = radiative corrections

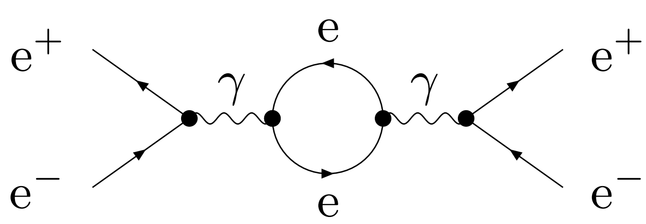
lowest order: tree diagrams





higher orders: loop diagrams

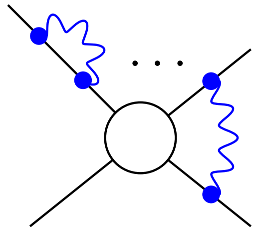




IR divergences and photon bremsstrahlung

Consider processes with charged external particles, e.g., $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$

- virtual corrections: loop diagrams

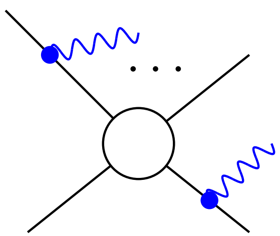


IR divergences from soft virtual photons ($q \to 0$)

$$\int \frac{\mathrm{d}^4 q \cdots}{(q^2 - m_\gamma^2)(2qp_1)(2qp_2)} \to C \ln(m_\gamma)$$

- "real" corrections: photon bremsstrahlung = "real" radiative corrections

$$\int \frac{\mathrm{d}^3 \mathbf{q}}{2q_0} \left| \quad \right|^2$$



IR divergences from soft real photons ($\mathbf{q} \to 0$)

$$\int \frac{\mathrm{d}^3 \mathbf{q} \cdots}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \to -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

$\hookrightarrow$ virtual and soft-photonic corrections cannot be discussed separately
are related due to limited experimental resolution of soft photons

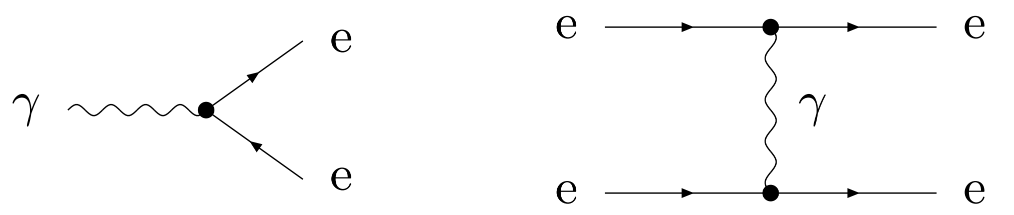
$\hookrightarrow$ cross-section predictions necessarily depend on treatment of photon emission (energy and angular cuts)

Perturbative evaluation of quantum field theories

Formulate theory:	Lagrangian
	$\Downarrow$
	quantization $\longrightarrow$ gauge fixing, Faddeev–Popov ghosts
	$\Downarrow$
perturbative evaluation:	Feynman rules
	$\Downarrow$
	Feynman graphs
	$\Downarrow$
	loop integrals $\longrightarrow$ technical problem: divergences (UV, IR)
	$\Downarrow$
	regularization $\longrightarrow$ divergences mathematically meaningful
	$\Downarrow$
define input parameters:	renormalization $\longrightarrow$ absorbs UV divergences
	$\Downarrow$
theoretical predictions:	calculation of observables (cross sections, decay widths, etc.)
	$\hookrightarrow$ IR divergences cancel for sufficiently inclusive quantities (e.g. inclusion of photon bremsstrahlung)

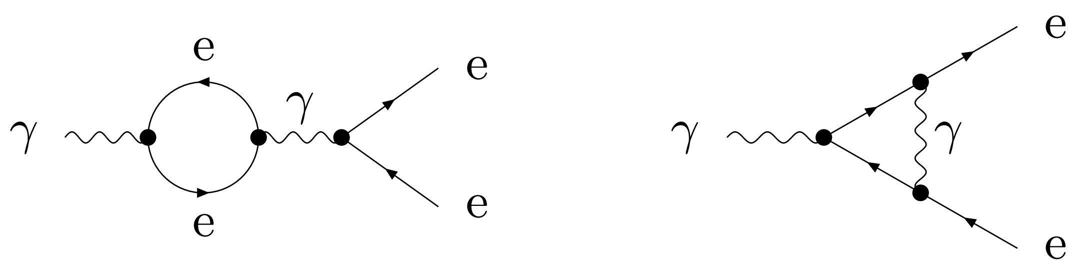
Relevance of radiative corrections in QED

Born approximation:



reproduces “classical” physics: $g = 2, \qquad V_{\text{Coulomb}} \sim \frac{\alpha}{r}, \qquad \ldots$

higher orders:



$\Rightarrow$ • anomalous magnetic moment: $g - 2 = 0.00231930 = \frac{\alpha}{\pi} + O(\alpha^2)$

- Lamb shift
- etc.

experiments with high accuracy require precise predictions from theory
including radiative corrections
$\hookrightarrow$ precision tests of theory

Electroweak precision experiments

- LEP1/SLC: $e^+e^- \to Z \to f\bar{f}$
  LEP1: $\sim 4 \times 10^6$ events/experiment, 4 experiments (1989 – 1995)

- LEP2: $e^+e^- \to W^+W^-$: $\mathcal{O}(10^4)$ W pairs (1996 – 2000)

- Tevatron: $q\bar{q}' \to W \to l\nu, q\bar{q}'$; $q\bar{q}' \to t\bar{t},\ t \to W^+b \to \ldots$

- low-energy experiments
  (μ decay, νN scattering, νe scattering, atomic parity violation, . . .)

some results:

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z [GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2\theta_{eff}^{lept}$	$= 0.23152 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.426 \pm 0.034$	0.04%
m_t [GeV]	$= 174.3 \pm 5.1$	2.9%
G_μ [GeV^{-2}]	$= 1.16637(1) \times 10^{-5}$	0.001%

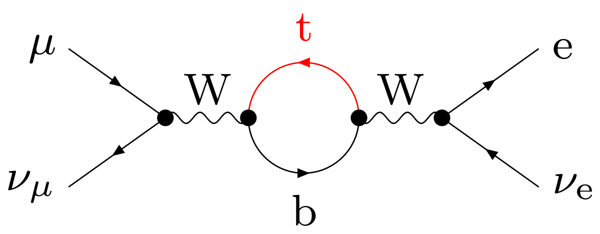
Relevance of quantum corrections

order of magnitude

$$\sim \left[\frac{\alpha}{\pi} \ldots \frac{\alpha}{\pi} \ln \frac{E^2}{m_\mathrm{e}^2}\right] \times \mathcal{O}(1) \sim [0.2\% \ldots 6\%] \times \mathcal{O}(1) \text{ for } E = 100\,\mathrm{GeV}$$

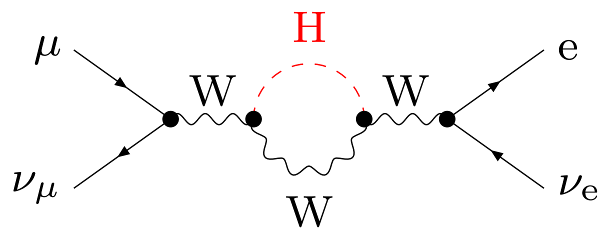
contain all details of the theory

- top quark



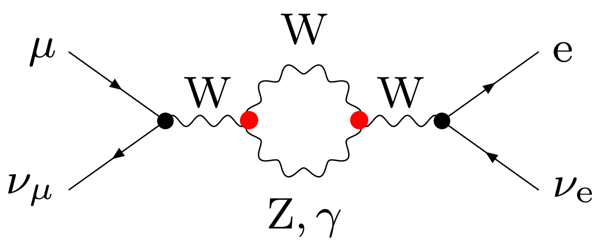
$\sim \frac{\alpha}{\pi} \frac{m_\mathrm{t}^2}{s_\mathrm{w}^2 M_\mathrm{W}^2} \approx 0.05 \; (m_\mathrm{t} = 175\,\mathrm{GeV})$

- Higgs boson



$\sim \frac{\alpha}{\pi} \ln \frac{M_\mathrm{H}}{M_\mathrm{W}}$

- gauge-boson self-couplings



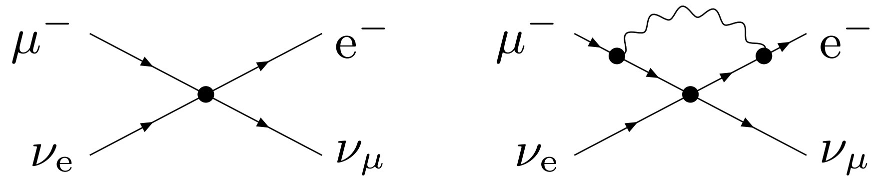
- New Physics (supersymmetry) ?

⇒ allow for indirect experimental tests of not directly accessible quantities

Gauge-boson-mass relation

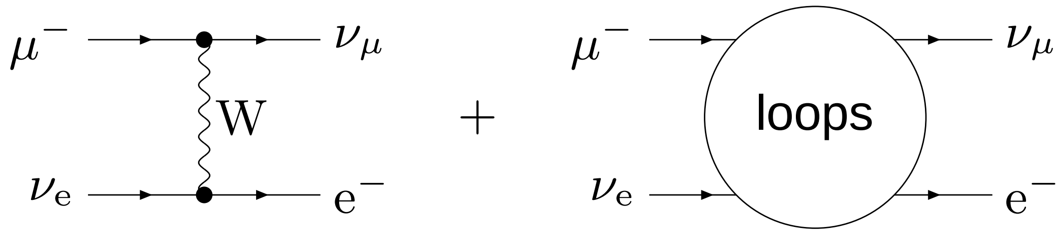
M_W and M_Z correlated via muon lifetime $\leftrightarrow$ Fermi constant G_μ

Fermi model



$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_\mathrm{e}^2}{m_\mu^2}\right)(1+\delta_\mathrm{QED})$$

Standard Model



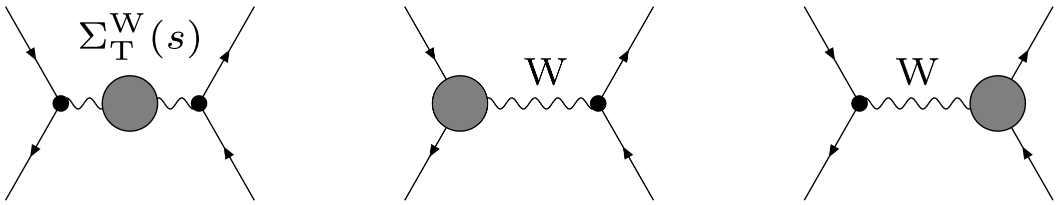
$$G_\mu = \frac{\pi\alpha}{\sqrt{2}}\frac{1}{M_\mathrm{W}^2(1-M_\mathrm{W}^2/M_\mathrm{Z}^2)}\frac{1}{1-\Delta r}$$

Δr: calculable quantum corrections

(beyond electromagnetic corrections to Fermi model)

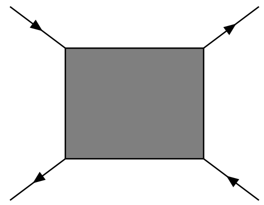
Electroweak corrections to μ decay

Virtual correction – one-loop diagrams:



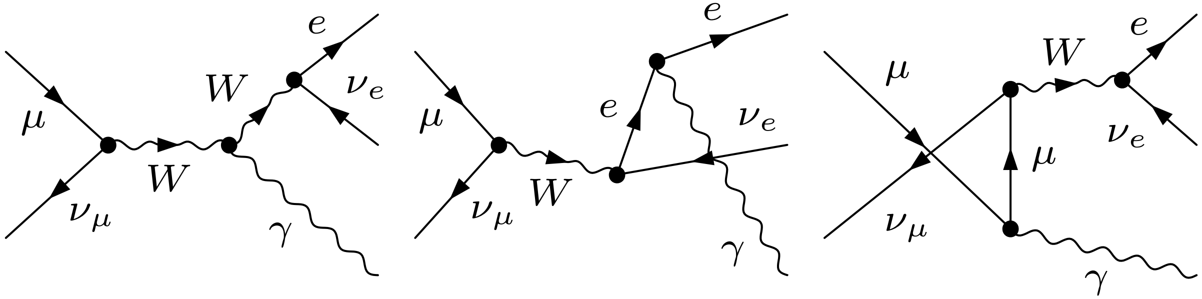
W self-energy

$Wl\nu_l$ vertex correction



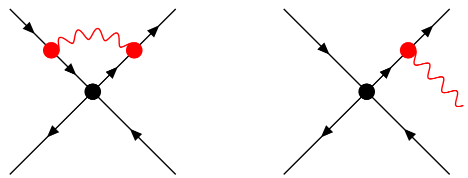
box diagrams

real correction – one-photon bremsstrahlung:



consistent use of G_μ:

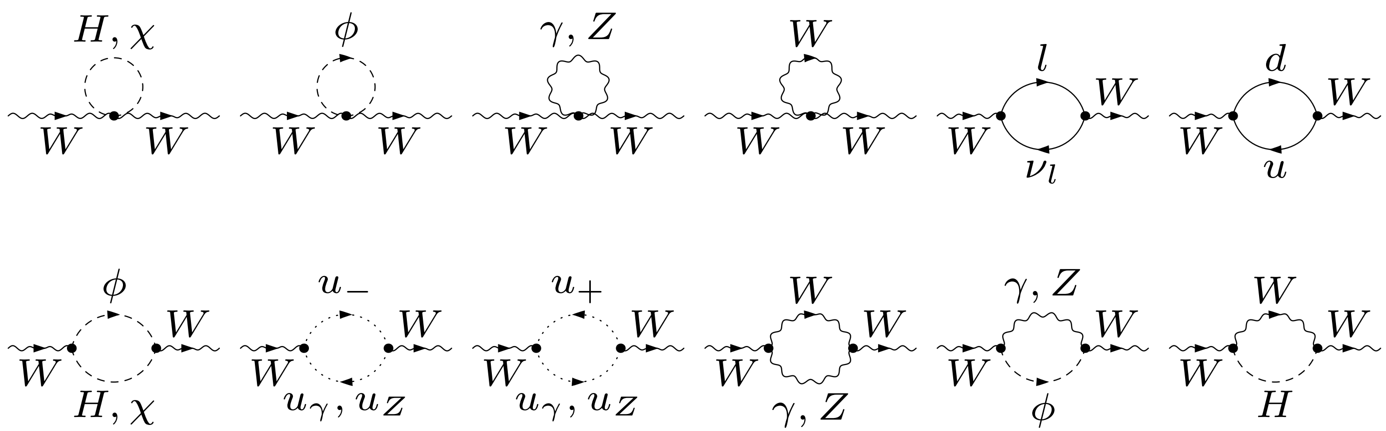
photonic QED corrections are treated in the Fermi model and subtracted from Δr



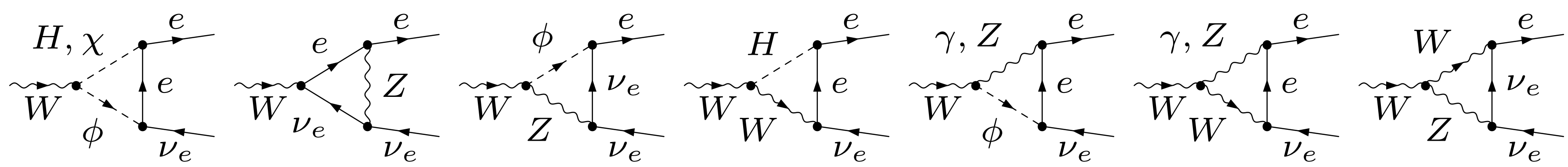
etc.

Feynman diagrams for vertex functions

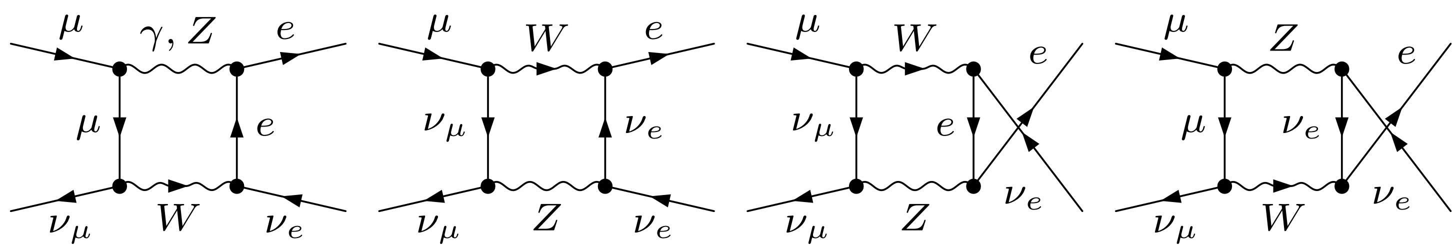
W-boson self-energy:



$We\nu_e$ vertex correction:



box diagrams:

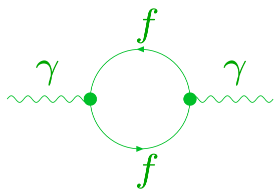


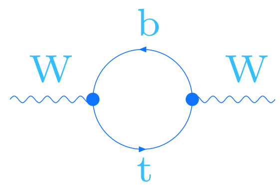
Structure of electroweak corrections to μ decay

$\mathcal{O}(\alpha)$ corrections: Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{\text{1-loop}} = \Delta\alpha(M_Z^2) - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}(M_H)$$

	$\Delta\alpha(M_Z^2)$	$\frac{c_W^2}{s_W^2}\Delta\rho$	$\Delta r_{\text{rem}}(M_H)$
	$\sim 6\%$	$\sim 3\%$	$\sim 1\%$
	$\alpha \ln(m_f/M_Z)$	$G_\mu m_t^2$	$\alpha \ln(M_H/M_Z)$





- $\Delta\alpha(M_Z^2)$: contribution of running electromagnetic coupling

  $$\Delta\alpha(M_Z^2) \sim \frac{\alpha}{3\pi}\sum_f Q_f^2 \ln\frac{M_Z^2}{m_f^2}$$

  $\hookrightarrow$ large effects from small fermion masses

- $\Delta\rho$: leading corrections to the ρ-parameter

  $$\Delta\rho_{\text{top}} \sim \left(\frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}\right)_{\text{top}} \sim \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2}$$

  $\hookrightarrow$ large effects from top–bottom loops in W self-energy Veltman '77

Hadronic contributions to running electromagnetic coupling

$$\begin{aligned}\Delta\alpha(s) &= -\,\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,ren}}(s)/s\} = -\,\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T}}(s)/s\} + (\Sigma^{AA}_{\mathrm{T}})'(0) \\ &= \Delta\alpha_{\mathrm{lept}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s) \sim \frac{\alpha}{3\pi}\sum_f Q_f^2 \ln\frac{M_Z^2}{m_f^2}\end{aligned}$$

$\Delta\alpha^{(5)}_{\mathrm{had}}$ becomes sensitive to unphysical quark masses m_q

$\hookrightarrow$ $\Delta\alpha^{(5)}_{\mathrm{had}}$ not calculable in perturbation theory

solution: $\Delta\alpha^{(5)}_{\mathrm{had}}$ obtainable from fit to experimental data with subtracted dispersion relation

$$\begin{aligned}\Delta\alpha^{(5)}_{\mathrm{had}} &= -\frac{\alpha}{3\pi}M_Z^2\,\mathrm{Re}\left\{\int_{4m_\pi^2}^{\infty}\mathrm{d}s'\,\frac{R(s')}{s'(s'-M_Z^2-\mathrm{i}\varepsilon)}\right\} \\ R(s) &= \frac{\sigma(\mathrm{e}^+\mathrm{e}^-\to\gamma^*\to\text{hadrons})}{\sigma(\mathrm{e}^+\mathrm{e}^-\to\gamma^*\to\mu^+\mu^-)}\end{aligned}$$

$R(s)$ is taken from perturbative QCD for high energies ($\sqrt{s}\gtrsim 13\,\mathrm{GeV}$)

$\Rightarrow$ $\Delta\alpha^{(5)}_{\mathrm{had}} = 0.027773 \pm 0.000354$

Jegerlehner; Burkhardt, Pietrzyk; Eidelman; Davier, Höcker,. . .

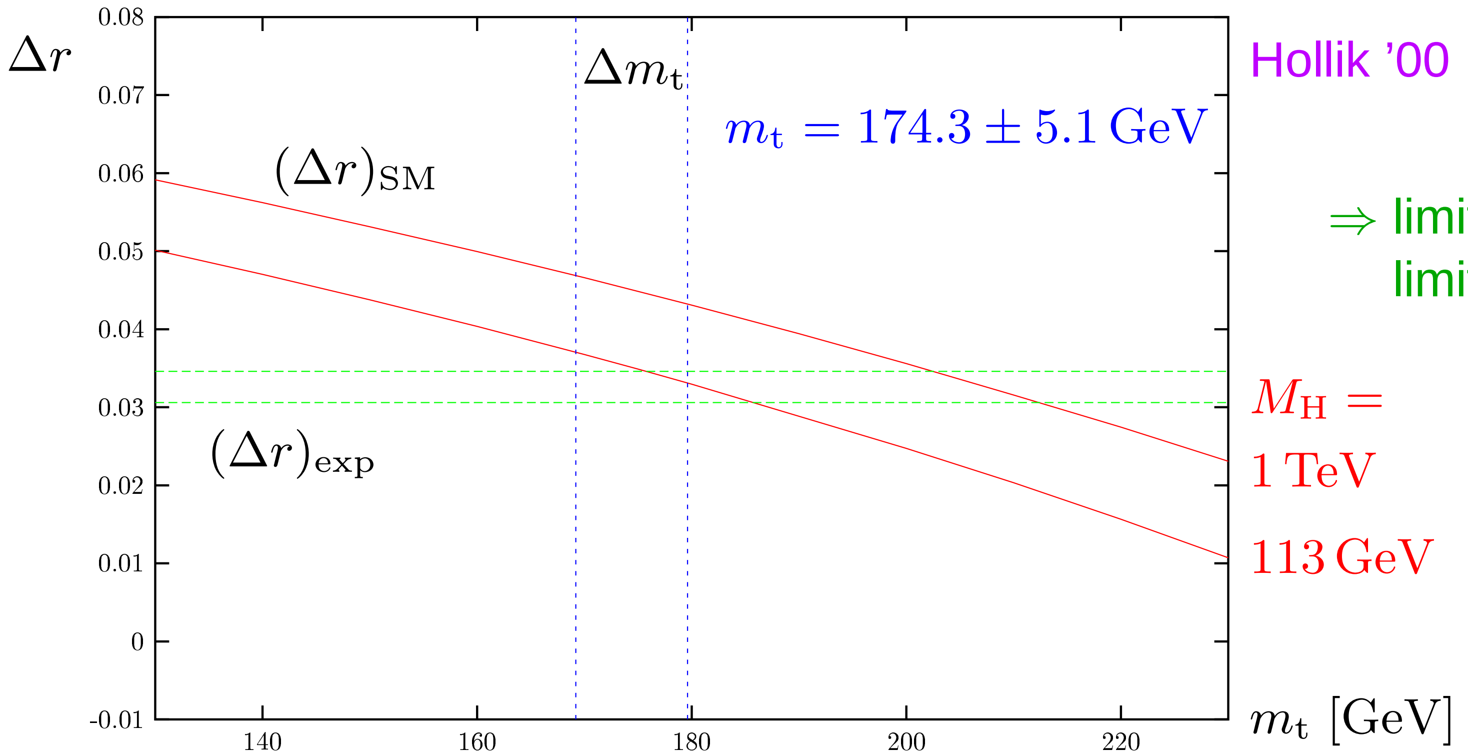
Experimental confirmation of quantum corrections

$$1 - \Delta r = \frac{\pi\alpha}{\sqrt{2}G_\mu} \frac{1}{M_\mathrm{W}^2(1 - M_\mathrm{W}^2/M_\mathrm{Z}^2)}$$

$\Rightarrow$ experimental determination of Δr $\quad$ $\Delta r = 0.0326 \pm 0.0020$

$\Delta r \neq 0$ with 16.3σ $\quad$ $\Rightarrow$ confirmation of quantum corrections

$\Delta r \neq \Delta\alpha \sim 0.0594$ wity 13.7σ $\quad$ $\Rightarrow$ confirmation of weak corrections



$\Rightarrow$ limits on m_t
limits on M_H

Uncertainties in prediction of M_W

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}} \frac{1}{M_\mathrm{W}^2(1 - M_\mathrm{W}^2/M_\mathrm{Z}^2)} \frac{1}{1-\Delta r}$$

α, M_Z, G_μ, $\Delta r(M_\mathrm{Z},\ldots) \Rightarrow$ prediction of M_W

experimental accuracy of M_W:

	now	Tevatron/LHC	Linear Collider
ΔM_W	$34\,\mathrm{MeV}$	15–$20\,\mathrm{MeV}$	$7\,\mathrm{MeV}$

parametric uncertainties:

	$\Delta m_\mathrm{t} = 5.1\,\mathrm{GeV}$	$\Delta(\Delta\alpha(M_\mathrm{Z})) = 3.6\times 10^{-4}$
ΔM_W	$31\,\mathrm{MeV}$	$6.5\,\mathrm{MeV}$

will go down to $2\,\mathrm{MeV}$ in the future

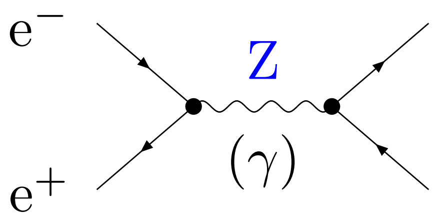
uncertainties/shifts from higher orders:

	2-loop bos.	2-loop ferm.	$\propto G_\mu^2 m_\mathrm{t}^4 \alpha_s$	unknown
ΔM_W	$1\,\mathrm{MeV}$	$5\,\mathrm{MeV}$	$\sim 6\,\mathrm{MeV}$	3–$4\,\mathrm{MeV}$

Known contributions to Δr

- complete one-loop corrections Sirlin,Marciano ’80
- two-loop QCD corrections Djouadi, Kniehl, . . . , ’88–’94
- three-loop QCD corrections
  Avdeeev, Fleischer, Mikhailov, Tarasov ’94; Chetyrkin, Kühn, Steinhauser ’95
- two-loop leading electroweak corrections $\propto G_\mu^2 m_\mathrm{t}^4, G_\mu^2 m_\mathrm{t}^2 M_\mathrm{Z}^2$
  Barbieri et al. ’92; Fleischer et al. ’93 ; Degrassi et al. ’96–’98, . . .
- two-loop fermionic corrections
  Freitas, Hollik, Walter, Weiglein ’00; Awramik, Czakon ’03
- two-loop Higgs-mass-dependent corrections
  Freitas, Hollik, Walter, Weiglein ’02
- full two-loop bosonic corrections Awramik, Czakon; Onishchenko, Veretin ’02
- three-loop leading electroweak corrections $\propto G_\mu^2 m_\mathrm{t}^4 \alpha_s, G_\mu^3 m_\mathrm{t}^6$
  Faisst, Kühn, Seidensticker, Veretin ’03

Z-boson resonance



$f = \mathrm{e}^-, \mu^-, \tau^-, q$

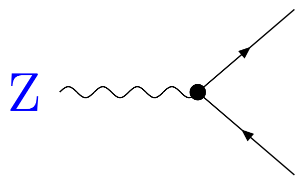
$\bar{f} = \mathrm{e}^+, \mu^+, \tau^+, \bar{q}$

LEP1: $\sim 16 \times 10^7$ events (1989–1995)

resonance cross-section (approximate Breit-Wigner) $\quad s = E_{\mathrm{CMS}}^2$

$$\sigma_f(s) = 12\pi \frac{s}{M_\mathrm{Z}^2} \frac{\Gamma(\mathrm{Z} \to \mathrm{e}^+\mathrm{e}^-)\,\Gamma(\mathrm{Z} \to f\bar{f})}{(s - M_\mathrm{Z}^2)^2 + s^2\Gamma_\mathrm{Z}^2/M_\mathrm{Z}^2}$$

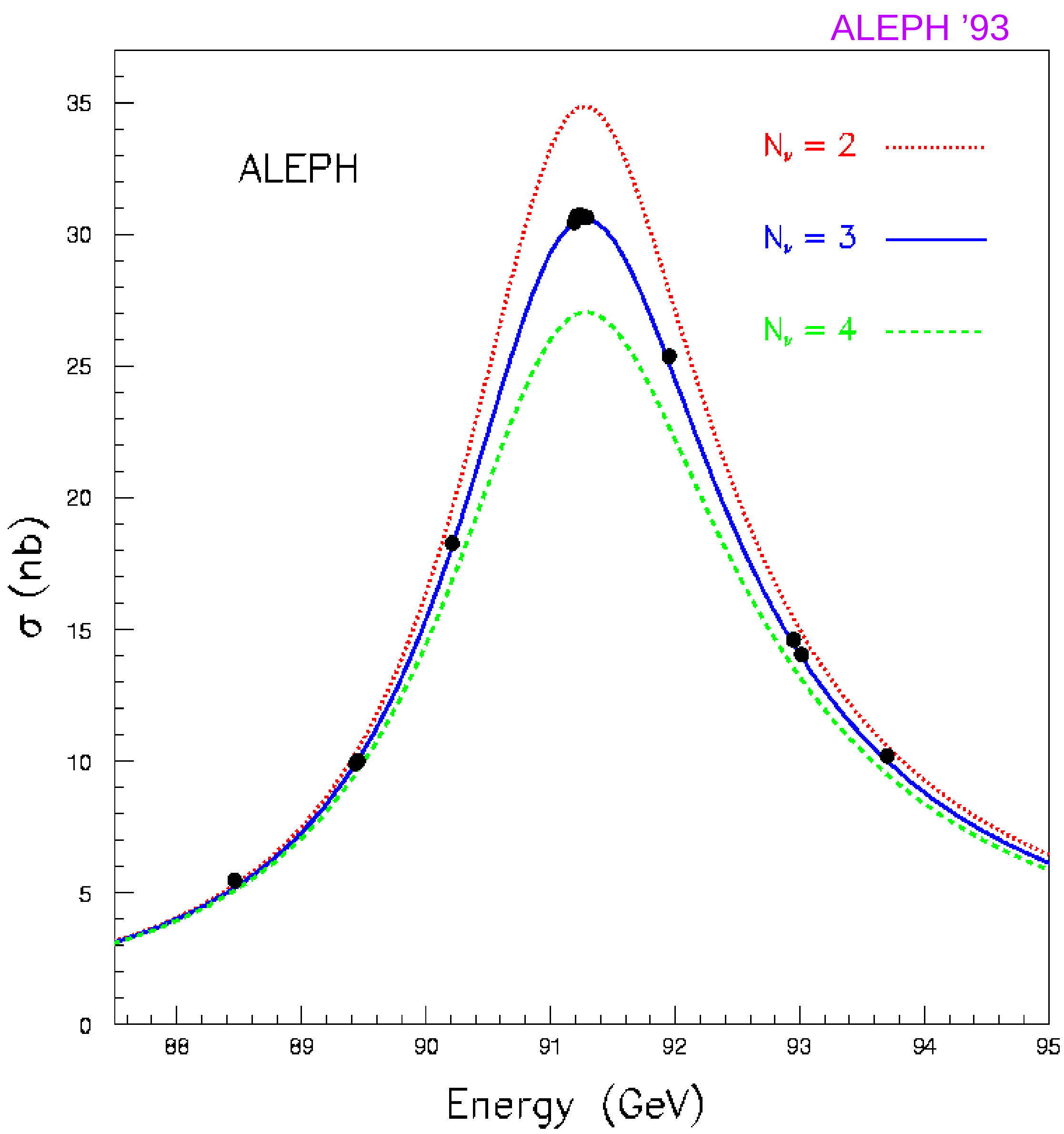
Z-boson width



$$\Gamma_\mathrm{Z} = \underbrace{\Gamma(\mathrm{e}^-\mathrm{e}^+, \mu^-\mu^+, \tau^-\tau^+)}_{\text{leptonic}} + \underbrace{\sum_q \Gamma(q\bar{q})}_{\text{hadronic}} + \underbrace{N_\nu \Gamma(\nu\bar{\nu})}_{\text{invisible}}$$

- line shape $\Rightarrow M_\mathrm{Z}, \Gamma_\mathrm{Z}$
- peak cross section $\Rightarrow \Gamma(\mathrm{Z} \to l^+l^-)/\Gamma_\mathrm{Z}, \Gamma(\mathrm{Z} \to \mathrm{hadrons})/\Gamma_\mathrm{Z}$
- angular distributions, polarization asymmetries
  $\Rightarrow$ effective $\mathrm{Z}f\bar{f}$ vector and axial-vector couplings g_{v_f}, g_{a_f}

Hadronic Z line shape



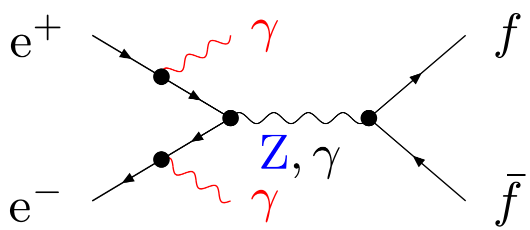
⇒ number of light neutrinos:

$N_\nu = 2.9841 \pm 0.0083$

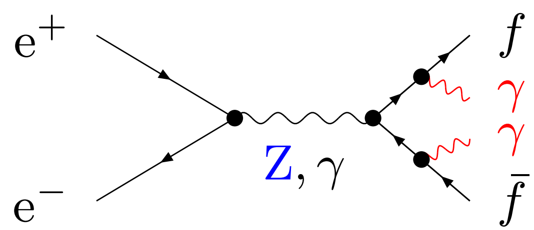
LEPEWWG '02

Electromagnetic corrections to $e^+e^- \longrightarrow f\bar{f}$

initial-state radiation (ISR)



final-state radiation (FSR)



ISR: contains large logarithms from collinear photon emission

$\Rightarrow$ large effects $\propto \frac{\alpha}{\pi}\log\frac{s}{m_e^2} \approx 6\%$

factorization theorem

$$\sigma_f(s) = \int_0^1 dx_+ \int_0^1 dx_- f_{ee}(x_+, Q^2) f_{ee}(x_-, Q^2) \sigma_{f,0}(x_+x_-s)$$

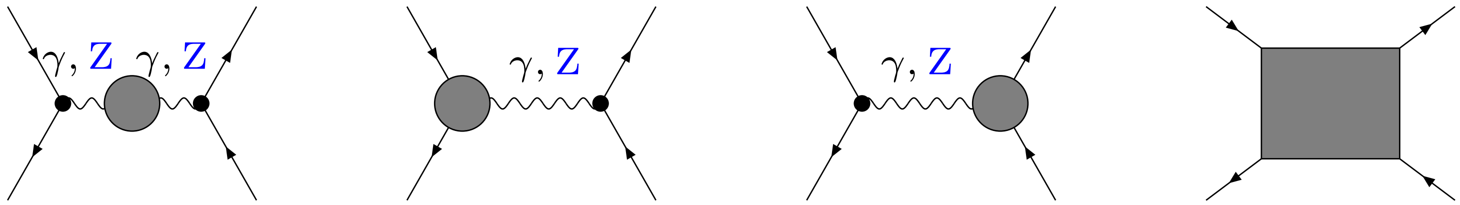
distribution functions f_{ee} contain mass-singular logarithms

FSR: mass-singular logarithms cancel between real and virtual contributions

$\Rightarrow$ correction factor $\quad 1 + \frac{3\alpha}{4\pi}Q_f^2$

interference: suppressed for total cross section $\propto \alpha\Gamma_Z/(\pi M_Z)$

Genuine weak corrections to $e^+e^- \longrightarrow f\bar{f}$



self-energy corrections   vertex corrections   box diagrams

self-energy corrections

Z-boson self-energy $\Rightarrow$ effective couplings and Z-boson width
γ–Z mixing energy $\Rightarrow$ effective couplings
photon self-energy $\Rightarrow$ effective couplings and running $\alpha(s)$

vertex corrections

$Zf\bar{f}$ vertex corrections $\Rightarrow$ effective couplings
$\gamma f\bar{f}$ vertex corrections $\sim 10^{-3}$ for $s \approx M_Z^2$

box corrections :

below 10^{-4} for $s \approx M_Z^2$ (non-resonant)

neglecting boxes and corrections to the $\gamma f\bar{f}$ vertex
$\Rightarrow$ improved Born approximation (accuracy $\lesssim 1\%$)

Realistic Z-boson resonance

improved Born approximation (accuracy $\lesssim 1\%$) for cross section including

- photon exchange contribution $\propto Q_f^2$
- photon–Z-boson interference R_f
- weak corrections in effective couplings
- electromagnetic and QCD final-state corrections
- no ISR ($\Rightarrow$ convolution)

$$\sigma_{f,\mathrm{kernel}}(s) = 12\pi \frac{s}{M_\mathrm{Z}^2} \frac{\Gamma(\mathrm{Z}\to\mathrm{e}^+\mathrm{e}^-)\Gamma(\mathrm{Z}\to f\bar{f})}{(s-M_\mathrm{Z}^2)^2 + \frac{s^2}{M_\mathrm{Z}^2}\Gamma_\mathrm{Z}^2} \left[1 + R_f \frac{s-M_\mathrm{Z}^2}{s}\right] + \frac{4\pi\alpha^2(s)}{3s} Q_f^2 N_\mathrm{C}^f (1+\delta_\mathrm{QED}^f)(1+\delta_\mathrm{QCD}^f)$$

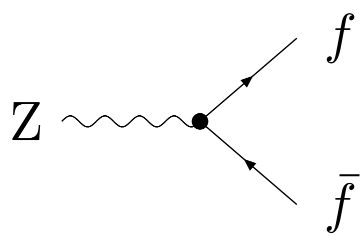
extraction of parameters:

- electromagnetic corrections are subtracted and
- photon contribution and interference is set to Standard Model value

$\Rightarrow$ extracted quantities $M_\mathrm{Z}, \Gamma_\mathrm{Z}, \Gamma(\mathrm{Z}\to f\bar{f}), \ldots$ are pseudo observables

Z-boson–fermion couplings

Z physics tests predominantly effective Z-boson–fermion couplings



$$= \mathrm{i}\frac{e}{2s_\mathrm{W}c_\mathrm{W}}\gamma_\mu(g_{v_f} - g_{a_f}\gamma_5)$$

effective couplings contain weak corrections to on-shell $Z\bar{f}f$ vertex

Z-boson width

$$\Gamma(\mathrm{Z}\to f\bar{f}) = \frac{G_\mu M_\mathrm{Z}^3}{6\pi\sqrt{2}}\left((g_{v_f})^2 + (g_{a_f})^2\right)(1+\delta_f^\mathrm{QED})$$

forward-backward asymmetry

$$A_\mathrm{FB}^f = \frac{\sigma_f(\theta<90^\circ) - \sigma_f(\theta>90^\circ)}{\sigma_f(\theta<90^\circ) + \sigma_f(\theta>90^\circ)}$$

contribution of Z-boson exchange (pseudo observable)

$$A_\mathrm{FB}^f(s=M_\mathrm{Z}^2) = \frac{3}{4}\frac{g_{v_\mathrm{e}}/g_{a_\mathrm{e}}}{(g_{v_\mathrm{e}}/g_{a_\mathrm{e}})^2+1}\frac{g_{v_f}/g_{a_f}}{(g_{v_f}/g_{a_f})^2+1}$$

$\Rightarrow g_{v_f},\ g_{a_f}$

Effective fermionic weak mixing angle

Lowest order:

$$g_{a_f,0} = I^3_{\mathrm{w},f}, \qquad g_{v_f,0} = (I^3_{\mathrm{w},f} - 2Q_f \sin^2\theta_\mathrm{w})$$

definition of effective fermionic mixing angle (pseudo observable)

$$g_{a_f} = \sqrt{\rho_\mathrm{f}}\, I^3_{\mathrm{w},f}, \qquad g_{v_f} = \sqrt{\rho_\mathrm{f}}(I^3_{\mathrm{w},f} - 2Q_f \sin^2\theta^\mathrm{f}_\mathrm{eff})$$

$$g_{v_f}, g_{a_f} \Rightarrow \sin^2\theta^\mathrm{f}_\mathrm{eff},\ \rho_\mathrm{f}, \qquad g_{v_f}/g_{a_f} \Rightarrow \sin^2\theta^\mathrm{f}_\mathrm{eff} = \frac{I^3_{\mathrm{w},f}}{2Q_f}\left(1 - \frac{g_{v_f}}{g_{a_f}}\right)$$

lowest-order perturbation theory:

$$\sin^2\theta^\mathrm{f}_\mathrm{eff} = \sin^2\theta_\mathrm{w} = 1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}, \qquad \rho_\mathrm{f} = 1$$

including weak corrections:

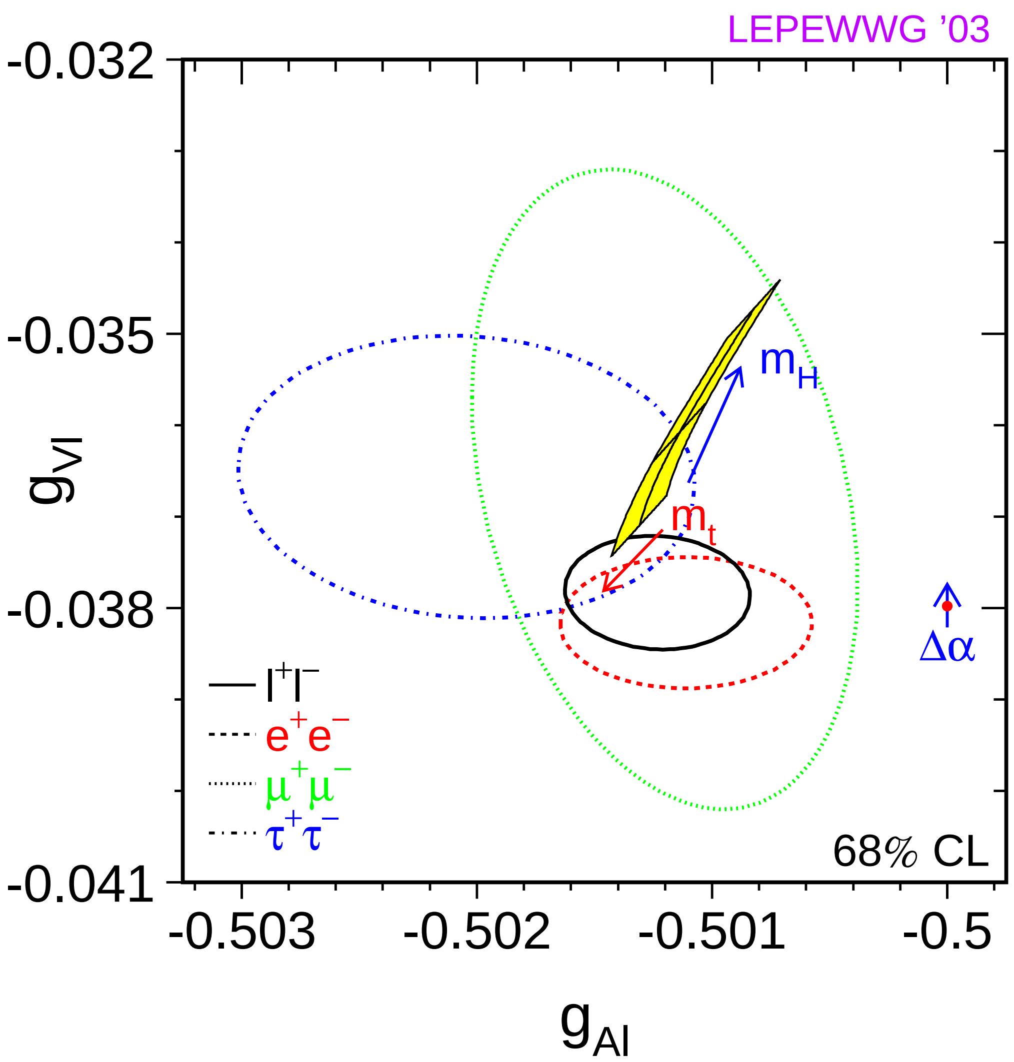
$$\sin^2\theta^\mathrm{f}_\mathrm{eff} \neq \sin^2\theta_\mathrm{w}, \qquad \rho_\mathrm{f} \neq 1$$

experiment:

$$\sin^2\theta^\mathrm{lept}_\mathrm{eff} \approx 0.2315, \qquad \sin^2\theta_\mathrm{w} \approx 0.2221, \qquad \sin^2\theta^\mathrm{lept}_\mathrm{eff}/\sin^2\theta_\mathrm{w} \approx 1.040$$

$$\rho_f \approx 1.005$$

Effective Z-boson couplings



parameters

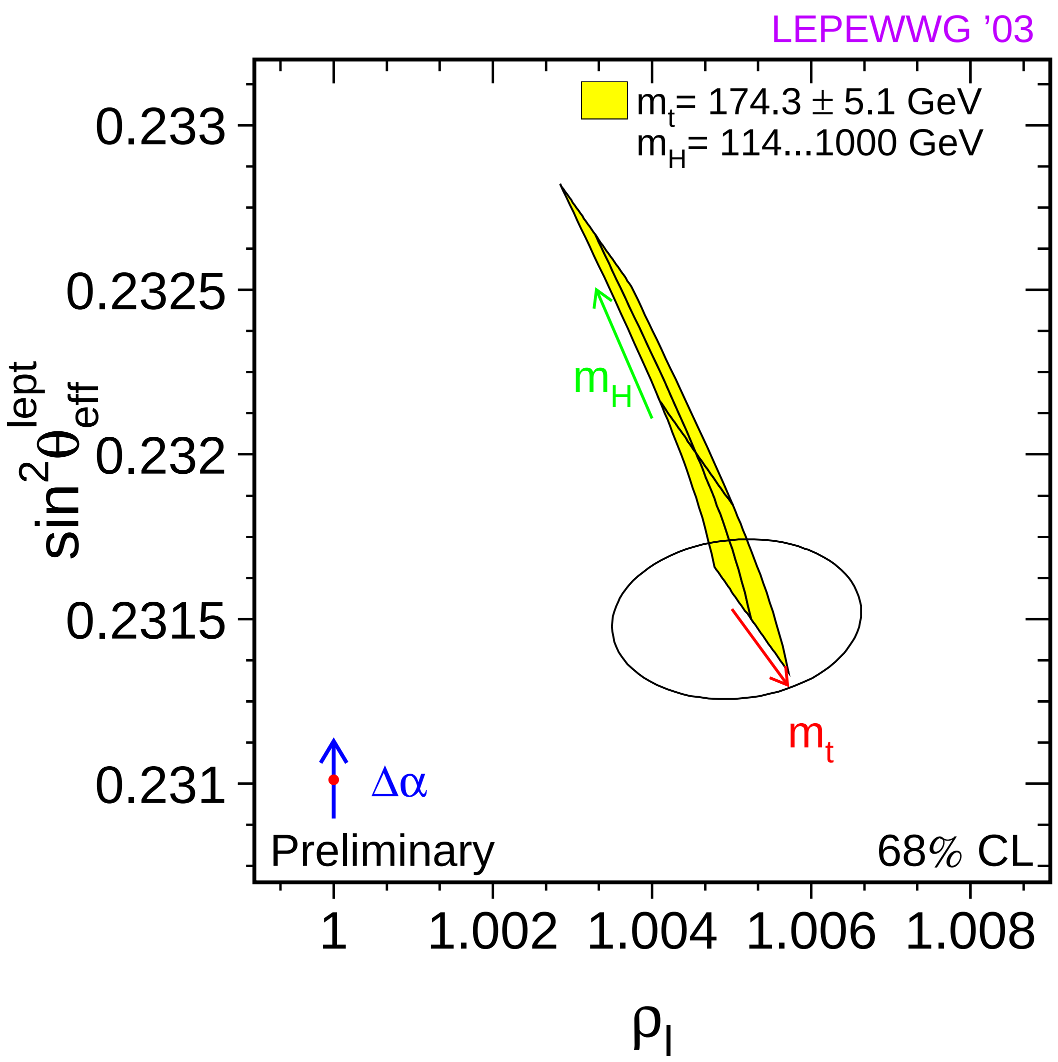
$$m_{\mathrm{t}} = 174.3 \pm 5.1\,\mathrm{GeV}$$
$$M_{\mathrm{H}} = 114 \ldots 1000\,\mathrm{GeV}$$
$$\Delta\alpha_{\mathrm{had}} = 0.02761 \pm 0.00036$$

confirmation of
lepton universality

experimental results
for couplings

$$g_{a_l} = -0.50123 \pm 0.00026$$
$$g_{v_l} = -0.03783 \pm 0.00066$$

Effective mixing angle versus leptonic ρ parameter



parameters

$$m_{\rm t} = 174.3 \pm 5.1\,\text{GeV}$$
$$M_{\rm H} = 114\ldots1000\,\text{GeV}$$
$$\Delta\alpha_{\rm had} = 0.02761 \pm 0.00036$$

preference for light Higgs boson

experimental results

$$\sin^2\theta_{\rm eff}^{\rm lept} = 0.23137 \pm 0.00033$$
$$\rho_{\rm l} = 1.0049 \pm 0.0010$$

Global electroweak fit

calculate all precision observables in SM including quantum corrections in terms of $\alpha(M_\mathrm{Z})$, G_μ, M_Z, $m_{f\neq t}$, m_t, M_H, $\alpha_s(M_\mathrm{Z})$

determine parameters by fit to all precision data

results

- good agreement between SM and data at the per-mille level (some exceptions)

- indirect determination of parameters

  all data except m_t: $\Rightarrow m_\mathrm{t} = 178\,^{+11}_{-8}\ \mathrm{GeV}$

  1994: $m_\mathrm{t} = 169\,^{+24}_{-27}$ (top discovery 1995)

  present experimental value: $m_\mathrm{t} = 174.3 \pm 5.1\,\mathrm{GeV}$

  all data

  $\Rightarrow M_\mathrm{H} = 91\,^{+58}_{-37}\ \mathrm{GeV}$

  $M_\mathrm{H} < 211\,\mathrm{GeV}$ (95% C.L.)

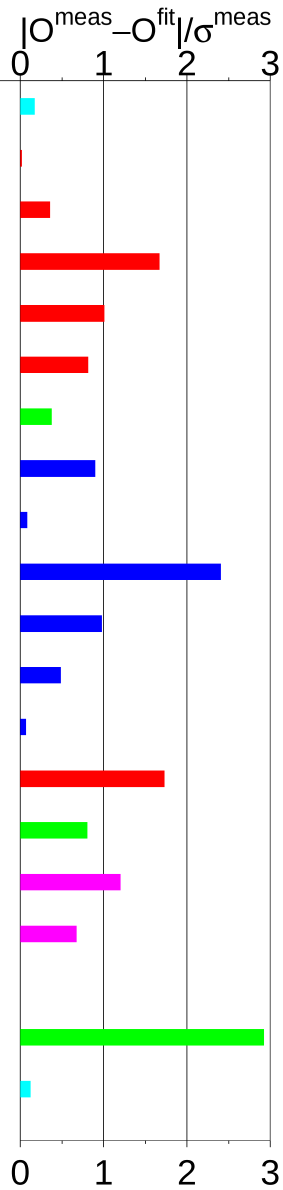
  $\alpha_s(M_\mathrm{Z}) = 0.1185 \pm 0.0027$

  preference for light Higgs boson

Global fit results

Summer 2003

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1875
Γ_Z [GeV]	2.4952 ± 0.0023	2.4960
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01636
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1477
R_b	0.21638 ± 0.00066	0.21579
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1036
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0740
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1477
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.426 ± 0.034	80.385
Γ_W [GeV]	2.139 ± 0.069	2.093
m_t [GeV]	174.3 ± 5.1	174.3
$\sin^2\theta_W(\nu N)$	0.2277 ± 0.0016	0.2229
$Q_W(Cs)$	-72.84 ± 0.46	-72.90



LEPEWWG '03

$\Rightarrow$ fair agreement

$\chi^2/\mathrm{d.o.f.} = 25.5/15 \quad (4.4\%)$

without NuTeV

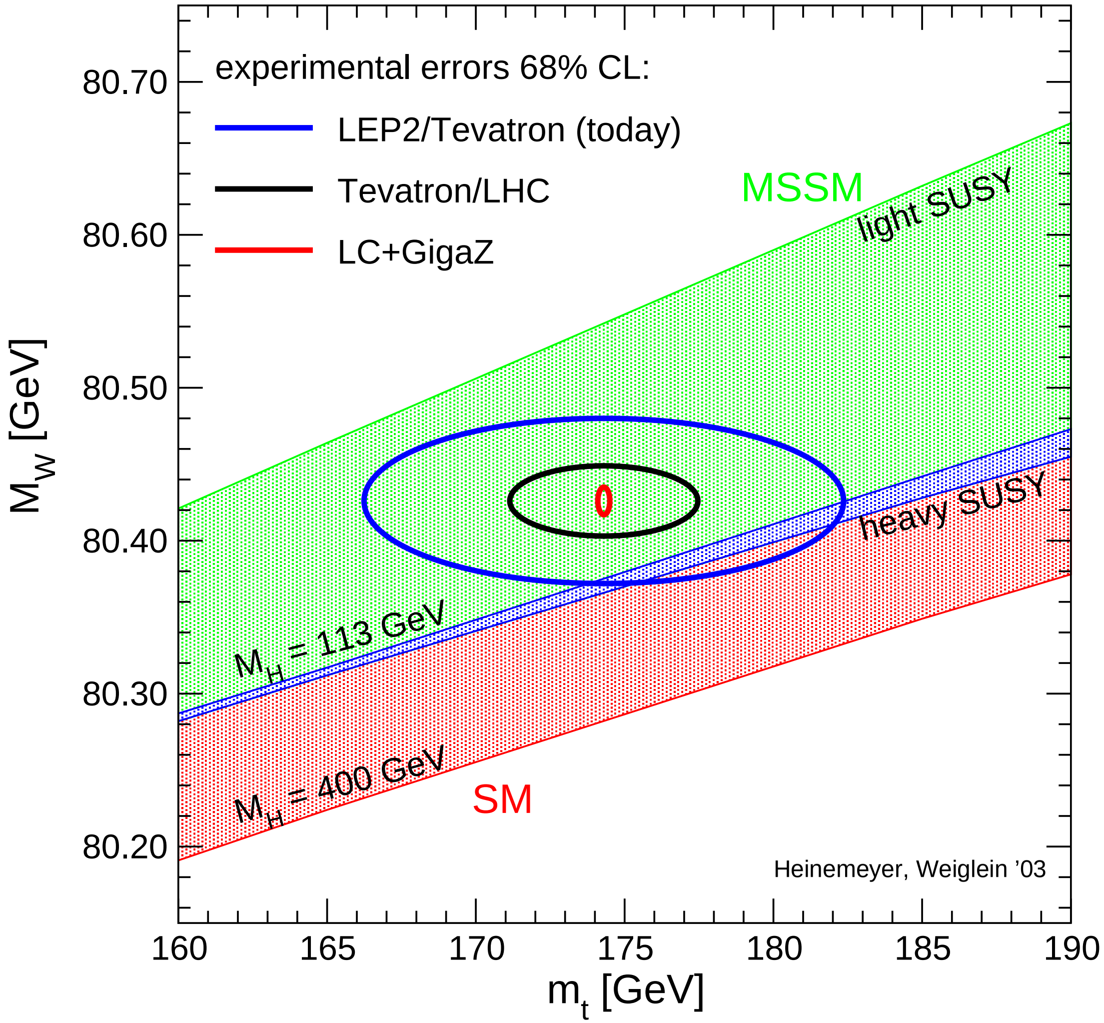
$(\sin^2\theta_\mathrm{w}(\nu\mathrm{N}))$

good agreement

$\chi^2/\mathrm{d.o.f.} = 16.7/14 \quad (27\%)$

M_W–m_t relation

Heinemeyer, Weiglein '03



SM: variation with M_H

Minimal Supersymmetric Standard Model (MSSM): variation with supersymmetric parameters

reduction of errors on M_W and m_t

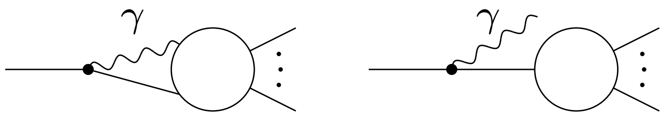
- further restriction of M_H in SM
- discrimination from supersymmetric models

Leading electroweak corrections at LEP energies

leading electromagnetic logarithms

from ISR and FSR: $\mathcal{O}(10\%)$ QED

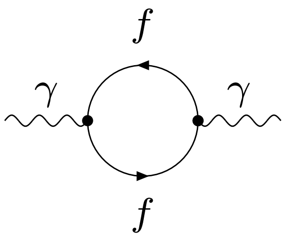
real and virtual corrections have to be added



$\sim \frac{\alpha}{\pi} \ln \frac{E^2}{m_e^2} \sim 6\%$ at $E \sim M_Z$

running of α: $\sim 6\%$ "QED"

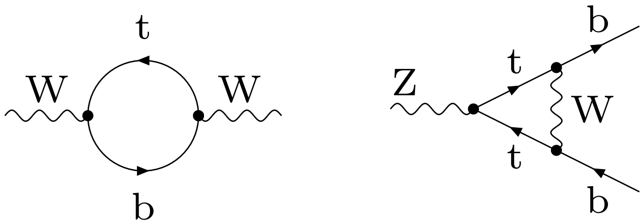
renormalization of electromagnetic coupling



$\sim \frac{\alpha}{3\pi} \sum_f Q_f^2 \ln \frac{E^2}{m_f^2}, \quad E \sim M_Z$

corrections $\propto m_t^2/M_W^2$: $\sim 3\%$

renormalization of weak mixing angle, ...



$\sim \frac{\alpha}{\pi} \frac{m_t^2}{s_w^2 M_W^2} \sim 0.05, \quad m_t = 175\,\mathrm{GeV}$

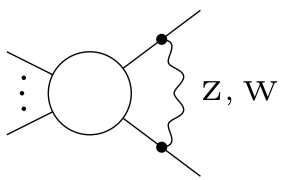
Leading electroweak (EW) corrections at TeV energies

$E \gg M \sim M_\mathrm{W} \sim M_\mathrm{Z}$: EW corrections are enhanced by large logarithms

- double logarithms
  $\frac{\alpha}{\pi s_\mathrm{w}^2} \ln^2 \frac{s}{M^2} = -26\% \,@\, 1\,\mathrm{TeV}$ for typical $2 \to 2$ process
- single logarithms
  $\frac{3\alpha}{\pi s_\mathrm{w}^2} \ln \frac{s}{M^2} = 16\% \,@\, 1\,\mathrm{TeV}$ for typical $2 \to 2$ process

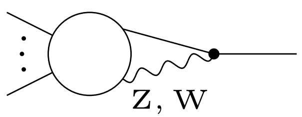
universal origin of leading EW logarithms

- mass singularities in virtual corrections related to external lines
  - ◇ soft and collinear virtual gauge bosons



double logarithms

  - ◇ collinear or soft virtual gauge bosons



single logarithms

- renormalization of parameters at scale $M^2 \ll s$
  $\Rightarrow$ running of electroweak couplings from M to $\sqrt{s}$ single logarithms

Properties of electroweak leading logarithms

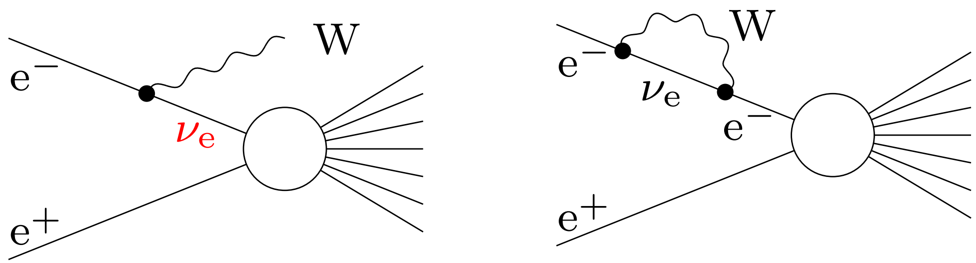
Differences to QED and QCD:

electroweak gauge bosons massive $\Rightarrow$ real radiation exp. distinguishable

electroweak double logarithms do not cancel in inclusive observables between real and virtual effects! M. Ciafaloni, P. Ciafaloni, Comelli '00

reason: free non-abelian charges (isospin) of initial states

example: W emission from incoming electron



$\Rightarrow$ different hard matrix elements in real and virtual corrections

$$\sigma^{\mathrm{LL}}_{\mathrm{inclusive,e^+e^-}} = (\sigma_{0,\mathrm{e^+}\nu_\mathrm{e}} - \sigma_{0,\mathrm{e^+e^-}})\frac{\alpha}{4\pi s_\mathrm{w}^2}\ln^2\frac{s}{M^2} = -\sigma^{\mathrm{LL}}_{\mathrm{inclusive,e^+}\nu_\mathrm{e}}$$

$\Rightarrow$ electroweak correction of 5% at $1\,\mathrm{TeV}$

(strong corrections for $\mathrm{e^+e^-} \to \mathrm{hadrons} \approx 3\%$)

cancellation recovered by summing over complete multiplets
(analogous to colour average in QCD!)

Universality of EW logarithms at one loop

Electroweak logarithms depend only on external-leg properties independent of process Denner, Pozzorini '00 '01

$$\mathcal{M}_1 = \underbrace{\mathcal{M}_0}_{g\to g(s)}\Big(1 + \underbrace{\delta^1_{\mathrm{EW}}}_{\mathrm{soft,coll.}}\Big), \qquad \delta^1_{\mathrm{EW}} = \frac{\alpha}{4\pi}\sum_{\mathrm{legs}\,k}\delta^1_{\mathrm{EW}}(k)$$

valid for

- high-energy limit: $(p_k + p_l)^2 \gg M^2 \gg m_f^2 \gg \lambda^2$
  $M \sim M_\mathrm{W} \sim M_\mathrm{Z} \sim M_\mathrm{H} \sim m_\mathrm{t}$
  mass-suppressed terms are neglected
- on-shell matrix elements $\mathcal{M}^{i_1\ldots i_n}$
- general electroweak processes with arbitrary polarized external particles
  $f_\mathrm{L}, f_\mathrm{R}, \;\; \gamma, \mathrm{Z}_\mathrm{T}, \mathrm{W}_\mathrm{T}, \;\; \mathrm{Z}_\mathrm{L}, \mathrm{W}_\mathrm{L}, \;\; \mathrm{H}$
  (External longitudinal gauge bosons have to be replaced by corresponding Goldstone bosons.)

External leg factors

Electroweak logarithms can be split in

- symmetric electroweak part
  resulting from EW theory with photon of mass M

$$\delta^1_{\mathrm{sew}}(k) = \delta^1_{\mathrm{EW}}(k)\big|_{\lambda=M} = -\frac{1}{2}C^{\mathrm{ew}}(k)\ln^2\frac{s}{M^2} + \gamma^{\mathrm{ew}}(k)\ln\frac{s}{M^2} + \sum_{l\neq k}\sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I^a(k)I^{\bar a}(l)\ln\frac{|(p_k+p_l)^2|}{s}\ln\frac{s}{M^2}$$

$C^{\mathrm{ew}}(k)$, γ^{ew}, $I^a(k)$ depend only on I^3_{w} and Y_{w} of external particles

- subtracted electromagnetic part
  resulting from mass gap between M and photon mass λ

$$\delta^1_{\mathrm{sem}}(k) = \delta^1_{\mathrm{EW}}(k) - \delta^1_{\mathrm{EW}}(k)\big|_{\lambda=M} = -\frac{1}{2}Q^2(k)\left[2\ln\frac{s}{m_k^2}\ln\frac{M^2}{\lambda^2} - \ln^2\frac{M^2}{m_k^2} - 2\ln\frac{M^2}{\lambda^2} - \ln\frac{M^2}{m_k^2}\right] + \sum_{l\neq k} Q(k)Q(l)\ln\frac{|(p_k+p_l)^2|}{s}\ln\frac{M^2}{\lambda^2}$$

depends only on charges $Q(k)$ of external particles

Electroweak correction in gauge-boson-pair production at LHC

$\mathrm{pp} \to \mathrm{WZ}$: cross section for $\mathrm{pp} \to l\nu_l l'\bar{l}'$

$P_{\mathrm{T}}^{\mathrm{cut}}(l'\bar{l}')$ [GeV]	σ_0 [fb]	σ [fb]	Δ [%]	$1/\sqrt{2L\sigma_0}$ [%]
300	0.899	0.811	−9.8	7.5
400	0.296	0.252	−14.9	13
500	0.114	0.092	−19.3	20.9

Accomando, Denner, Pozzorini '01

$L = 100\,\mathrm{fb}^{-1}$

2 experiments

$\mathrm{pp} \to \mathrm{W}\gamma$: cross section for $\mathrm{pp} \to l\nu_l \gamma$

$P_{\mathrm{T}}^{\mathrm{cut}}(\gamma)$ [GeV]	σ_0 [fb]	σ [fb]	Δ [%]	$1/\sqrt{2L\sigma_0}$ [%]
300	3.180	2.940	−7.6	4.0
400	1.100	0.966	−12.2	6.7
500	0.437	0.366	−16.2	10.7
600	0.190	0.152	−19.8	16.2
700	0.089	0.068	−23.3	23.7

Conclusions

Standard Model established as a quantum field theory

- in agreement with all experiments (accuracy $\gtrsim 0.1\%$) (some exceptions)
- quantum corrections = radiative corrections are established
- indirect and direct determinations of m_{t} agree
- constraints on the Higgs-boson mass $\Rightarrow$ light Higgs boson
- triple-gauge-boson self-interactions established at per-cent level

not yet directly tested

- existence of Higgs boson
- Higgs-boson gauge couplings
- Higgs-boson self-interaction $\Rightarrow$ Higgs potential
- Yukawa interaction

$\Rightarrow$ future experiments needed (LHC, linear $\mathrm{e}^{+}\mathrm{e}^{-}$ collider. . . .)

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  - ◇ Bardin/Passarino, “The Standard Model in the Making”
  - ◇ Hollik/Duckeck, “Electroweak Precision Tests at LEP”
  - ◇ P. Langacker, ed., “Precision tests of the Standard Model”
  - ◇ G. Altarelli, R. Kleiss, C. Verzegnassi, Z-physics at LEP 1, CERN-89-08
  - ◇ D. Bardin, M. Grünewald, G. Passarino, “Precision Calculation Project Report”, hep-ph/9902452
- Experimental results:
  - ◇ P.B. Renton, Rept. Prog. Phys. **65** (2002) 1271
  - ◇ The LEP Collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavour Group, LEPEWWG/2003-01, http://lepewwg.web.cern.ch/LEPEWWG/